ANALYSIS AND BASIC DEPENDENCIES OF PULSE WIDTH MODULATE CONTROLLED SERIES RESONANT DC-DC CONVERTER

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A series resonant DC-DC converter is examined at pulse width modulated control. The operation modes are presented. The conditions for soft commutation of the power devices are discussed. Based on the existing mathematic model of the converter the dependencies of the basic quantities are received. The received results support the analysis and design.

Keywords: Series Resonant DC-DC Converter, PWM Control

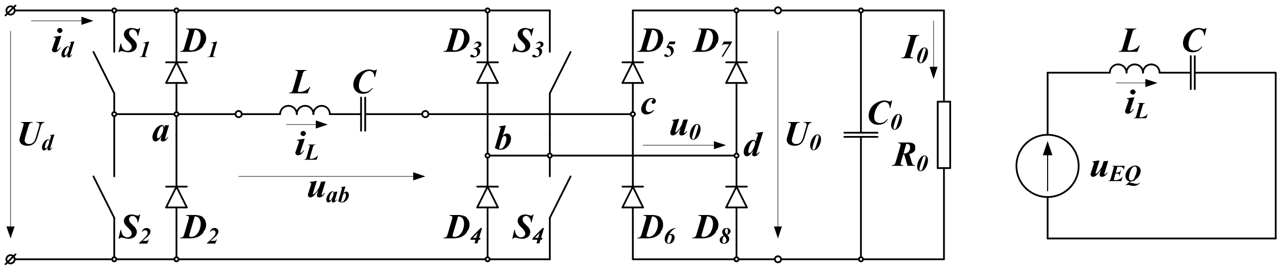
1. INTRODUCTION

Pulse Width Modulated Control (PWMC) is the most frequent used method to change the output power of the series resonant DC-DC converters for constant operation frequency above resonant. Two main modes are observed with this method, defining the different commutation mechanisms of the power devices [1]. They are defined mainly by the loading of the converter. The existing analyses of these converters use approximate methods or require iteration calculation procedures [2]. In [3] and [4] there were proposed mathematic models of series resonant DC-DC converter with PWMC, allowing easy and precise defining of some of its basic quantities for all operation modes.

The purpose of the present work is on the basis of the mathematic models, proposed in [3] and [4] to achieve the dependences for all converter qualities, necessary for its analysis and design.

2. OPERATION MODES AND SOFT COMMUTATION

The examined converter (fig.1.a) consists of a bridge inverter ($S_1 \div S_4$, $D_1 \div D_4$), a series resonant circuit (L, C), an uncontrollable bridge rectifier ($D_5 \div D_8$), a filter capacitor C_0 and a load resistor R_0.



a) b)

Fig.1. Circuits of the series resonant DC-DC converter

In [1] the conditions, under which is possible soft commutation of the power devices of the converter are explained. Two main operation modes are defined – with zero voltage switching and zero current and zero voltage switching. But the analysis, presented in [2] divides the second of these modes to two sub-modes that is connected with the appearance of current pauses (discontinued current mode) in the operation of the rectifier and the inverter. The wave forms of the normalized inverter voltage $\boldsymbol{u_{ab}}$ and the current through the resonant circuit $\boldsymbol{i_L}$ for the three cases are shown at fig. 2. The sequences of conducting of the inverter power devices are presented also for one half-cycle.

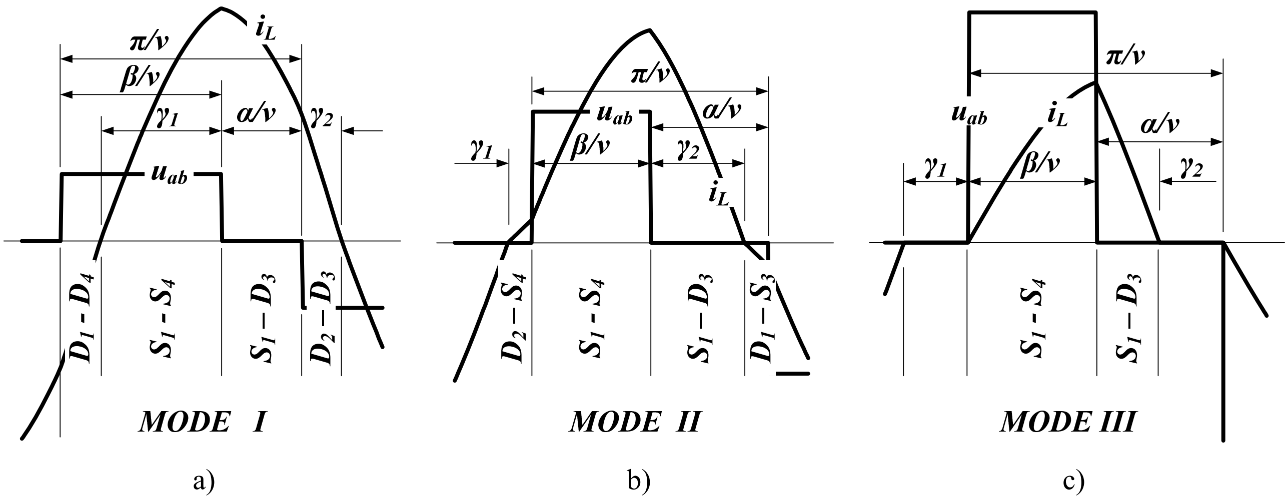


Fig.2. Wave forms of inverter voltage $\boldsymbol{u_{ab}}$ and resonant tank current $\boldsymbol{i_L}$ under alternative operating modes

In fig. 2 $\boldsymbol{\alpha}$ marks the control angle for the operation frequency; $\boldsymbol{\pi}$ – the angle of the half-cycle for the operation frequency; $\boldsymbol{\beta}$ – the angle, corresponding to the interval, in which is achieved an energy exchange with the power supply source, for the operation frequency. Accepting angle $\boldsymbol{\alpha}$ for a control parameter is conditional, but it is the angle of phase-shift between the controlling impulses of the power switches. The three angles are reduced to the resonant frequency of the storage circuit by the means of the frequency distraction $\boldsymbol{v}$.

For the first case (***MODE I***) it is necessary all power switches ($\boldsymbol{S_1 \div S_4}$) to have one and the same commutation mechanisms – spontaneous switching on at zero voltage and forced switching off. When the resonant current $\boldsymbol{i_L}$ changes its direction, the voltage on the corresponding couple of opposite switches ($\boldsymbol{S_1}$, $\boldsymbol{S_4}$ or $\boldsymbol{S_2}$, $\boldsymbol{S_3}$) becomes zero and they switch on with minimum loss. Their switching off becomes consequent in an interval of time, corresponding to angle $\boldsymbol{\alpha}$ and defining the duty ratio. Only in this case there is energy return towards the power supply source achieved (in interval γ_2).

For the realization of soft commutation on the second (***MODE II***) and the third (***MODE III***) cases it is necessary that the switches $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ switch on forcibly some time after the zeroing of the resonant current and to switch off by zero current. The switches $\boldsymbol{S_3}$ and S_4 have the same commutation mechanism as the first case (***MODE I***). Now, the duty ratio is defined by the time between the switching on of the first switches couple (***S₁***, ***S₂***) and the switching off of the second couple (***S₃***, ***S₄***). In these two cases no energy is returned to the power supply source.

3. DEPENDENCIES OF BASIC QUANTITIES

The wave forms at fig.2 show that within the ranges of one half-period the converter circuit realizes three turns, defining three operation intervals. Therefore, each time the resonant circuit is put under different conditions. Knowing the processes, running in it, allows the dependences between the converter's quantities to be defined. In [3] and [4] a modeling of the resonant circuit operation is realized, based on the equivalent substituting circuit of fig. 1 b. It is presumed that all elements, constructing the converter, are ideal and the ripples of the output voltage U_0 are ignored. Thus the resonant frequency, the characteristic impedance and the frequency distraction of the resonant circuit are:

$$\omega_0 = 1/\sqrt{LC}\,; \qquad \rho_0 = \sqrt{L/C}\,; \qquad \nu = \omega_S/\omega_0 \tag{1}$$

The equivalent voltage u_{EQ} is a sum of the inverter voltage u_{ab} and the voltage u_0 at the input of the rectifier. For unification all quantities are represented in relative units: the voltages towards U_d; currents towards U_d/ρ_0. Thus the current through the inductance L and the voltage across capacitor C for each one of the intervals are expressed as follows:

$$\begin{aligned} i'_L(\Theta) &= I'_L \cos\Theta - \left(U'_C - U'_{EQ}\right)\sin\Theta \\ u'_C(\Theta) &= I'_L \sin\Theta + \left(U'_C - U'_{EQ}\right)\cos\Theta + U'_{EQ} \end{aligned} \tag{2}$$

In the above expressions I'_L and U'_C are the initial quantities of the current i'_L and the voltage u'_C; U'_{EQ} - the equivalent voltage, acting upon the resonant circuit within the range of the interval; $\Theta = 0 \div \Theta_W$; Θ_W – angle, corresponding to the interval duration. On fig. 2 with γ_1 is marked the angle of the first interval, and with γ_2 – the angle of the third. Under ***MODE I*** the angle of the second interval is α/ν, and by the cases ***MODE II*** and ***MODE III*** - β/ν.

In Table 1 there are represented the conditions, under which the resonant circuit is put for each one of the three consequent intervals, at the different cases, examined above. It is shown that the interval angles, reduced towards the resonant frequency of the tank circuit; expressions for calculation of the quantities of the integration constants from equations (2); coefficients K_C, giving the conducting of the power devices of the inverter and the rectifier for each of the intervals. The coefficients K_C can be used for defining the loading of the power devices. Since the half-cycles are symmetric, these coefficients relate to a couple of alternative conducting devices (for example S_1, S_2 or D_3, D_4).

In Table 1 there are provided the expressions from [3] and [4] to determine the output voltage U'_0 and the maximum value of the capacitor voltage U'_{CM}.

Table1. Work conditions for the three consequent intervals

Case	Parameter		Interval		
			1	2	3
MODE I	Θ_W		γ_1	α / ν	γ_2
	U'_{EQ}		$1-U'_0$	$-U'_0$	$-1-U'_0$
	I'_L		0	$\left(U'_{CM}+1-U'_0\right)\sin\gamma_1$	$\left(U'_{CM}+1+U'_0\right)\sin\gamma_2$
	U'_C		$-U'_{CM}$	$-\left(U'_{CM}+1-U'_0\right)\cos\gamma_1+1-U'_0$	$\left(U'_{CM}+1+U'_0\right)\cos\gamma_2-1-U'_0$
	K_C	S_{12}	1	1	0
		S_{34}	1	0	0
		D_{12}	0	0	1
		D_{34}	0	1	0
		D_{5678}	1	1	1
	U'_0		$0{,}5\left[\sin\gamma_1+\sin\left(\alpha/\nu+\gamma_1\right)-\sin\left(\alpha/\nu+\gamma_2\right)-\sin\gamma_2\right]/\sin\left(\pi/\nu\right)$		
	U'_{CM}		$0{,}5\left[\sin\gamma_1+\sin\left(\alpha/\nu+\gamma_1\right)+\sin\left(\alpha/\nu+\gamma_2\right)+\sin\gamma_2\right]/\sin\left(\pi/\nu\right)-1$		
MODE II	Θ_W		γ_1	β / ν	γ_2
	U'_{EQ}		$-U'_0$	$1-U'_0$	$-U'_0$
	$I'_L(0)$		0	$I'_{L2}\left(U'_{CM}-U'_0\right)\sin\gamma_1$	$\left(U'_{CM}+U'_0\right)\sin\gamma_2$
	$U'_C(0)$		$-U'_{CM}$	$-\left(U'_{CM}-U'_0\right)\cos\gamma_1-U'_0$	$\left(U'_{CM}+U'_0\right)\cos\gamma_2-U'_0$
	K_C	S_{12}	1	1	0
		S_{34}	1	0	1
		D_{12}	0	0	1
		D_{34}	0	1	1
		D_{5678}	1	1	1
	U'_0		$0{,}5\left[\sin\left(\beta/\nu+\gamma_1\right)+\sin\left(\beta/\nu+\gamma_2\right)-\sin\gamma_1-\sin\gamma_2\right]/\sin\left(\pi/\nu\right)$		
	U'_{CM}		$0{,}5\left[\sin\left(\beta/\nu+\gamma_1\right)-\sin\left(\beta/\nu+\gamma_2\right)-\sin\gamma_1+\sin\gamma_2\right]/\sin\left(\pi/\nu\right)$		
MODE III	Θ_W		γ_1	β / ν	γ_2
	U'_{EQ}		$-U'_{CM}$	$1-U'_0$	$-U'_0$
	$I'_L(0)$		0	0	$\left(U'_{CM}+U'_0\right)\sin\gamma_2$
	$U'_C(0)$		$-U'_{CM}$	$-U'_{CM}$	$\left(U'_{CM}+U'_0\right)\cos\gamma_2-U'_0$
	K_C	S_{12}	1	1	0
		S_{34}	1	0	0
		D_{12}	0	0	0
		D_{34}	0	1	0
		D_{5678}	0	1	1
	U'_0		$0{,}5\left[\sin\left(\beta/\nu+\gamma_2\right)+\sin\left(\beta/\nu\right)-\sin\gamma_2\right]/\sin\left(\beta/\nu+\gamma_2\right)$		
	U'_{CM}		$0{,}5\left[\sin\left(\beta/\nu+\gamma_2\right)+\sin\left(\beta/\nu\right)+\sin\gamma_2\right]/\sin\left(\beta/\nu+\gamma_2\right)-1$		

The first operation mode of the converter (***MODE I***) is possible for a change of angle γ_1 from $\beta/2v$ (short circuit $\gamma_2=\beta/2v$) up to β/v. The increase of angle γ_1 above β/v leads to transit into the second mode (***MODE II***). In relation to it angle γ_1 begins to change from zero (Fig. 2b). Another boundary condition for ***MODE II*** (in relation to ***MODE III***) is given with the expressions:

$$\gamma_1 \le \left[(\pi+\alpha)/\nu-\pi\right]/2 \qquad \gamma_2 \ge (\pi-\beta/\nu)/2 \tag{3}$$

The above presented dependences allow the modeling of the converter processes. With the help of the data provided in Table 1 it is possible to obtain expressions for all other quantities, necessary to analyze and design the converter. So by integrating i'_L, for the output current I'_0 we receive:

$$\begin{aligned} I'_0 &= \frac{\nu}{\pi}\int_0^{\pi/\nu} i'(\Theta)d\Theta = \frac{\nu}{\pi}\sum_{i=1}^{3}\int_0^{\Theta_{Wi}}\left[I'_{Li}\cos\Theta-\left(U'_{Ci}-U'_{EQi}\right)\sin\Theta\right]d\Theta = \\ &= \frac{\nu}{\pi}\left[\sum_{i=1}^{3} I'_{Li}\sin\Theta_{Wi}-\sum_{i=1}^{3}\left(U'_{Ci}-U'_{EQi}\right)\left(1-\cos\Theta_{Wi}\right)\right] \end{aligned} \tag{4}$$

Due to expression (4) the average value of currents trough the power devices of the inverter and the rectifier can be determined. For this purpose the conducting coefficients K_C (Table 1) are used. Presuming that each of the power devices conducts no more of one half-cycle we receive:

$$\begin{aligned} I'_{SD\_AVG} &= \frac{\nu}{2\pi}\int_0^{\pi/\nu} i'_{SD}(\Theta)d\Theta = \frac{\nu}{2\pi}\sum_{i=1}^{3}\int_0^{\Theta_{Wi}} K_{Ci}\left[I'_{Li}\cos\Theta-\left(U'_{Ci}-U'_{EQi}\right)\sin\Theta\right]d\Theta = \\ &= \frac{\nu}{2\pi}\left[\sum_{i=1}^{3} K_{Ci}I'_{Li}\sin\Theta_{Wi}-\sum_{i=1}^{3} K_{Ci}\left(U'_{Ci}-U'_{EQi}\right)\left(1-\cos\Theta_{Wi}\right)\right] \end{aligned} \tag{5}$$

Since S_1 and S_2 work in higher number of intervals than S_3 and S_4, it can be concluded that the currents through the first couple are bigger than those through the second. In a similar way, analyzing the operation of the anti-parallel diodes the conclusion is made that the loading of D_3 and D_4 is bigger than of ***D1*** and D_2.

The root-mean-square value of the current through the inductance can be determined as:

$$\begin{aligned} I'_{L\_RMS} &= \sqrt{\frac{\nu}{\pi}\int_0^{\nu/\pi} i'^2_L(\Theta)d\Theta} = \sqrt{\frac{\nu}{\pi}\sum_{i=1}^{3}\int_0^{\Theta_{Wi}}\left[I'_{Li}\cos\Theta-\left(U'_{Ci}-U'_{EQi}\right)\sin\Theta\right]^2 d\Theta} = \\ &= \sqrt{\frac{\nu}{\pi}\left[\begin{array}{l} \sum_{i=1}^{3}\left(\frac{I'_{Li}}{2}\right)^2\left(2\Theta_{Wi}+\sin 2\Theta_{Wi}\right)+ \\ +\sum_{i=1}^{3}\left(\frac{U'_{Ci}-U'_{EQi}}{2}\right)^2\left(2\Theta_{Wi}-\sin 2\Theta_{Wi}\right)- \\ -\sum_{i=1}^{3} I'_{Li}\frac{U'_{Ci}-U'_{EQi}}{2}\left(\sin 2\Theta_{Wi}-1\right) \end{array}\right]} \end{aligned} \tag{6}$$

The root-mean-square values of the currents through the switches and diodes are obtained in analogical way as the average values, using the conducting coefficients K_C and based on equation (5):

$$I'_{SD\_RMS} = \sqrt{\frac{\nu}{2\pi}\int_0^{\nu/\pi} i'^2_{SD}(\Theta)d\Theta} = \sqrt{\frac{\nu}{2\pi}\sum_{i=1}^{3}\int_0^{\Theta_{Wi}} K_{Ci}\left[I'_{Li}\cos\Theta - \left(U'_{Ci} - U'_{EQi}\right)\sin\Theta\right]^2 d\Theta} =$$

$$= \sqrt{\frac{\nu}{2\pi}\left[\begin{array}{l} \sum_{i=1}^{3} K_{Ci}\left(\frac{I'_{Li}}{2}\right)^2\left(2\Theta_{Wi} + \sin 2\Theta_{Wi}\right) + \\ + \sum_{i=1}^{3} K_{Ci}\left(\frac{U'_{Ci} - U'_{EQi}}{2}\right)^2\left(2\Theta_{Wi} - \sin 2\Theta_{Wi}\right) - \\ - \sum_{i=1}^{3} K_{Ci} I'_{Li}\frac{U'_{Ci} - U'_{EQi}}{2}\left(\sin 2\Theta_{Wi} - 1\right) \end{array}\right]} \tag{7}$$

The output power of the converter can be expressed as:

$$P'_0 = U'_0 I'_0 \tag{8}$$

3. CONCLUSION

Analysis of the operation modes of series resonant DC-DC converter at pulse width modulated control is presented. It is accepted that in the range of one half-cycle the operation of the converter is divided into three intervals, defined by the turning in the power circuit. On the basis of the analysis and existing mathematic model the dependences for the calculation of the converter quantities are obtained. All expressions are functions of the angles of the abovementioned three operation modes of the converter.

The results can be used to analyze the converter. The load characteristics as well as the trajectories in the state-space can be constructed easily.

The obtained dependences can also be used to design this converter or other types of converters with similar behavior (for example, a vector control converter).

2. REFERENCES

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