TRANSIENT RESPONSE AND TIME CHARACTERISTIC OF ONE CAPACITIVE INTERFACE CIRCUIT IN CASE OF FRACTALITY

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Recent studies have represented many physical phenomena such as dielectric absorption, electrical double-layers, geometrical fractality and etc. by constant parameter differential equations which order is fractional (FODE). The paper reviews transient response of a classical capacitive interface circuit based on non inverting amplifier if one term FODE describes behavior of sensing element. The obtained results are applicable at design and tuning of the capacitive transmitters especially in area of mechanical, electrical, chemical and biological measurements.

Keywords: fractality, capacitive measurements, transient response

1. INTRODUCTION

Commonly capacitive sensors are built with conductive sensing electrodes in a dielectric and signal conditioning circuits which turn capacitance variations into a voltage, frequency, or pulse width modulation. The phenomena in the dielectric may have strong effect on the work of the capacitive sensor. For example as shown by Jonscher [1, 2] and many others, in all cases, real dielectric display a degree of „fractance" and admittance of sensing element has a form

$$Y_X = \alpha . s^{\beta} \tag{1}$$

where $\alpha, \beta \in \Re$, $0 \le \beta \le 1$,

or it includes the terms of this kind. Element with admittance (1) is described by two independent parameters and it is known as fractional capacitor (FC) of order β. In simplest cases β closes to unity and the FC has properties similar to ideal capacitor.

In nowadays several techniques are available to convert impedance and capacity. The Fig.1 presents a simple condition circuit that converts ordinary capacity in voltage. It is very suitable for applications with grounded sensing electrodes. If the amplifier $A1$ and the capacitors C_X, C_F are ideal

$$u = \left(\frac{C_X}{C_F} + 1 \right) . v \tag{2}$$

where $u = u(t)$ -input voltage, $v = v(t)$ -output voltage, t -time, the output is scaled by C_X copy of the input [3]. In case of FC it is not at all.

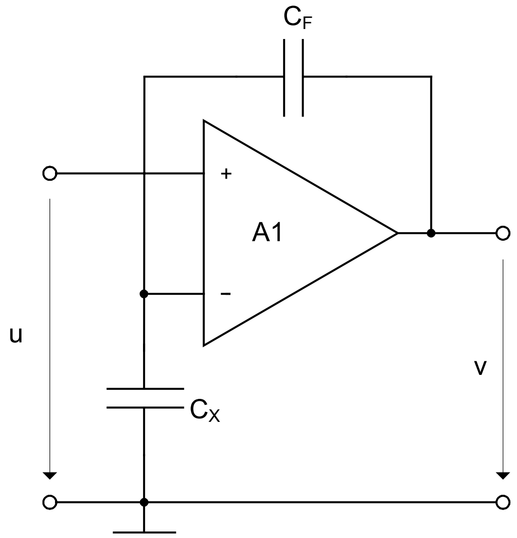

Fig.1 Capacitive interface circuit

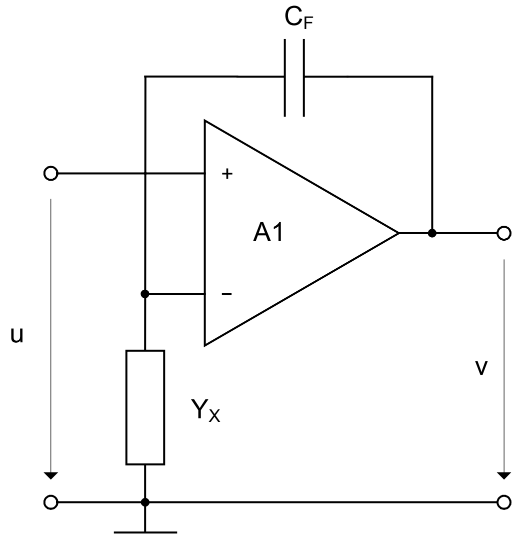

Fig.2 FC interface circuit

2. UNIT STEP RESPONSE IN CASE OF FC

Let the input u is a unit step:

$$u(t) = H(t) \tag{3}$$

where $H(t) = \begin{cases} 0, t \le 0 \\ 1, t > 0 \end{cases}$

is the Heaviside step function and the eq. (1) described impedance Z_X. Now response of circuit shown on Fig.2 in s-domain can be expressed as:

$$V(s) = \left(1 + \frac{\alpha}{C_F} . \frac{1}{s^{1-\beta}}\right) . U(s) = U(s) + \frac{\alpha}{C_F} . \frac{1}{s^{1-\beta}} . U(s) \tag{4}$$

where $V(s)$ is Laplace transform of v, $U(s)$ is Laplace transform of u [4].

Above equation is valid at zero initial conditions, ideal amplifier $A1$ and capacitor C_F. Taking the inverse Laplace transform of (4) produces:

$$v(t) = u(t) + w(t) = 1 + \frac{1}{\Gamma(2-\beta)} . \frac{\alpha}{C_F} . t^{1-\beta}, \quad t > 0 \tag{5}$$

where

$$w(t) = \frac{1}{\Gamma(2-\beta)} . \frac{\alpha}{C_F} . t^{1-\beta}, \tag{6}$$

and $\Gamma(.)$ is the Gamma function [5].

A typical response with respect to β appears in Fig.3. If the $\beta = 1$ the output voltage is became constant value α / C_F. The behavior of the interface is the same as the circuit shown on Fig.1. In all other cases the u increases with time and additional care to be taken against saturation of amplifier.

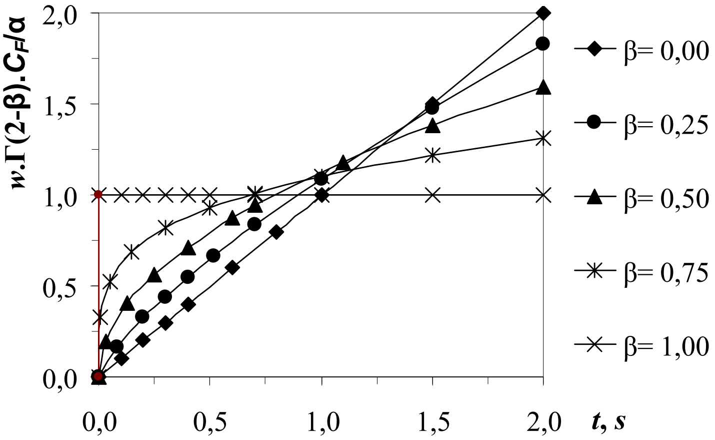

Fig.3 Response w of interface circuits with respect to β.

Expression (5) can be generalized in case input is a series of steps:

$$u(t) = \sum_i A_i.H(t - t_i) \tag{7}$$

where i -th step delayed by t_i has height $|A_i|$, $i \in N$, $A_i \in \Re$.
Because the Laplace transform is linear

$$L[a.f(t) + b.g(t)] = a.L[f(t)] + b.L[g(t)] \tag{8}$$

the response to input (7) can find consider (5):

$$v(t) = u(t) + w(t) = \sum_i A_i.H(t - t_i) + \sum_i \frac{A_i}{\Gamma(2-\beta)}.\frac{\alpha}{C_F}.(t - t_i)^{1-\beta}.H(t - t_i) \tag{9}$$

$$w(t) = \frac{1}{\Gamma(2-\beta)}.\frac{\alpha}{C_F}.\sum_i A_i.(t - t_i)^{1-\beta}.H(t - t_i) \tag{10}$$

3. RESPONSE TO SERIES OF TWO STEPS IN CASE OF FC

Now input is a series of two steps

$$u(t) = A_1.H(t - t_1) + A_2.H(t - t_2) \tag{11}$$

Let us accept the following conditions:

$$A_1 = A,\ A_2 = -2.A,\ t_1 = 0. \tag{12}$$

Than

$$u(t) = A.H(t) - 2.A.H(t - t_2) \tag{13}$$

and for all $t > t_2$ the output becomes

$$v(t) = u(t) + w(t) = A - 2.A.H(t - t_2) + \frac{A}{\Gamma(2-\beta)}.\frac{\alpha}{C_F}.t^{1-\beta} - \frac{2.A}{\Gamma(2-\beta)}.\frac{\alpha}{C_F}.(t - t_2)^{1-\beta} \tag{14}$$

$$w(t) = \frac{A}{\Gamma(2-\beta)} \cdot \frac{\alpha}{C_F} . t^{1-\beta} - \frac{2.A}{\Gamma(2-\beta)} \cdot \frac{\alpha}{C_F} . (t - t_2)^{1-\beta} \tag{15}$$

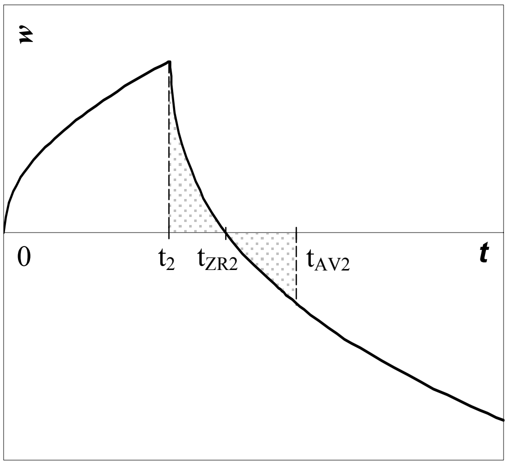

Fig.4 Plot of response w to series of two steps

The above eq. allow us to find some important characteristic of the time response:

1. Time t_{ZR2}, $t_2 < t_{ZR2}$ when

$$w(t_{ZR2}) = 0 \text{ or } v(t_{ZR2}) = -A \tag{16}$$

This is the moment when w is zeroing.

2. Time t_{AV2}, $t_2 < t_{AV2}$ when

$$\int_{t_2}^{t_{AV2}} w(t) = 0 \tag{17}$$

This moment define the interval where the average value of w is zero.

From (15) and (16)

$$\frac{A}{\Gamma(2-\beta)} \cdot \frac{\alpha}{C_F} . \left(t_{ZR2}{}^{1-\beta} - (t_{ZR2} - t_2)^{1-\beta} \right) = 0 \tag{18}$$

$$t_{ZR2}{}^{1-\beta} - (t_{ZR2} - t_2)^{1-\beta} = 0$$

and

$$t_{ZR2} = \frac{\sqrt[1-\beta]{2}}{\sqrt[1-\beta]{2} - 1} . t_2 \text{ or } \frac{t_{ZR2}}{t_2} = \frac{\sqrt[1-\beta]{2}}{\sqrt[1-\beta]{2} - 1} \tag{19}$$

Note that the ratio t_{ZR2} / t_2 depends only in order β of FC and the carve $t_{ZR2} / t_2 = f(\beta)$ (see Fig.4) assumed to be approximately linear over the large range of values of β .

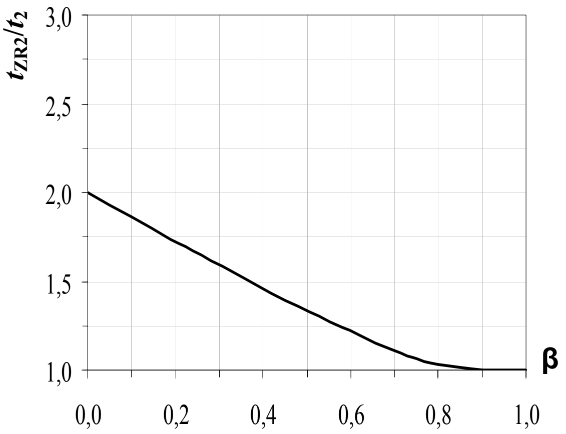

Fig.4 Plot of function $t_{ZR2} / t_2 = f(\beta)$

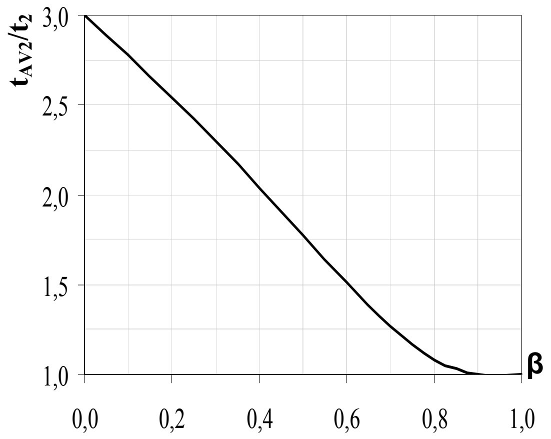

Fig.5 Plot of function $t_{AV2} / t_2 = f(\beta)$

As we know

$$\int t^b .dt = \frac{t^{b+1}}{b+1} + C \,, b \neq -1 \,, C \in \Re \tag{20}$$

From eq. (15), (17) and (20) we obtain:

$$\int_{t_2}^{t_{ZR2}} w(t) = \frac{A}{\Gamma(2-\beta).(2-\beta)} . \frac{\alpha}{C_F} . \left(t_{ZR2}{}^{1-\beta} - t_2{}^{1-\beta} - 2.(t_{ZR2} - t_2)^{1-\beta} \right) = 0 \tag{21}$$

$$t_{ZR2}{}^{1-\beta} - t_2{}^{1-\beta} - 2.(t_{ZR2} - t_2)^{1-\beta} = 0$$

Divided by the above eq. into t_2 and substation

$$a = t_{ZR2} / t_2 \tag{22}$$

we obtain a transcendental eq.

$$a^{1-\beta} - 2.(a-1)^{1-\beta} - 1 = 0 \tag{23}$$

The numerical solution of (23) shown in Fig.5. Like the case above the ratio t_{AV2} / t_2 depends only in order β of FC and the carve $t_{AV2} / t_2 = f(\beta)$ assumed to be approximately linear over the large range of values of β .

4. Conclusion

This paper reviews transient response of a capacitive interface circuit based on non-inverting amplifier in case of fractality. The results obtained above can be used at design and tuning of such transmitters. Briefly we point out some of their possible application.

The equation (19) predicts that the circuit Fig.2 can convert order β of FC to ratio of times. Moreover it can be done linearity and independent of α .

Let us consider one more complicated case. Usually [6] synchronous detector and LP filter (integrator) following circuit in Fig.2. Now in the filter stores an average of output signal $v(t)$. Let us assume that the admittance of the sensing element can be expressed in form as sum:

$$Y_X = Y_1(s) + \alpha . s^{\beta} \tag{24}$$

where $Y_1(s)$ - some function of s . The response in s-domain and t-domain can be described by

$$V(s) = \left(1 + \frac{Y_1}{s.C_F} + \frac{\alpha}{C_F} . \frac{1}{s^{1-\beta}} \right) . U(s) \tag{25}$$

$$v(t) = u(t) + w_1(t) + w(t) \tag{26}$$

where $w_1(t) = L\left[\frac{Y_1}{s.C_F} \right]$.

Consider (17) easy to see that detection in interval $[t_2 ; t_{AV2}]$ gives average value independent of second term of (24). That means that the measuring system has zero sensitivity to fractional term of order β.

Whole above results can also adapted to use with interface circuits based on inverting amplifier. They have been validated in SPACE.

5. References

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