TRACKING FILTERS IN MULTISENSOR FUSION

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There is an important decision in multisensor problem which fusion type we choose: centralized or decentralized, inside a sensors net. The scenario is to tracking a target with a Kalman filter, having a sensors net as the information sources. We make a comparison between centralized and decentralized schemes by observing the accuracy of tracking, namely MSE (Mean Square Error) of the state estimation for some situations.

Keywords: Kalman filtering, multisensor fusion

1. INTRODUCTION

Data fusion from a number of sensors represents the available sensor data processing method in an optimal manner relating to specific criterion to obtain an enhanced information quality about a target.

An important problem is: the fusion should be made in a central computing node or in a distributed manner. *Centralized fusion* means that we can access to all the measurements of the optimal filter. Contrary in the *decentralized fusion* a filter is applied to every measurement and the global fusion process has access only to the estimates and their errors covariances. Decentralized fusion has particular advantages such as the case of detecting a fault; the various subsystems may apply a "vote" strategy to detect the fault. An obvious disadvantage is the growing number of signaling, in the view of the state, in particular its covariance matrix, often has bigger size relative to measurements vector. There are arguments that the global processor may process the measurements in a decentralized manner to obtain the advantages of both variants. However many sensor systems have embedded filters.

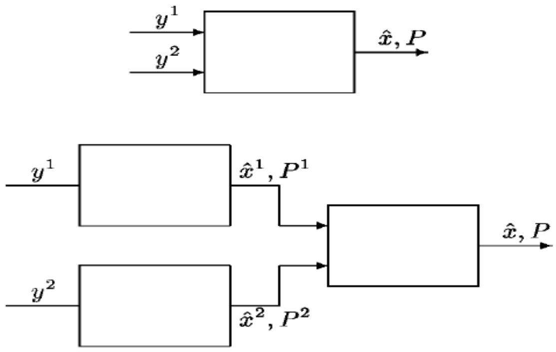

Fig. 1 Centralized and decentralized filtering concepts

2. CENTRALIZED KALMAN FILTERING – CKF

This section deals with CKF technique and model. The models are built according to [2]. In centralized Kalman filtering, signals of sensors are transferred through the communication network to the central processor to generate the optimal central estimate $x(n|n)$. The all information is sent to the fusion center to yield $x(n|n)$ and minimize state estimation error.

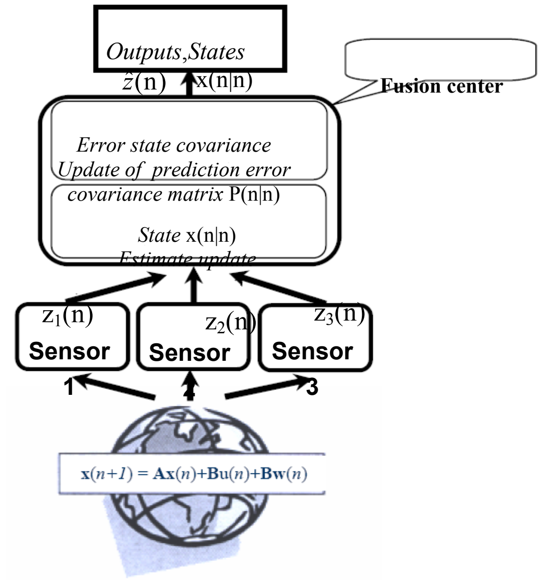

Fig. 2 Centralized Kalman filter topology

All sensors are measuring outputs of plant – of course in the multiradar case these outputs are represented by the data streams from each radar system. We'll consider the same value for the control input $u(n)$ to every estimator of CKF. Three sensors presented in Figure 2 produce observations $z_1(n)$, $z_2(n)$ and $z_3(n)$ which are necessarily not the same. The observations are sent from each sensor to fusion centre that performs central state estimation, see Figure 3-1. The concept of CKF technique was analyzed in the paper works [2], [5].

The initial time n_0 is the formal time when processor does not process first sample but starts an initial program (it loads initial values which we give it).

3. Decentralized Kalman Filtering - DKF

The decentralized Kalman filter processes data from many sensors to provide a global state estimation in multi-sensor fusion. A DKF model was built according to references [1], [3] – [5]. Every DKF contains a local and a global filter that emphasises double-estimation in a node. The local filter uses its own data and observation $z(n)$ to perform an optimal local estimates $\boldsymbol{P}(n|n)$ and $\boldsymbol{x}(n|n)$.

These estimates are obtained in a parallel processing mode implicitly; thus, each node takes observation (possibly asynchronously) from a plant of an environment. With this observation (and its associated variance) the DKF is able to compute a partial state estimate. Then each node broadcasts one vector and a matrix of error information to the others and it receives other information being broadcasted to it. Those two (one vector and a matrix) as state error information $\boldsymbol{SEI}(n)$ and variance error information $\boldsymbol{VEI}(n)$ are used for global state and covariance estimate in global filter of every node. Finally, all nodes performed same global state estimates $\boldsymbol{x}(n|n)$ because $\boldsymbol{SEI}(n)$ and $\boldsymbol{VEI}(n)$ matrices are fused in the same way. The DKF been recently described corresponds to Figure 3. Generally, the number of sensors is not restricted as well as Figure 3 described; all depends only by the network transmission capacity and local processing power in each node.

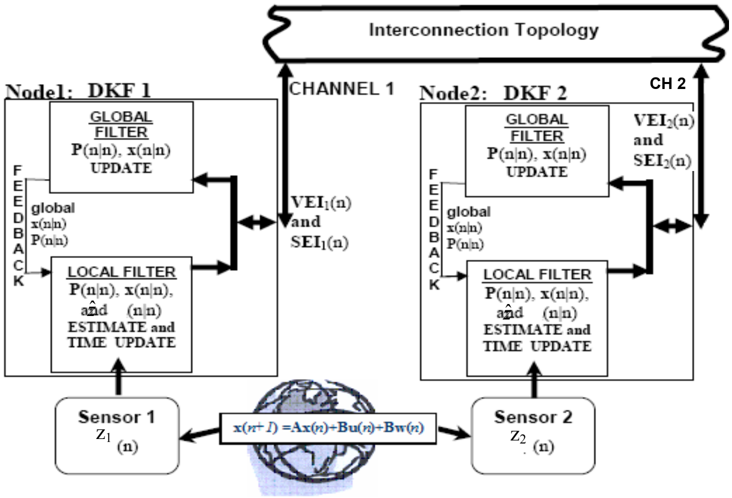

Fig. 3 Decentralized Kalman filter topology

The local filter of the *i*-th node computes its own local estimate updates using residuals and an innovation KF gain, respectively. The time updates are also performed by a local filter of each *i*-th DKF node. The local filter of the *i*-th DKF node works independently and indirectly from the other nodes and all their residuals differ. After the local filter was processed, the ***SEI***(*n*) and ***VEI***(*n*) are computed by assimilation equations. So, every nodal DKF computes its own part such as the *i*-th part that is called fractional matrix. These fractional matrices are sent both to other nodes where are collected to get global ***SEI***(*n*) and ***VEI***(*n*).

In each processor (node), a feedback process is running, where the global filter sends a global updated estimates $\boldsymbol{x}(n|n)$ and $\boldsymbol{P}(n|n)$ covariance matrix into its own local filter. This way, the $\boldsymbol{x}(n|n)$ and $\boldsymbol{P}(n|n)$ are interchanged with $\boldsymbol{x}_i(n|n)$ and $\boldsymbol{P}_i(n|n)$, respectively. Finally, the state estimate updates are evaluated in the same manner in all nodes.

Each DKF estimator can be embedded in a sensor. The advantage of DKF against CKF is an embedded processor of DKF in sensor, hence no central fusion is needed. In DKF node the observations are used directly. In sense of estimation, the advantage of DKF is also the less sensitive estimator to corrupted SEI(n) and VEI(n) when corrupted broadcasting happens. In other words, each sensor node has its own processing element, and its own communication facilities. Each node must communicate one SEI(n) vector and one matrix VEI(n) to each other node. Assuming there are No sensing nodes and each node estimates a full state vector of dimension l, then a total of (l2 + l)(N0 - 1) numbers need to be communicated in each cycle. In CKF systems (central fusion) only No numbers are equal to the number of sensors need to be broadcasted, [1].

4. Centralized and Decentralized Kalman Filter Comparison

In this section we will dedicate the effort to explore identity between decentralized, DKF, and centralized, CKF, Kalman filter which can be measured by mean square error MSE in state estimates in an experimental simulation study. Both models are created according to [1], [3-5].

In order to evaluate the KF performance, the MSE factor will be used to measure state estimation accuracy and it has the following expression:

$MSE = E[\boldsymbol{a}^2(n)] - \{E[\boldsymbol{a}(n)]\}^2$

Variable $\boldsymbol{a}(n)$ will depends of

$\boldsymbol{a}(n) = \boldsymbol{x}(n) - \boldsymbol{x}(n|n)$

A stochastic system model took from the literature will be describe by the following:

$$x(n+1) = \begin{bmatrix} 1{,}1269 & -0{,}4940 & 0{,}1129 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(n) + \begin{bmatrix} 0{,}3832 \\ 0{,}5919 \\ 0{,}5191 \end{bmatrix} [u(n) + w(n)]$$

$$z(n) = [1 \quad 0 \quad 0] x(n) + v(n)$$

where $w(n)$ şi $v(n)$ are the process and observation white noise (zero mean), respectively. This is the third order filter of linear finite-dimensional stochastic system model defined in discrete time domain. Constant variances of process and sensor noise are namely given by (took from the literature as an example):

Q = $3{,}669 \times 10^{-2}$ $\boldsymbol{I}$, high power of a process noise w(n)

Q = $3{,}669 \times 10^{-4}$$\boldsymbol{I}$, low power of a process noise w(n)

$\boldsymbol{R} = 8{,}69 \times 10^{-4}$

Those noises are assigned to realistic approach, where both are usually dominated in a low frequency band. The observation $\boldsymbol{H}$ matrix will be considered as

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Time n will be considered from 0 to 100 in Matlab simulation.

A relative MSE of a difference between DKF and CKF state estimates $\boldsymbol{x}(n|n)$ is shown at last row of the following table. The value in first row is related to difference between $\boldsymbol{x}(n)$ and $\boldsymbol{x}(n|n)$ of CKF state estimates. The second item explains a relative MSE of DKF computed in such way as the previous MSE in the first row.

(MSE of CKF) / ($\mathbf{x}^2$(n) of plant)	$3{,}420 \times 10^{-4}$
(MSE of DKF) / ($\mathbf{x}^2$(n) of plant)	$3{,}416 \times 10^{-4}$
($\mathbf{x}(n\|n)_{DKF}$ - $\mathbf{x}(n\|n)_{CKF}$)^2 / ($\mathbf{x}^2$(n) of plant)	$3{,}429 \times 10^{-7}$

Table 1 MSE of DKF; MSE of CKF; MSE of estimated state **x**(n|n) (DKF-CKF) comparison

Both values are very similar to satisfy the numerical identity among CKF and DKF. To make relevant comparison on estimators is to compute a difference between state vectors $\boldsymbol{x}(n|n)$ of DKF and CKF, see the last value. This relative MSE is about 10^3 times lower than the first two values. Actually, this value is expected to be small, because the DKF and CKF provide mathematically equal states. As an example, the Table 2 shows some sampled values of state vectors at $n = 2$.

x(n|n) of CKF	[-1,457; -0,719; -0,061]
x(n|n) of DKF	[-1,457; -0,719; -0,061]
x(n|n) of plant	[-1,471; -0,729; -0,057]

Table 2 State vectors of Model 3 of CKF, DKF and plant at n = 2

One can see no numerical difference in state elements of both estimators due to low precision. The last item shows the ***x***(*n*) of plant that is on numerical comparability with ***x***(*n*|*n*) of CKF and DKF. In this simulation, two sensors and two nodes of DKF estimator called DKF1 and DKF2 are shown. As shown in the last item of Table 1, the relative MSE is measured in the case of DKF1 and DKF2 and shown below in Table 3.

$\Sigma[(\mathbf{x}(n\|n)_{DKF1} - \mathbf{x}(n\|n)_{CKF})^2]$ / ($\mathbf{x}^2$(n) of plant)	$3{,}429 \times 10^{-7}$
$\Sigma[(\mathbf{x}(n\|n)_{DKF2} - \mathbf{x}(n\|n)_{CKF})^2]$ / ($\mathbf{x}^2$(n) of plant)	$3{,}429 \times 10^{-7}$

Table 3 DKF verification toward to CKF result based on stated measurement

We have measured the relative MSE about 10-2 in state estimates of CKF and DKF. Measured MSE of ***x***(*n*|*n*) difference between DKF1, DKF2 and CKF is about 10^3 smaller than the value mentioned firstly.

In the practical point, we can say that both estimators perform on the same state estimation despite the unimportant difference in our MATLAB simulations. In our simulations all signals are broadcasted without any unknown latency or transmission failure in multi-sensor fusion.

5. One Biased Sensor in DKF Model

The aim of this unit is to make some experiments on DKF estimator that contains a sensor affected by a bias. Then an error seen on such affected state estimation would be measured in MATLAB. Thus a relation of a total number of sensors and MSE will be explored. We have the hypothesis that the system noises as v_1(n), v_2(n) etc. are uncorrelated their observation covariance ***R***(n) are equal. Will be considered DKF model that contains two or five sensors, respectively; The first sensor noise is additively coupled with noise v_1(n) and an existed bias. The level of constant bias equals to 10, with n = 0, 1, …, 1000.

5.1 Two sensors in DKF model

A bias handling ability is identified by measuring the MSE of state estimation optionally with two power levels of process noise mentioned in tables below.

Constant bias = 10	
MSE of DKF state vector	1,014

Table 4 Two nodes decentralized Kalman filter result, high power of w(n)

Constant bias = 10	
MSE of DKF state vector	$4{,}312 \times 10^{-2}$

Table 5 Two nodes decentralized Kalman filter result, low power of w(n)

The setting of value in ***Q***(*n*) must correspond truly to covariance of *w*(*n*). If not so, the state estimation may be not optimal. Moreover as we can see in the figure, the bias influence works against the optimal state estimation. Making the ***Q***(*n*) lower than the appropriate covariance of *w*(*n*), the state estimation gives low accuracy.

5.2 Five sensors in DKF model

This experiment is related to the one presented above but five sensors are taken. Here the ***H*** matrix has the size of [5x3] with all rows been the same.

Constant bias = 10	
MSE of DKF state vector	$1{,}622 \times 10^{-1}$

Table 6 Five nodes decentralized Kalman filter result, high power of w(n)

Constant bias = 10	
MSE of DKF state vector	$7{,}033 \times 10^{-3}$

Table 7 Five nodes decentralized Kalman filter result, low power of w(n)

Here the goal is to find a relation of MSE and the total number of sensors in DKF model affected by bias. We would like to compare the MSE_{2DKF} of DKF model consist of two sensors and five sensors MSE_{5DKF}. Hence the MSE_{2DKF} model is higher than MSE_{5DKF} and their ratio

$$\sqrt{MSE_{2DKF} / \ MSE_{5DKF}} = \sqrt{6{,}252} = 2{,}5$$

The ratio of total number of sensors is 2.5, which gets numerically near the computed value. In MATLAB simulation with low power of $w(n)$, the ratio is equal to 2.476 been near the ratio of total number of sensors. The point is that the fusion in DKF model works as an averaging sum.

So far we tried to answer to the question "How does the MSE depend on the total number of sensors?". In this experimental study we explored the problem of one biased sensor occurred in DKF model that contains two and five sensors as a total. This relation can be explained using the example with two and five sensors models, where $\sqrt{MSE_{2DKF} / \ MSE_{5DKF}}$ is equal to number of sensors ratio given by 5/2. The DKF model works as an averaging sum in fusion. The power of plant state was 1.25, power of constant bias was 100. In other words, the power of bias was higher than the power of state. High power of bias may cause catastrophic state estimation in Kalman-Filter estimation. The 100-times big change of $\boldsymbol{Q}(n)$ makes 23-times changed MSE for the test with constant bias.

6. One Broken Sensor in DKF Model

The purpose of this unit is to obtain a relation of MSE of state estimation measured in DKF model and a total number of sensors. Only one sensor can be broken within total number of sensors. In our model simulations a broken sensor provides zero value instead of its eventual correct filtered output $z(n)$, where n = 0, 1, ..., 1000 . This zero provided value is accepted by DKF estimator.

The model of DKF comprises of two and five sensors, respectively. In each section, two experiments are optionally performed with high and low power level of process noise. The first section shows some results of a problem mentioned above, where two sensors are associated with one system. The second section presents some results with five sensors. Matrix $\boldsymbol{H}$ has [5×2] size. MSE and $\boldsymbol{a}(n)$ are computed according to modality in section 3.

6.1 Testing the DKF with two sensors

This section presents two tables assuming different power levels of process noise.

Broken sensor	
MSE of DKF state vector	$3{,}597 \times 10^{-1}$

Table 8 DKF result in two sensors test of broken sensor, high power of w(n)

Broken sensor	
MSE of DKF state vector	$1{,}380 \times 10^{-1}$

Table 9 DKF result in two sensors test of broken sensor, low power of w(n)

6.2 Testing the DKF with five sensors

In the same way with previous section, the MSE is measured on five sensors system. We are about to compare the MSE in Table 8 called MSE_{2DKF} and MSE_{5DKF} in Table 10.

Broken sensor	
MSE of DKF state vector	$5{,}772 \times 10^{-2}$

Table 10 DKF result in five sensors test of broken sensor, high power of w(n)

Broken sensor	
MSE of DKF state vector	$2{,}206 \times 10^{-2}$

Table 11 DKF result in five sensors test of broken sensor, low power of w(n)

There MSE_{2DKF} is higher than MSE_{5DKF} of five sensors model and their ratio $\sqrt{MSE_{2\,DKF}/MSE_{5\,DKF}}$ yields $\sqrt{6{,}232}$ = 2.496. It is computed by the assumption of high power of $w(n)$ where its covariance is time-invariant. This ratio is numerically close to theoretical value given by 5/2 as the five to two ratio (a ratio of total number of sensors). This rule is valid also in a case of low power of process noise with its covariance matrix $\boldsymbol{Q}$, where the $\sqrt{MSE_{2\,DKF}/MSE_{5\,DKF}}$ ratio is $\sqrt{6{,}232}$ = 2.501.

7. Conclusions

In the practical point, we can say that both estimators perform on the same state estimation despite the unimportant difference in these MATLAB simulations. In my simulations all signals are broadcasted without any unknown latency or transmission failure in multi-sensor fusion. The MSE relation for constant bias case can be explained using the example with two and five sensors models, where $\sqrt{MSE_{2DKF}/\ MSE_{5DKF}}$ is equal to number of sensors ratio given by 5/2. The DKF model works as an averaging sum in fusion.

It was also presented in which way MSE depends of total number of plant sensors for the case of DKF model.

8. References

[1] Rao B. S., Durrant-Whyte H. F., *Fully decentralized algorithm for multisensor Kalman filtering*, IEEE PROCEEDINGS-D, Vol. 138, No. 5, pp.413-420, September 1991.

[2] Pao L. Y., Baltz N. T., *Control of Sensor Information in Distributed Multisensor Systems*, Proc. American Control Conference, San Diego, CA,pp. 2397-2401, June 1999.

[3] Durrant-Whyte H. F., Leonard J. J., *Toward a fully decentralized architecture for multi-sensor data fusion*, Robotics and Automation, 1990. Proceedings., IEEE International Conference, Vol.2, pp. 1331 – 1336, 13-18 May 1990.

[4] Durrant-Whyte H. F., Rao B. Y. S.,HU H., *Toward a fully decentralized architecture for multi-sensor data fusion*, Principles and Applications of Data Fusion, IEEE Colloquium, pp.2/1 - 2/4, 4 Feb. 1991.

[5] Durrant-Whyte H. F., *Elements of sensor fusion*, Intelligent Control, IEE Colloquium, pp.5/1 - 5/2, 19 Feb. 1991.