# Electrical Design of Three-Phase Bridge Diode Rectifiers with Active-Capacitive Load by MATLAB 


#### Abstract

Evgeniy Ivanov Popov, Liliya Ivanova Pindeva Department of Power Electronics, Technical University of Sofia, K1. Ohridski Bul. No.8, 1000 Sofia, Bulgaria, phone: +359 2 9652642, e-mail: epopov@tu-sofia.bg Three-phase bridge diode rectifiers with active-capacitive load can be applied for energy conversion in simple wind power generators or in controlled a.c. electrical drives with squirrel-cage induction motors. They pose specific problems to the engineers. The known methods (the "eastern", based on the assumption that the voltage across the filtering capacitor is almost even and the "western") are generally created for electrical design of single phase diode rectifiers. They are graphic-analytical and of course suffer from limited accuracy and waste of time. The r.m.s. value of the rectifier supplying current is specified. The standard supplying power related to the d.c. power, the output (load) characteristic, the harmonic components related to the first harmonic and the distortion coefficient of the supplying current are also obtained. MATLAB programs for exact graphical representation of the functions, participating in the full electrical design of the rectifiers and for computeraided design implementation are created. PSPICE simulations confirm the results for some practical examples.


Keywords: MATLAB design, three-phase, bridge rectifier, capacitive load.

## 1. INTRODUCTION

Three-phase bridge diode rectifiers with active-capacitive load can be applied for rectification of the a.c. power, produced by the wound rotor induction generators in simple and cheap variants of renewable energy resources, based on wind turbines or in controlled a.c. electrical drives with squirrel-cage induction motors. In such rectifiers after the three-phase bridge diode configuration a filtering (smoothing) capacitor is connected. The capacitor participates in the rectifying action and of course influences the principle of operation that becomes completely different compared to the other types of loads. The parameters of the three-phase bridge diode rectifier with active-capacitive load (Fig.1) depend on the load resistance $R_{d}$, on the capacitance of the filtering capacitor $C$ and the last but not the least on the internal resistance of the rectifier $R_{\text {rect }}$. The last includes the ohmic resistance of the a.c. conductors and the windings of the rotor, the resistance introduced by the diodes and the resistance of the protective resistor, if such exists.

The method for the electrical design of such rectifiers published in the Russian and Bulgarian language technical literature [1], [2], [3], [4] is based on the simplifying but close to the practice assumption that the rectifier load voltage (across the filtering capacitor) is almost even (rippling factor $0.05 \div 0.2$ ). Thus the load can be also an accumulator battery. Other assumptions include the negligence of the leakage inductances and the linearity of the V-A diode characteristics. This method in
previous versions with the exception of [4] is generally graphic-analytical and it naturally suffers from limited accuracy and waste of time. Especially for the case of three-phase bridge diode rectifiers the method is not fully developed.

Hence, the aims of this paper is to further develop the method for design of threephase bridge diode rectifiers assuming almost constant output voltage; to specify the analytical expressions of all the variables; to create two MATLAB programs: (a) for exact graphical representation of the functions of the necessary variables and (b) for precise design implementation on a computer; to prove the correctness of the design.


Fig.1. Three-phase bridge diode rectifier with active-capacitive load.

## 2. Problem Statement

In the method under consideration [1] first the coefficient $A$ is calculated

$$
\begin{equation*}
A=\frac{\pi R_{r e c t} I_{d}}{m V_{d}}=\frac{\pi R_{r e c t}}{m R_{d}}=\tan \theta-\theta \tag{1}
\end{equation*}
$$

where $I_{d}$ and $V_{d}$ are the average values of the rectified current and voltage, $m$ is the number of phases ( $m=6$ for three-phase bridge diode rectifier), $\theta$ is the cut-off angle known from [1], [2] and [4] (half of the interval in angular units when a particular diode conducts). The shape of the rectifier supplying current is shown in Fig.2.


Fig.2. The shape of the rectifier supplying current.
Equation (1) expresses the implicit function $\theta=f(A)$ (Fig.3). After its calculation the following parameters are determined from:

The r.m.s. value of the transformer secondary winding phase voltage $V_{2 \text { ph.r.m.s. }}$

$$
\begin{equation*}
\frac{V_{2 p h . r . m . s .}}{V_{d}}=\frac{V_{2 p h . m}}{\sqrt{2} V_{d}}=\frac{B}{\sqrt{3}}=\frac{1}{\sqrt{3} \sqrt{2} \cos \theta}=\frac{f_{B}(\theta)}{\sqrt{3}} \tag{2}
\end{equation*}
$$

The peak current through the diodes $I_{m}$

$$
\begin{equation*}
\frac{I_{m}}{I_{d}}=\frac{F}{m}=\frac{\pi(1-\cos \theta)}{m(\sin \theta-\theta \cos \theta)}=f( \tag{3}
\end{equation*}
$$

The r.m.s. value of the transformer secondary winding phase current $I_{2 \text { ph.r.m.s. }}$

$$
\begin{equation*}
\frac{I_{2 p h . r . m . s .}}{I_{d}}=\frac{2 D}{m}=\frac{2 \sqrt{\pi[\theta(1+0.5 \cos 2 \theta)-0.75 \sin 2 \theta]}}{(\sin \theta-\theta \cos \theta) \cdot m}=f(A) \tag{4}
\end{equation*}
$$

Expression (4) can be explained as follows: (a) in the single- or double- phase half-wave rectifiers the current flows through the conductor of the secondary winding once a period without changing its direction, hence $I_{2 r . m . s .} / I_{d}=D / m$; (b) in the single-phase bridge connection the current flows through the conductor of the secondary winding twice a period changing the direction, hence $I_{2 \text { r.m.s. }} / I_{d}=D_{b r} / m=\sqrt{2} D / m$; (c) in the three-phase bridge rectifier the pulses of the secondary phase current are doubled once again (Fig.2) and hence $I_{2 \text { ph.r.m.s. }} / I_{d}=\sqrt{2} D_{b r} / m=\sqrt{2} \sqrt{2} D / m$. Applying the rules for finding a r.m.s. value leads to the result, expressed by (4).

The capacitance of the capacitor for a given rippling factor $S_{\text {ripple }}=(0.05 \div 0.2)$

$$
\begin{equation*}
C[\mu F] \geq H_{m=6} /\left(S_{\text {ripple }} \cdot R_{\text {rect }}\right) \tag{5}
\end{equation*}
$$

where $\omega=2 \pi f$ is the mains angular frequency

$$
\begin{equation*}
H_{m=6}=\frac{10^{6} .2(\sin m \theta \cos \theta-m \cos m \theta \sin \theta)}{m \omega \pi\left(m^{2}-1\right) \cos \theta}=f(A) \text { (Fig.7) } \tag{6}
\end{equation*}
$$

The standard supplying power $S_{2}=3 . V_{2 \text { ph.r.m.s. }} I_{2 \text { ph.r.m.s. }}$ related to the rectified power $P_{d}=V_{d} \cdot I_{d}$, namely $S_{2} / P_{d}=f(A)$ is found from (2), (4) and is drawn in Fig. 8.

The normalized output (load) characteristic is (Fig.9)

$$
\begin{equation*}
V_{d} / \sqrt{6} V_{2 \text { ph.r.m.s. }}=\cos \theta=f\left[\pi R_{\text {rect }} I_{d} /\left(6 \sqrt{6} V_{2 \text { ph.r.m.s. }}\right)\right]=f(\sin \theta-\theta \cos \theta) \tag{7}
\end{equation*}
$$

The distortion coefficient of the supplying current, equal to the power factor of the rectifier and connected to the effectiveness of the rectification process (important nowadays) is (Fig.10)

$$
\begin{equation*}
\kappa=\frac{I_{2(1) \text { ph.r.m.s. }}}{I_{2 \text { ph.r.m.s. }}}=\frac{\sqrt{3}(\theta-0.5 \sin 2 \theta)}{\sqrt{2} D(\sin \theta-\theta \cos \theta)}=f(A) \tag{8}
\end{equation*}
$$

The ratio between the r.m.s. values of the $n$-th order harmonic component of the supplying current $(n \neq 1, n=5,7,11,13, \ldots)$ and its first order harmonic component is given by (9) (Fig. 11 - Fig.14). It can be seen that in the Fourier series of the supplying current there are no harmonic components whose order number is divisible
by 3. This fact is of certain importance for filtering the higher order harmonic components of the current and for power losses assessment in the electrical machine when the circuit under consideration is applied for rectification of the a.c. power, produced in the rotor windings by the wound rotor induction generators in simple and cheap variants of renewable energy resources, based on wind turbines.

$$
\begin{align*}
& \frac{I_{2(n) r . m . s .}^{n=5, \ldots, 1,13, \ldots}}{I_{2(1) r . m . s .}}=\frac{2 \cdot \sin (n \pi / 2) \cdot \sin (n \pi / 3)}{\sqrt{3}}  \tag{9}\\
& \quad . \frac{\sin [(n-1) \theta] /(n-1)+\sin [(n+1) \theta] /(n+1)-2 \cdot \cos (\theta) \cdot \sin (n \theta) / n}{\theta-0.5 \sin (2 \theta)}
\end{align*}
$$

## 3. Results

MATLAB programs for exact graphical representation of the main functions and for precise design implementation on a computer are created. The graphics are presented in Fig. 3 to Fig.14, as it has been already explained. A special mark shows the position of the parameters on the graphics for the concrete example (Table 1). The numerical results for a practical example of the circuit (Fig.1) from the second MATLAB program and from the PSPICE simulations are compared in Table 1.

Table 1 (Fig.1).

| $V_{d}$ | $I_{d}$ | $R_{\text {rect }}$ | $S_{\text {ripple }}$ | $P_{d}$ | $\theta$ | $A$ | $B$ | $F$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | A | $\Omega$ | $\%$ | W | deg | - | - | - |
| 506.78 | 19.372 | 1.0273 | 20 | 10000 | 22.308 | 0.0294 | 0.7643 | 12.134 |
| 506.0072 | 19.702 | 1.0273 | 19.8 | 9969.37 | 22.977 | - | - | - |
| $D_{b r}$ | $H$ | $E_{2}$ | $I_{m}$ | $I_{2}$ | $S_{2}$ | $C$ | $\kappa$ | Method |
| - | - | V | A | A | W | $\mu F$ | $\%$ |  |
| 4.4038 | 23.415 | 223.63 | 39.906 | 20.482 | 13741 | 1139.6 | 77.479 | MATLAB |
| - | - | 223.63 | 39.237 | 20.144 | 13514.4 | 1139.6 | 78.594 | PSPICE |

## 4. Conclusions

The method for electrical design of three-phase bridge diode rectifiers, assuming almost constant output voltage, is further developed. The analytical expressions of all the variables are specified. MATLAB programs for exact graphical representation of the functions, participating in the full electrical design of the rectifiers and for precise design implementation on a computer are created. They can be downloaded from: http://mail.dir.bg/~epopov/rectrc06.m (for the graphics generation) or http://mail.dir.bg/~epopov/rectrc09.m (for the computer-aided design implementation). Simulations and other methods confirm the results.

## 5. References

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Fig.3. $\theta=f(A)$.


Fig.4. $V_{2 \text { ph.r.m.s. }} / V_{d}=f(A)$.


Fig.5. $I_{m} / I_{d}=f(A)$.


Fig.6. $I_{2 \text { ph.r.m.s. }} / I_{d}=f(A)$.


Fig.7. $H_{m=6}=f(A)$.


Fig.8. $S_{2} / P_{d}=f(A)$.

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Fig.9. $\cos \theta=f(\sin \theta-\theta \cos \theta)$


Fig.10. $\kappa=f(A)$.


Fig.11. $I_{2(5) r . m . s .} / I_{2(1) r . m . s .}=f(A)$.


Fig.12. $I_{2(7) \text { r.m.s. }} / I_{2(1) \text { r.m.s. }}=f(A)$


Fig.13. $I_{2(11) r . m . s .} / I_{2(1) r . m . s .}=f(A)$


Fig.14. $I_{2(13) r . m . s .} / I_{2(1) r . m . s .}=f(A)$

