A SIMPLE METHOD FOR STABILIZATION OF ARBITRARY SPECTRA

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A method is proposed for stabilization of spectral data with arbitrary form. The method is based on the hypothesis that the successive measurements of a spectrum represent the same statistical distribution but in different X-scales. The transformation between X-scales is found by calculation of the moments of the distributions. It can be further used for either recalculation of the spectral data in order to be processed together, or as a feedback signal for online stabilization of the data acquisition system. The calculations are extremely simple in the case of a linear transformation between both X-scales; in that case they can be performed directly on a microcontroller. The method is illustrated for long series of spectral records of the gamma-radiation using scintillation detectors.

Keywords: spectra stabilization, long-term behaviour, scintillation detectors

1. INTRODUCTION

The problem for stabilization of the measuring equipment characteristics is one of the common problems in the experimental science and engineering. It is of crucial importance in the case of long term measurements.

Several factors lead to change of the transfer function of the system, the most important ones being the temperature influence (drift) and the ageing of the elements. The progress in the instrumentation reduces the problem but it does not overcome it.

Numerous of methods have been (and are being) developed to reduce the system instability. Some of them are passive, i.e. they rely upon preserving the ambient conditions (such as temperature stabilization); others are active, and based on a feedback loop which adjusts some parameters of the measuring system with the aim to retain its transfer characteristic.

In this work we propose a general numerical method to assess the bias of the transfer function of a spectrometric system in a manner which is independent on the particular form of the spectral data. Its results can be then used for either a subsequent recalculation of the investigated spectrum in order to be processed together with the previous records, or as a signal in a feedback loop for active stabilization of the system.

2. METHOD

The formal statement of the problem is as follows:

We have two sets of data (hereafter referred as 'spectra'):

$$F_i = \int_{x_i}^{x_{i+1}} f(x)dx\,; \quad \Phi_i = \int_{\xi_i}^{\xi_{i+1}} \varphi(\xi)d\xi\,, \qquad i = 0,...,n-1\,. \tag{1}$$

They can be e.g. successive records from the same experimental series, or, perhaps, two measurements made at different conditions. The integrals in (1) reflect the fact that $\{F_i, \Phi_i\}$ result from a quantization of continuously changing functions f, φ , which is the common situation if an analog-to-digital converter (ADC) is used to digitize the signal, and integration is made within the energy range corresponding to the same digital code (the 'channel width').

The basic assumption is that the signal functions f, φ are essentially identical, apart from a trivial scaling factor and some noise with known statistical characteristics. We suppose that the main difference between f and φ is due to an unknown X-scale transformation (drift) which had occurred between the records $\{F\}$ and $\{\Phi\}$. It has the general form:

$$\xi = \Xi(x) \tag{2}$$

The transformation (2) is supposed to be unique and reasonably smooth, so the inverse dependence $x = X(\xi)$ is also defined. Once $x = X(\xi)$ is known, then the data set $\{\Phi\}$ can be back transformed into the scale x and then processed together with $\{F\}$.

In order to find $X(\xi)$ certain parameterization is useful; the easiest one is the polynomial form:

$$X(\xi) = \sum_{k=0}^{r-1} a_k \xi^k\,, \tag{3}$$

where the coefficients $\{a_k\}$ are to be found. The present method, however, is not restricted to that case.

Our idea is to use the fact that if two statistical distributions are identical, then all their corresponding raw moments (i.e. moments taken about 0) are equal, i.e:

$$\int_{-\infty}^{\infty} x^k f(x)dx \equiv \int_{-\infty}^{\infty} \xi^k \varphi(\xi)d\xi, \;\; k = 0,1,...\,, \tag{4}$$

Indeed, the data sets are not normalized, so, we have to use (4) in the form:

$$\frac{\int_{-\infty}^{\infty} x^k f(x)dx}{\int_{-\infty}^{\infty} f(x)dx} \equiv \frac{\int_{-\infty}^{\infty} \xi^k \varphi(\xi)d\xi}{\int_{-\infty}^{\infty} \varphi(\xi)d\xi}\,, \;\; k = 1,... \tag{4'}$$

or, in discrete form:

$$\frac{\sum_{i=0}^{n-1} x_i^k F_i \Delta x_i}{\sum_{i=0}^{n-1} F_i \Delta x_i} \equiv \frac{\sum_{i=0}^{n-1} \xi_i^k \Phi_i \Delta \xi_i}{\sum_{i=0}^{n-1} \Phi_i \Delta \xi_i}, \; k = 1,... \qquad (4")$$

Note that the above expression is further simplified if the data are equidistant, which is the usual case in the spectroscopy.

The system (4") can be used to find the argument transformation $x = X(\xi)$. In practice, the minimal number of equations, m in (4") has to be equal to the number of the parameters r in $X(\xi)$, $m \geq r$, while a maximal rank of the system $k = m$ is limited mostly by the noise in the data. If $m = r$ then (4") has a unique solution; if $m > r$, then a least-squares solution can be obtained, which minimizes some measure for the difference between left and right side of the above equations.

Let us suppose equidistant measurements and denote abscissas ξ_i simply by the bin number i. Then we need to minimize:

$$Q_k = \left(\frac{\sum_{i=0}^{n-1} x(i)^k F_i}{\sum_{i=0}^{n-1} F_i} - \frac{\sum_{i=0}^{n-1} i^k \Phi_i}{\sum_{i=0}^{n-1} \Phi_i} \right)^2, \; k = 1,..,m \qquad (5)$$

with respect to $x(i)$. The minimum of Q_k is at the point when its gradient is 0; so, we have:

$$\boxed{\frac{\partial Q_k}{\partial a_l} = 2 \left(\frac{\sum_{i=0}^{n-1} x(i)^k F_i}{\sum_{i=0}^{n-1} F_i} - \frac{\sum_{i=0}^{n-1} i^k \Phi_i}{\sum_{i=0}^{n-1} \Phi_i} \right) \sum_{i=0}^{n-1} F_i k.x(i)^{k-1} \frac{\partial x(i)}{\partial a_l} = 0, \quad k = 1,..,m; l = 0,...,r-1} \qquad (6)$$

A great simplification of this procedure can be achieved if

$$X(i) = a_0 + a_1 i \qquad (7)$$

is a linear function, which is the most common situation, at least as a first-order approximation. Then, using the following notations:

$$m_k \overset{def}{=} \sum_{i=0}^{n-1} i^k F_i, \; \mu_k \overset{def}{=} \sum_{i=0}^{n-1} i^k \Phi_i \qquad (8)$$

we obtain from (4") for the first and the second order moment of the distribution:

$$\begin{aligned} \frac{a_0 m_0 + a_1 m_1}{m_0} &= \frac{\mu_1}{\mu_0} \\ \frac{a_0^2 m_0 + 2a_0 a_1 m_1 + a_1^2 m_2}{m_0} &= \frac{\mu_2}{\mu_0} \end{aligned} \qquad (9)$$

The solution of (9) is:

$$\boxed{\begin{aligned} a_0 &= \frac{\mu_1}{\mu_0} - a_1 \frac{m_1}{m_0} \\ a_1^2 &= \frac{\frac{\mu_2}{\mu_0} - \frac{\mu_1^2}{\mu_0^2}}{\frac{m_2}{m_0} - \frac{m_1^2}{m_0^2}} \end{aligned}} \qquad (10)$$

3. EXPERIMENTAL RESULTS

We have applied the method described above to a sequence of NaI scintillation spectra of several weak radioactive sources (^{241}Am - 26keV, 60keV; ^{137}Cs - 662keV; ^{60}Co - 1173keV, 1332keV) taken at different temperatures. As it is well known, the overall gain of a scintillation detector has a strong temperature dependence due to both the light yield of the scintillator and to the photomultiplier gain.

Fig.1 shows a typical change in the 1332keV ^{60}Co peak position during a temperature cycle: +20^{0}C (+1^{0}C/h) +55^{0}C (-1^{0}C/h) -25^{0}C (+1^{0}C/h) +20^{0}C [1]. Besides the typical temperature drift, a deep hysteresis is observed that makes the system behavior ambiguous.

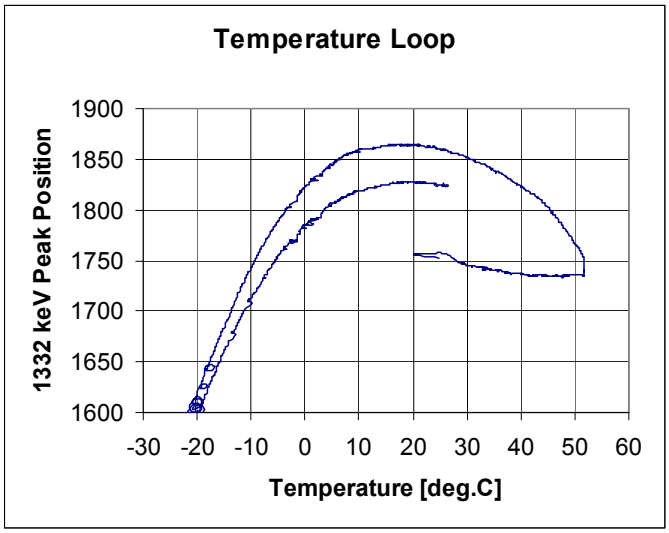

Fig.1.

In our previous work [1] we have used a method [2] based on a least-squares comparison of the profiles of successive spectral records. One of the records (say, the first one) is kept fixed, while a linear transformation of the X-scale of each other record is calculated so that the difference between the two spectral forms is minimal.

Fig.2 shows the ^{137}Cs peak position from the Fig.1 cycle before and after the application of the two x-scale adjustment algorithms. The results are summarized in Table 1. The previous method [1] diminishes about 5 times the peak position variation of the original data, but a bias still remain due to the lower sensitivity of the minimization procedure nearby the minimum. The advantage of the present method is clearly seen: the peak position variation is decreased three times more and now falls well within the instrumental linewidth.

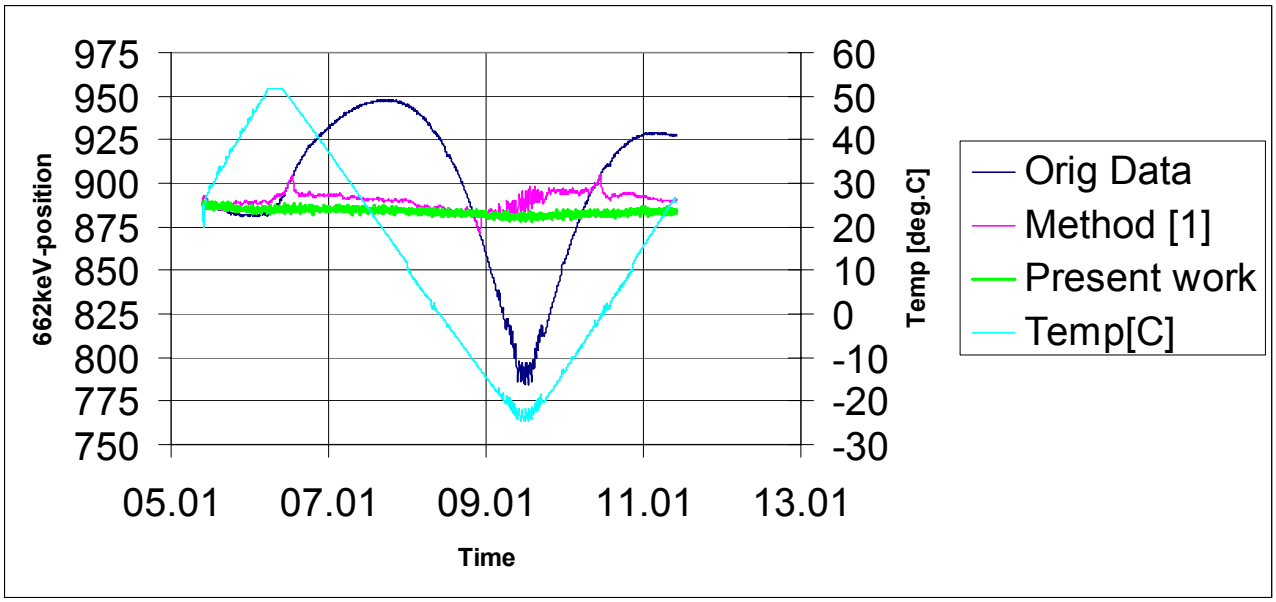

Fig.2

Table 1

Data	Average	Variation
Original	897.3	163.7
Method [1]	890.2	33.7
Present work	883.7	11.3

4. CONCLUSION

A method for numerical evaluation of the X-scale change between two records of statistically identical data sets ('spectra') is developed in the present work. It is based on the concept for the equality of the corresponding raw moments of the statistical distributions and is independent on the particular spectrum profile. The calculations are very simple in the most common case of a linear X-scale change and can be performed directly on a small microcontroller. The results (X-scale slope and shift) can be used further in two ways:

- for recalculation of the spectral data in order several records to be processed together; again, spline techniques are adequate for rebinning the data;
- or, as a feedback signal for online stabilization of the data acquisition system.

A typical application of the method can be for stabilization of long-term spectral measurements of the environmental radiation using scintillation detectors [3]. The present method, however, is not restricted to nuclear data or to one-dimensional data sets.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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