TIME DOMAIN MODELING OF BANDPASS RECURSIVE DIGITAL FILTER USING RECURENT NEURAL NETWORK

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In this paper one approach for modeling of 4-th order bandpass recursive digital filter based on two layer recurrent neural network is proposed. The Lagrange multipliers method has been applied to the training process of the neural network. The set of time domain data is generated and used as a target function in the training procedure. To demonstrate the effectiveness of the proposed neural network model some simulations are realized using harmonic signals with different frequencies in the filter's passband and stopband. The analysis of the behaviour of neural network model and target filter frequency responses shows good approximation results.

Keywords: recursive digital filters, neural networks, optimization

Neural networks have increasingly been used in many areas of signal processing. In last year various approaches for modeling and simulation of the analog and discrete dynamic systems based on different type of neural networks have been developed [1-5]. Basic results related to the discrete dynamical systems approximation using neural networks are discussed in [1]. Some of these methods are successfully applied to the design problem in time domain for analog [2] the non-recursive, recursive and adaptive digital filters [3-5]. One approach for the 1-D FIR digital filter design based on the weighted mean square method and neural network to state the approximation problem is proposed in [3]. Some methods for the non-linear digital filters design using neural networks are considered in [4]. Time domain recursive digital filter model, based on recurrent neural network is proposed in [5].

In this paper a 4-th order bandpass recursive digital filter model based on two layer recurrent neural network is proposed. The Lagrange multipliers method has been applied to the training process of the neural network. The set of time domain data is generated and used as a target function in the training procedure. The digital filter model has been trained in such a way that with given predetermined input signal, the output variable approximates the target function in mean square sense. To demonstrate the effectiveness of the proposed neural network model some simulations are realized using harmonic signals with different frequencies in the filter's passband and stopband. The analysis of the behaviour of neural network model and target filter frequency responses shows good approximation results.

1. ALGORITHM OF LAGRANGE MULTIPLIERS

Two layer recurrent neural network is considered. The structure of neural network is shown in Fig. 1. The Lagrange multipliers algorithm is used as a training procedure. This algorithm has been written in matrix form that allows it's generalization for the same type two layer structure with more neurons in each layer.

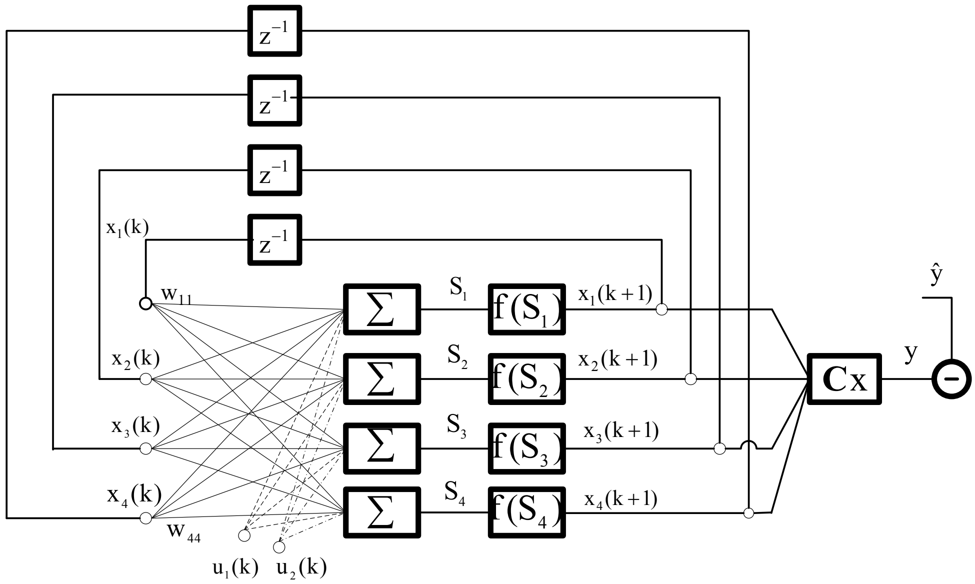

Figure 1. Recurrent neural network structure with four neurons

The recurrent neural network is described with the recurrent system of equations as follows:

- for the first layer of the neural network

$$S_1(k) = w_{11}x_1(k) + w_{12}x_2(k) + w_{13}x_3(k) + w_{14}x_4(k) + w_{15}u_1(k) + w_{16}u_2(k)$$
$$S_1(k) = w_{211}x_1(k) + w_{22}x_2(k) + w_{23}x_3(k) + w_{24}x_4(k) + w_{25}u_1(k) + w_{26}u_2(k)$$
$$S_3(k) = w_{31}x_1(k) + w_{32}x_2(k) + w_{33}x_3(k) + w_{34}x_4(k) + w_{35}u_1(k) + w_{36}u_2(k)$$
$$S_4(k) = w_{41}x_1(k) + w_{42}x_2(k) + w_{43}x_3(k) + w_{44}x_4(k) + w_{45}u_1(k) + w_{46}u_2(k)$$
$$x_1(k+1) = f(S_1) \qquad x_2(k+1) = f(S_2)$$
$$x_3(k+1) = f(S_3) \qquad x_4(k+1) = f(S_4)$$

$$\begin{aligned} x_1(k+1) &= f(w_{11}x_1(k) + w_{12}x_2(k) + w_{13}x_3(k) + w_{14}x_4(k) + w_{15}u_1(k) + w_{16}u_2(k)) \\ x_2(k+1) &= f(w_{211}x_1(k) + w_{22}x_2(k) + w_{23}x_3(k) + w_{24}x_4(k) + w_{25}u_1(k) + w_{26}u_2(k)) \\ x_3(k+1) &= f(w_{31}x_1(k) + w_{32}x_2(k) + w_{33}x_3(k) + w_{34}x_4(k) + w_{35}u_1(k) + w_{36}u_2(k)) \\ x_4(k+1) &= f(w_{41}x_1(k) + w_{42}x_2(k) + w_{43}x_3(k) + w_{44}x_4(k) + w_{45}u_1(k) + w_{46}u_2(k)) \end{aligned} \quad (1)$$

$$\mathbf{x}(0) = x_0 \quad (2)$$

- for the second layer of the neural network

$$\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} = \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \end{bmatrix} \qquad k = 0,1,2,\ldots,N\text{-}1 \quad (3)$$

The mean square error objective function can be stated in the form:

$$J(\mathbf{p}) = \frac{1}{2}\sum_{k=1}^{N-1}\left[(y_1(k) - \hat{y}(k))^2\right] \tag{4}$$

where $\mathbf{p} = \left[w_{11}, w_{12}, ..., w_{46}\right]$ is a vector of the neural network weighting coefficients that are obtained by optimization of the objective function (4)

The following vectors and matrixes are defined:

- a vector of the neural network outputs

  $\mathbf{y}(k) = [y_1(k)\ \ y_2(k)\ \ y_3(k)\ \ y_4(k)]^T$, where $\hat{y}\ (k)$ is the target function;
- a vector of the first layer inputs of neural network;

  $\mathbf{x}(k) = [x_1(k)\ \ x_2(k)\ \ x_3(k)\ \ x_4(k)]^T$
- an error vector;

  $\mathbf{e}(k) = [y_1(k) - \hat{y}\ (k)\quad 0\quad 0\quad 0]^T$
- a vector of Lagrange multipliers;

  $\boldsymbol{\lambda} = [\lambda_1(k)\ \ \lambda_2(k)\ \ \lambda_3(k)\ \ \lambda_4(k)]^T;\quad \boldsymbol{\Gamma} = [\Gamma_1(k)\ \ \Gamma_2(k) ... \Gamma_{24}(k)]^T$
- sub-matrix of weighting coefficients matrix

$$\mathbf{W}_x = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{13} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{34} \\ w_{41} & w_{42} & w_{43} & w_{44} \end{bmatrix}$$

The Hamiltonian of the optimization problem (1)-(3) and objective function (4) is stated in the form:

$$\begin{aligned} &H(x(k), p(k)) = \frac{1}{2}(y_1(k) - \hat{y}(k))^2 + \\ &[\boldsymbol{\lambda}_1(k+1)\,\boldsymbol{\lambda}_2(k+1)\,\boldsymbol{\lambda}_3(k+1)\,\boldsymbol{\lambda}_4(k+1)][f(S_1)\,f(S_2)\,f(S_3)\,f(S_4)\,]^T \qquad (5) \\ &[\Gamma_1(k+1)\,\Gamma_2(k+1) ... \Gamma_{46}(k+1)\,][\,w_{11}, w_{12}, ..., w_{46}\,]^T \end{aligned}$$

Using (5) the conjugated system with appropriate boundary conditions is composed in the form:

$$\boldsymbol{\lambda}(k) = \frac{\partial H}{\partial \mathbf{x}} \qquad \boldsymbol{\lambda}(N) = 0 \tag{6}$$

$$\boldsymbol{\Gamma}(k) = \frac{\partial H}{\partial \mathbf{p}}\ ;\quad \boldsymbol{\Gamma}(N) = 0\quad k=N,N-1,\ldots,2,1 \tag{7}$$

For the problem (1)-(3) the equations (6)-(7) can be written as follows:

$$\begin{bmatrix} \lambda_1(k) \\ \lambda_2(k) \\ \lambda_3(k) \\ \lambda_4(k) \end{bmatrix} = \begin{bmatrix} y_1(k) - \hat{y}(k) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} w_{11} & w_{21} & w_{31} & w_{41} \\ w_{12} & w_{22} & w_{32} & w_{42} \\ w_{13} & w_{23} & w_{33} & w_{43} \\ w_{14} & w_{24} & w_{34} & w_{44} \end{bmatrix} \begin{bmatrix} f'(S_1) & 0 & 0 & 0 \\ 0 & f'(S_2) & 0 & 0 \\ 0 & 0 & f'(S_3) & 0 \\ 0 & 0 & 0 & f'(S_4) \end{bmatrix} \begin{bmatrix} \lambda_1(k+1) \\ \lambda_2(k+1) \\ \lambda_3(k+1) \\ \lambda_4(k+1) \end{bmatrix} \tag{8}$$

and in the matrix form

$$\boldsymbol{\lambda}(k) = \mathbf{e} + \mathbf{W}_x^T \operatorname{diag}(f'(S_1)\, f'(S_2)\, f'(S_3)\, f'(S_4))\, \boldsymbol{\lambda}(k+1) \quad (8.1)$$

$$\begin{bmatrix} \Gamma_{11}(k) & \Gamma_{12}(k) & \Gamma_{13}(k) & \Gamma_{14}(k) & \Gamma_{15}(k) & \Gamma_{16}(k) \\ \Gamma_{21}(k) & \Gamma_{22}(k) & \Gamma_{23}(k) & \Gamma_{24}(k) & \Gamma_{25}(k) & \Gamma_{26}(k) \\ \Gamma_{31}(k) & \Gamma_{32}(k) & \Gamma_{33}(k) & \Gamma_{34}(k) & \Gamma_{35}(k) & \Gamma_{36}(k) \\ \Gamma_{41}(k) & \Gamma_{42}(k) & \Gamma_{43}(k) & \Gamma_{44}(k) & \Gamma_{45}(k) & \Gamma_{46}(k) \end{bmatrix} =$$

$$\begin{bmatrix} \Gamma_{11}(k+1) & \Gamma_{12}(k+1) & \Gamma_{13}(k+1) & \Gamma_{14}(k+1) & \Gamma_{15}(k+1) & \Gamma_{16}(k+1) \\ \Gamma_{21}(k+1) & \Gamma_{22}(k+1) & \Gamma_{23}(k+1) & \Gamma_{24}(k+1) & \Gamma_{25}(k+1) & \Gamma_{26}(k+1) \\ \Gamma_{31}(k+1) & \Gamma_{32}(k+1) & \Gamma_{33}(k+1) & \Gamma_{34}(k+1) & \Gamma_{35}(k+1) & \Gamma_{36}(k+1) \\ \Gamma_{41}(k+1) & \Gamma_{42}(k+1) & \Gamma_{43}(k+1) & \Gamma_{44}(k+1) & \Gamma_{45}(k+1) & \Gamma_{46}(k+1) \end{bmatrix} +$$

$$\begin{bmatrix} f'(S_1) & 0 & 0 & 0 \\ 0 & f'(S_2) & 0 & 0 \\ 0 & 0 & f'(S_1) & 0 \\ 0 & 0 & 0 & f'(S_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_1(k+1) \\ \boldsymbol{\lambda}_2(k+1) \\ \boldsymbol{\lambda}_3(k+1) \\ \boldsymbol{\lambda}_4(k+1) \end{bmatrix} \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) & x_4(k) & u_1(k) & u_2(k) \end{bmatrix} \quad (9)$$

$$\boldsymbol{\Gamma}(k) = \boldsymbol{\Gamma}(k+1) + \operatorname{diag}(f'(S_1)\, f'(S_2), f'(S_3), f'(S_3))\lambda(k+1)\mathbf{z}^T(k) \quad (9.1)$$

$$\boldsymbol{\lambda}(N) = 0\,; \quad \boldsymbol{\Gamma}(N) = 0 \quad k=N,N-1,\ldots,2,1 \quad (10)$$

The main problem in the neural network training process is the gradient calculation of the mean square objective function (4) with respect to weights w.

The Lagrange multipliers algorithm is realized in three steps:

Step 1. Solve the system (1) using the initial condition (2) and store the set of values $x_1(k), x_2(k), x_3(k), x_4(k)$ for k=0,1,…,N-1 and calculate the objective function (4)

Step 2. Solve the conjugate system (8) и (9) using the boundary condition (10) for k=N,N-1,….,2,1 (backwards in time)

Step 3. Obtain the objective function gradient from the solution of (9) for k=0

$$\operatorname{grad} J(\mathbf{p}) = [\Gamma_{11}(0), \Gamma_{12}(0), ..., \Gamma_{46}(0)]^T$$

The objective function (4) is minimized applying the standard optimization procedure.

2. Neural Network Model of Recursive Digital Filter

The recursive digital filter can be considered as a linear discrete dynamic system described in time domain with n – order difference equation that has been stated using the delayed samples of the input excitation and the response signal at the output. Transforming the difference equation the state space description of the recursive digital filter can be obtained as a linear system of n first order recurrent equations. The advantages of the state space representation are the simplicity of time domain analysis, the matrix form description of digital filter impulse and step responses and possibility of parallel calculations.

The neural network structure shown in Fig. 1 must be modified to linear structure. In this case the activation function f(x) is linear. This linear network is used to create a model of 4th order recursive digital filter. The corresponding system of equations (1)-(3) is linear recurrent system. The Lagrange multipliers algorithm modified for linear network is used as a training procedure of the neural network model. The training sequences are generated using the impulse response of target recursive digital filter that is designed by Filter Design Toolbox of MATLAB. The following requirements to the magnitude response of the target bandpass digital filter are specified: bandpass [300–3400] Hz, passband loss – 3 dB; bandstops [0–100] Hz, [3600–4000] Hz, bandstop loss – 15 dB; sampling frequency – 8000 Hz.

3. MODELING RESULTS

The effectiveness of the proposed algorithm is demonstrated by simulation with neural network model. The simulation is realized applying input harmonic excitations with frequencies in the passband and stopband of digital filter. These sinusoidal signals are different from the set of data using in the training process of neural network model. The impulse responses and magnitude responses of the target digital filter and the neural network model are shown in Fig. 2 and Fig. 3 respectively.

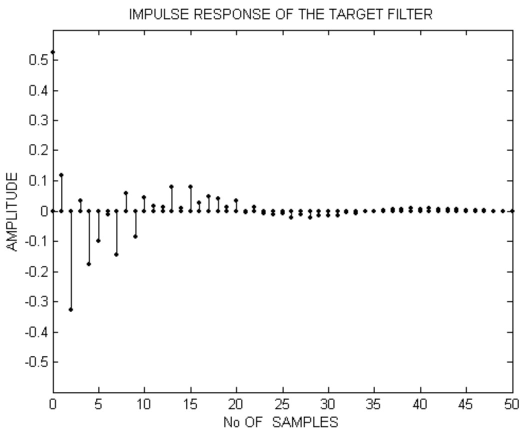

Fig 2a. Impulse response of target filter

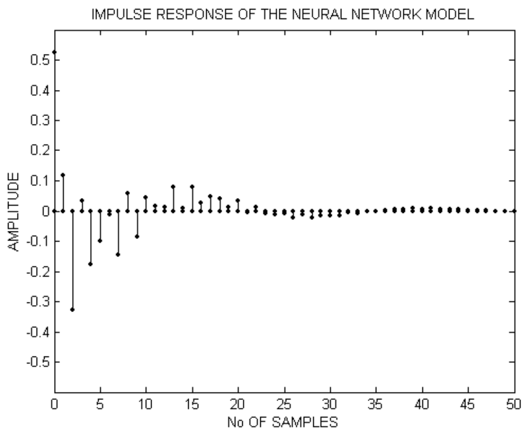

Fig. 2b. Impulse response of neural network model

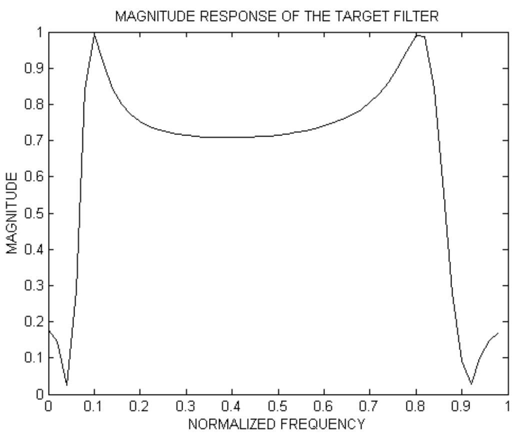

Fig 3a. Magnitude response of target filter

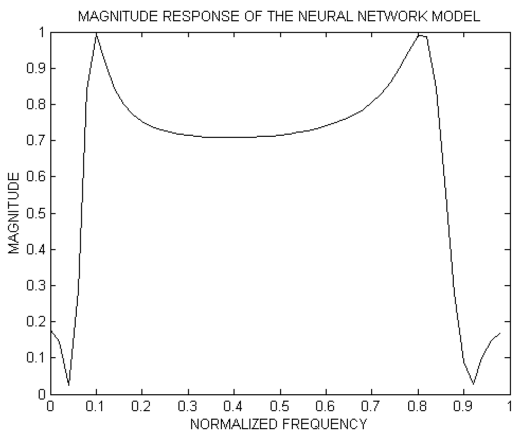

Fig. 3b. Magnitude response of neural network model

The simulation experiments are implemented when at the input of neural network model have been applied sinusoidal signals with the following frequencies - 50 Hz from the filter stopband and 1000 Hz – from the filter passband. The comparison between the input harmonic excitation and the output signal of neural network model is illustrated in Fig. 4a and Fig. 5a respectively. Fig. 4b and Fig. 5b demonstrate the time responses of target digital filter and neural network model.

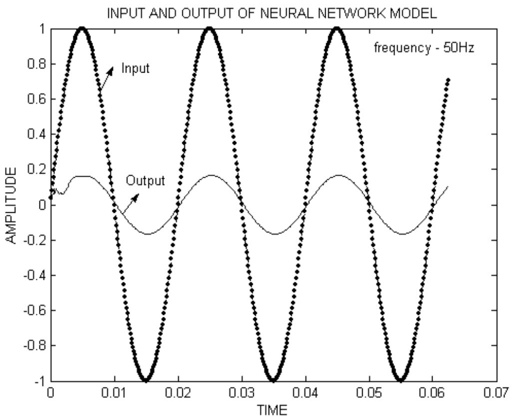

Fig 4a. Input and output of neural network model – 50 Hz

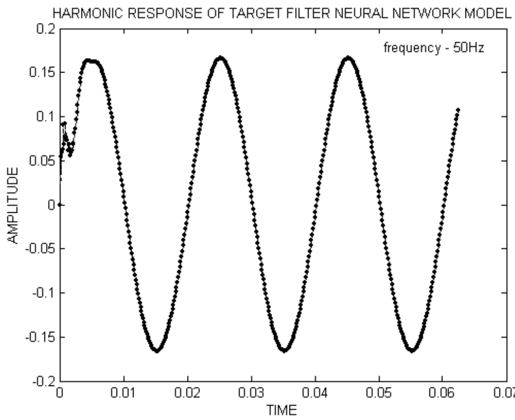

Fig. 4b. Harmonic response of target filter and neural network model

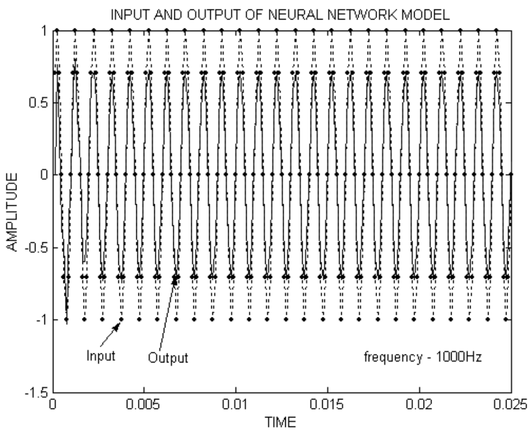

Fig 5a. Input and output of neural network model – 1000 Hz

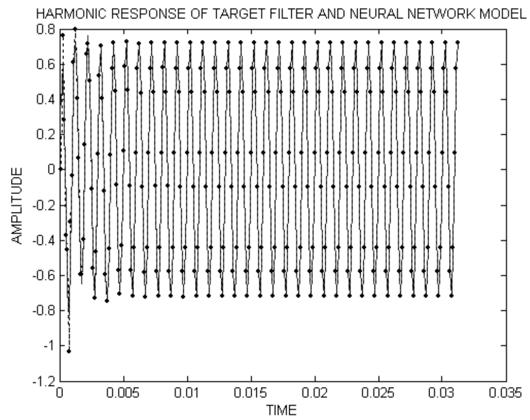

Fig. 5b. Harmonic response of target filter and neural network model

4. REFERENCES

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