Development of Computer-Aided Approach to Parameter Extraction of Spiral Inductor Model

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Abstract – **Computer-aided extraction procedure is developed for obtaining the parameters of spiral inductor model. The procedure is of direct type and can be easily realized using the possibilities of standard circuit simulators such as *Cadence PSpice* and graphical analyzer *Cadence Probe*. The computer realization of the extraction procedure is performed using macro-definitions in the graphical analyzer *Cadence Probe*.**

Keywords – **Spiral Inductors, Parameter Extraction, Computer Model, PSpice Simulator, Macro-Definitions**

I. Introduction

The on-chip spiral inductors are of significant importance in wireless communication applications such as VCOs, mixers, RF amplifiers, passive filters, impedance-matching circuits, etc. Recently, a number of paper are devoted on on-chip spiral inductors modeling and parameter extraction of the equivalent circuit. Spiral inductor models are developed with geometry-dependent model parameters and frequency-dependent series resistance, taking into account the skin effect [1, 2]. The advantage of these models is that the geometry parameters can be optimized in order to improve the electrical characteristics of the inductor. The wide-band spiral inductor models [3, 4] are valid in a wide frequency range. The model parameters are frequency - and geometry-independent. The general-purpose analysis programs such as *PSpice* and MATLAB can be successfully used in the circuit analysis and optimization. Computer approaches to modeling, parameter extraction and optimization of planar inductors using MATLAB are developed in [5, 6].

In the present paper, a computer-aided extraction procedure is developed for obtaining the parameters of spiral inductor model. The procedure is of direct type and can be easily realized using the possibilities of standard circuit simulators such as *Cadence PSpice* and graphical analyzer *Cadence Probe*. The automated procedure is based on measurement data for the *S*-parameters. The computer realization of the extraction procedure is performed using macro-definitions in the graphical analyzer *Cadence Probe*. The comparison between the simulation results and the measured data confirms the validity of the developed extraction procedure, as well as the high accuracy of the computer model.

II. Equivalent Circuit of the Spiral Inductor Model

The equivalent circuit of on-chip spiral inductor is presented in Fig. 1. This circuit also represents the parasitic elements associated with the bonding pads and interconnect lines (parasitic elements outside the subcircuit *N*). They are eliminated from the measurements using de-embedding technique [8]. After de-embedding, the inductor model can be represented as shown in Fig. 2.

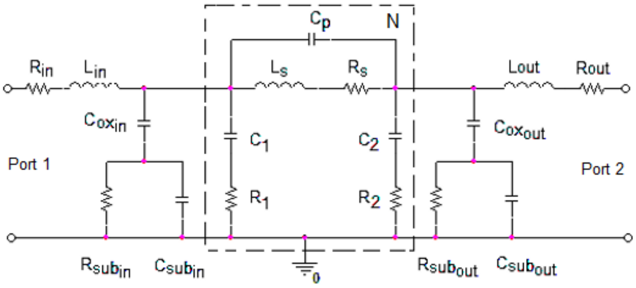

Fig. 1. Equivalent circuit of spiral inductor

The extraction procedure is based on measured data for *S*-parameters. After converting to *Y*-parameters, the parameter values of the elements of the model shown in Fig. 2 can be obtained:

$$Y = Y_R + \mathrm{j}Y_I \tag{1}$$

$$Y_R = \frac{R_s}{R_s^2 + \omega^2 L_s^2} \tag{2}$$

$$Y_I = \omega \frac{C_p R_s^2 - L_s^2 + \omega^2 L_s^2 C_p}{R_s^2 + \omega^2 L_s^2} \tag{3}$$

$$Q = -\left[\frac{Y_{11I}}{Y_{11R}}\right] \tag{4}$$

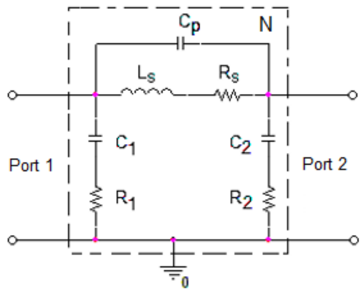

Fig. 2. De-embedded equivalent circuit of spiral inductor

In order to take into account the skin-effect, the series resistance R_s of the inductor model is assumed constant up to frequency f_o and then increases as $\sqrt{f}$ [7]:

$$R_s(f) = \begin{cases} R_{so} & \text{for } f < f_o \\ A\sqrt{f} & \text{for } f \geq f_o \end{cases} . \tag{5}$$

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At high frequencies, the following expression can be used [7]:

$$R_s(f) = A\sqrt{f} \ . \tag{6}$$

III. Parameter Extraction Procedure for Model M$_1$

In model M_1 the resistance R_s is frequency independent. The parameter extraction procedure P_1 consists of the following steps:

Step 1. Introducing input data for the de-embedded parameters Y_R and Y_I

As the measurement data for the parameters $Y_R(f)$ and $Y_I(f)$, do not correspond to the same frequency point, they have to be introduced separately using different frequency tables of EFREQ type (Fig. 3).

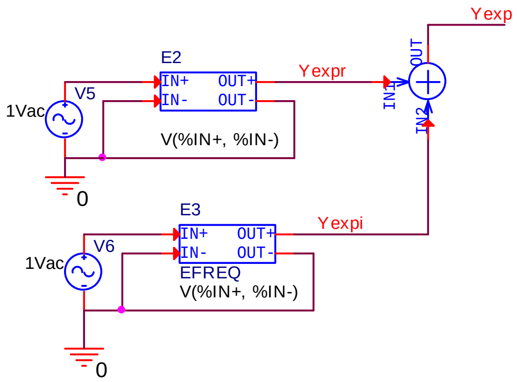

Fig. 3. Introducing input data in *Cadence Capture*

Step 2. Determination of R_s

$$Z = 1/Y \ . \tag{7}$$

R_s is obtained from $\mathrm{Re}[Z]$ at lower frequency:

$$R_s = \mathrm{Re}[Z] \ \text{ for } f = f_{low} \ . \tag{8}$$

Step 3. Determination of L_s

Ls is obtained from $\mathrm{Im}[Z]$ at lower frequency:

$$L_s = \mathrm{Im}[Z] \ \text{ for } f = f_{low} \ . \tag{9}$$

Step 4. Determination of C_p

C_p is obtained from $\mathrm{Im}[Y - Y_{RL}]$ at higher frequency:

$$Z_{RL} = R_s + j\omega L_s \quad ; \quad Y_{RL} = 1/Z_{RL} \ , \tag{10}$$

$$C_p = \frac{\mathrm{Im}[Y - Y_{RL}]}{\omega} \ . \tag{11}$$

IV. Parameter Extraction Procedure of Model M$_2$

In the developed computer model M_2 the resistance R_s is frequency dependent. In order to model the skin effect more precisely at high frequencies, two coefficients K_1 and K_2 are introduced to describe the frequency dependence $R_s(f)$:

$$R_s(f) = K_1 f^{K_2} \ . \tag{12}$$

The extraction procedure P_2 for model M_2 is the same as the extraction procedure P_1 for model M_1 except *Step* 2: determination of R_s. The expression (12) is used to describe R_s (f) in model M_2. The coefficients K_1 and K_2 are obtained using the values of $\mathrm{Re}[Z]$ for two frequencies f_1 and f_2:

$$f_1 = f_{low} \quad ; \quad f_2 = (f_{low} + f_{hi})/2 \ , \tag{13}$$

$$R_{f_1} = \mathrm{Re}[Z] \ \text{ for } f = f_1 \ , \tag{14}$$

$$R_{f_2} = \mathrm{Re}[Z] \ \text{ for } f = f_2 \ . \tag{15}$$

Using (12) , the following coefficients are obtained:

$$K_2 = \frac{\ln(R_{f_1}/R_{f_2})}{\ln(f_1/f_2)} \ , \tag{16}$$

$$K_1 = \frac{R_{f_1}}{f_1^{K_2}} \ . \tag{17}$$

V. Computer Realization of the Parameter Extraction Procedures

The parameter extraction procedure for model M_1 is realized using corresponding macro-definition in the graphical analyzer *Probe*. The following macros are defined:

```
* Step 1: Determination of the lower and higher frequencies of the range (Flow and Fhi) , determination of Y, Z, Re(Z) and Im(Z)
Flow = MIN(frequency)
Fhi = MAX(frequency)
w = 2*pi*frequency
wlow=2*pi*Flow
pi = 3.14159265
jj = sqrt(-1)    ! complex number j
Ym = V(Yexp)
Zm = 1/Ym
* Step 2: Determination of Rs
Rf=R(Rm)
Rs =MIN( Rf*frequency/Flow)
* Step 3: Determination of  Ls
Xm = IMG(Zm)
Lf=IMG(1/V(Yexp))/w
Ls=MIN(Lf*Frequency/Flow)
* Step 4: Determination of Zrl, Yrl, Yc and Cp
Zrl = Rs+jj*w*Ls
Yrl = 1/Zrl
Ycm = Ym-Yrl
Bcm = IMG(Ycm)
Cp = max( Bcm)/(2*pi*Fhi)
```

Step 2 of parameter extraction procedure P_2 is realized using the following macro-definition in the graphical analyzer *Probe*:

```
K1 = Rf1/pwr(F1,k2)
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```
K2 = LOG(Rf1/Rf2)/LOG(F1/F2)
Rs=K1*pwr(Frequency,K2)
```

V. ACCURACY INVESTIGATION OF THE EXTRACTION PROCEDURES

The quality factor Q is obtained by the equation:

$$Q = -\frac{\mathrm{Im}\left[Y_{11}\right]}{\mathrm{Re}\left[Y_{11}\right]}. \qquad (18)$$

The values Y_{11m} obtained from the measurements, are introduced in the model using EFREQ elements. The corresponding quality factor Q_m is:

$$Q_m = -\frac{\mathrm{Im}\left[Y_{11m}\right]}{\mathrm{Re}\left[Y_{11m}\right]} \; ; \; Q_{\max} = \max(Q_m). \qquad (19)$$

The simulated quality factor Q_{s1} of the model M_1 is:

$$Q_{s1} = -\frac{\mathrm{Im}\left[Y_{11,s1}\right]}{\mathrm{Re}\left[Y_{11,s1}\right]}, \qquad (20)$$

where $Y_{11,s1}$ is obtained from Fig. 4 in the form:

$$Y_{11,s1} = \frac{I_{1a}}{V_{1a}}. \qquad (21)$$

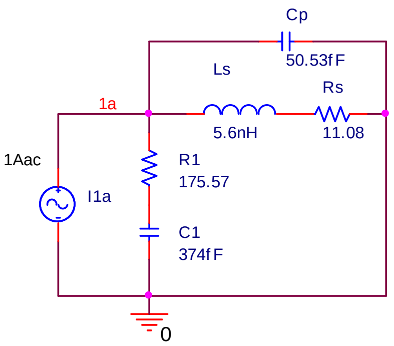

Fig. 4. Circuit for simulation the quality factor Q of model M_1

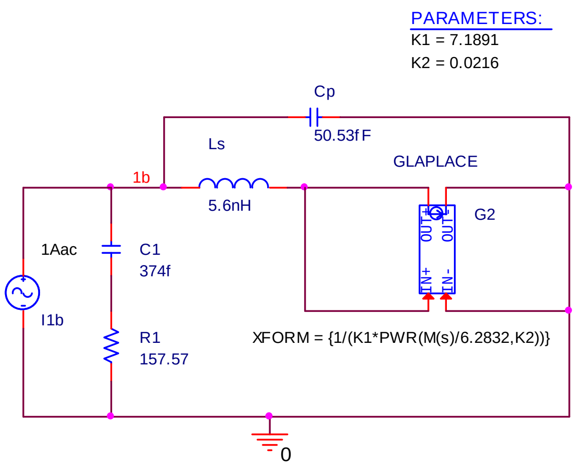

Fig. 5. Circuit for simulation the quality factor Q of model M_2

Similarly, the simulated quality factor Q_{s2} of the model M_2 has the form:

$$Q_{s2} = -\frac{\mathrm{Im}\left[Y_{11,s2}\right]}{\mathrm{Re}\left[Y_{11,s2}\right]}, \qquad (22)$$

where $Y_{11,s2}$ is obtained from Fig. 5 in the form:

$$Y_{11,s2} = \frac{I_{1b}}{V_{1b}}. \qquad (23)$$

The resistor R_s with the frequency dependent resistance $R_s(f)$ (12) is modeled using voltage controlled current source G2 of GLAPLACE type. The admittance $G_s(f)=1/R_s(f)$ is described in the attribute XFORM:

```
{1/(K1*PWR(M(s)/6.2832,K2))}
```

The corresponding macro-definitions in *Probe* for determination of the quality factor Q are in the form:

```
*eqn. (19)
Qm =- IMG(V(Y11m))/R(V(Y11m))
Qmax=MAX(Qm)
*eqn. (20) and (21): model M1
Y11s1 = -I(I1a)/V(1a)
Qs1 = - IMG(Y11s1)/R(Y11s1)
*eqn. (22) and (23): model M2
Y11s2 = -I(I1b)//V(1b)
Qs2 = - IMG(Y11s2)/R(Y11s2)
```

The simulation results for the quality factor Q of circular inductor of 4 ½ turns [8] are presented in Fig. 6, where

- Curve 1: Q_m - based on measurement data;
- Curve 2: Q_s - simulated using model M_1 with extracted parameters applying optimization approach [8]: L_s=6.31nH, $R_s = 11.68\,\Omega$, C_p=33.4 fF, C_1=374fF $R_1 = 175.57\,\Omega$;
- Curve 3: Q_{s1} - simulated using extracted parameters for model M_1 using the developed direct extraction procedure;
- Curve 4: Q_{s2} - simulated using extracted parameters for model M_2 using the developed direct extraction procedure.

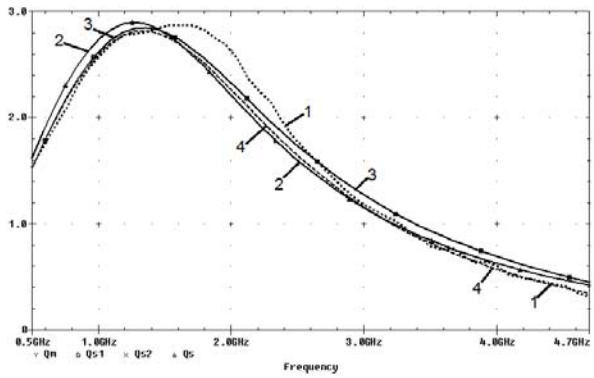

Fig.6. Simulation results for the quality factor Q of circular inductor of 4 ½ turns

The average relative errors $\varepsilon_{Qs}, \varepsilon_{Qs1}$ and ε_{Qs2} of the corresponding simulated Q factors Q_s, Q_{s1} and Q_{s2}, are calculated for the whole frequency range [f_{low}, f_{hi}] using the following macro-definitions in *Probe*:

```
DF=Fhi-Flow
EPS_Q = S(100*abs((Qm-Qs)/(0.5*Qmax)))/DF
EPS_Q1=S(100*abs((Qm-Qs1)/(0.5*Qmax)))/DF
EPS_Q2=S(100*abs((Qm-Qs2)/(0.5*Qmax)))/DF
```

The results are shown in Fig. 7, where:

- *Curve* 1: ε_{Qs} - calculated using extracted parameters in [8] for model M_1: $\varepsilon_{Qstot} = 2.6\%$;

– *Curve* 2: ε_{Qs1} - calculated using extracted parameters for model M_1 using the developed direct extraction procedure: $\varepsilon_{Qs1tot} = 1.96\%$;

– *Curve* 3: ε_{Qs2} - calculated using extracted parameters for model M_2 using the developed direct extraction procedure: $\varepsilon_{Qs2tot} = 1.58\%$.

The measurement data [8] as well as the simulated characteristics for $Y_R(f)$ and $Y_I(f)$ of the model M_1, obtained using the direct extraction procedure, are shown in Fig. 8 and Fig. 9 respectively. Curve 1 corresponds to the measurement data and curve 2 – to the simulated characteristics.

The simulation results for the quality factors Q_s, Q_{s1} and Q_{s2} and the relative errors of circular inductor of 7 ½ turns [8] are obtained. The corresponding relative errors are:

$$\varepsilon_{Qstot} = 5.06\%\,,\quad \varepsilon_{Qs1tot} = 4.67\%\,,\quad \varepsilon_{Qs2tot} = 4.85\%\,.$$

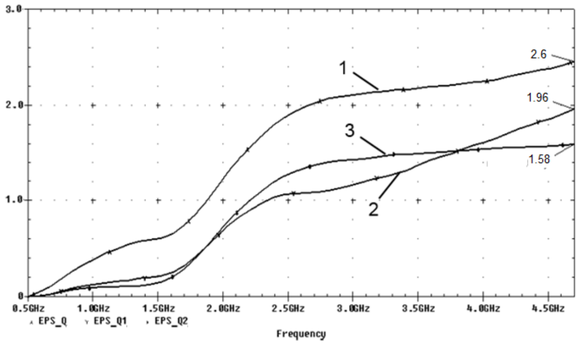

Fig.7. Average relative errors of the simulated Q factors calculated for the frequency range [f_{low}, f_{hi}],

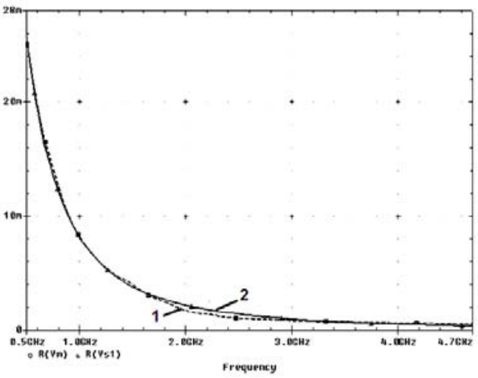

Fig. 8. Measured and simulated characteristic Y_R

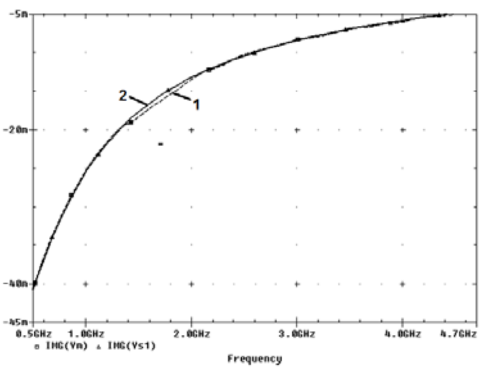

Fig. 9. Measured and simulated characteristic Y_I

The obtained results confirm the validity and the accuracy of the developed parameter extraction procedure and computer models.

VI. CONCLUSION

A computer-aided approach to parameter extraction of on-chip spiral inductor models is proposed, based on measurement data for *S*-parameters. The realization is performed in the environment of the *Cadence Capture, Cadence PSpice* and graphical analyzer *Cadence Probe*. The corresponding macro-definitions in *Probe* are given. The developed extraction procedure is characterized by a high accuracy.

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