Subtraction Method for Removing a Powerline Interference from ECG : Case of Powerline Frequency Deviation and Non-Multiple Samlping

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Extension of the subtraction method for removing from ECG signals a power-line interference, which deviates around rated value and non-multiple sampling rate, is proposed. A permanent criterion for finding linear segments in ECG is used and a new modification of the linear criterion is introduced that retains needed features for non-multiple sampling. Simplified formulas for extrapolating the interference in non-linear segments of ECG and for dynamic recalculation of filter's coefficients are derivate too. A simplified algorithm is elaborated and experimented. The existing version is improved and adapted to the dynamic deviations around the rated power-line frequency. Experiments with 60 Hz power-line frequency show that the proposed procedure successfully compensates abrupt and gradual changes of the power-line frequency allowed by the standards.

Keywords: Digital filtering, ECG filtering, Interference rejection.

1. Introduction

The subtraction method for removing the power-line (PL) interference from ECG signals [1, 2, 4] shows high efficiency when the sampling rate frequency Φ and the PL frequency F are synchronized, i.e. when $n = \Phi/F$ is an integer. The algorithm of the method includes three main stages:

1. Each current sample X_i of the ECG signal is checked whether it belongs to a linear segment. The introduced criterion of linearity is $|D| \le M$, where $|D|$ represents a second difference between signal samples. The threshold M is practically chosen value. More often the second difference is used

$$D_i = \left(X_{i-n} - X_i\right) - \left(X_i - X_{i+n}\right) = X_{i-n} - 2X_i + X_{i+n}, \tag{1}$$

where first differences are taken by samples located at distance of one period of the PL frequency, thus eliminating the interference influence.

2. If the current sample belongs to a linear segment, the interference is removed by a 'moving average' filter. The sample Y_i (free of interference) is obtained by:

$$Y_i = \frac{1}{n}\sum_{j=-m}^{m} X_{i+j}, \quad n = 2m+1; \quad Y_i = \frac{1}{n}\left(\sum_{j=-m+1}^{m-1} X_{i+j} + \frac{X_{i-m} + X_{i+m}}{2}\right), \quad n = 2m. \tag{2}$$

A simple subtraction of filtered from non-filtered sample gives the value of the current interference $B_i = X_i - Y_i$, which is stored in a temporal buffer, i.e.

$$B_i = X_i - Y_i. \tag{3}$$

3. If the current sample does not belong to linear segment, a preceding value B_{i-n} which corresponds to the same interference phase is taken from the buffer

$B_{i-1}, B_{i-2}, \ldots, B_{i-n}$. This value is subtracted from the contaminated current sample

$$Y_i = X_i - B_{i-n}. \tag{4}$$

and is stored back at the current position in the buffer.

The theoretic base of applying the subtraction method to the general case of non-multiplicity between F and Φ is consecutively developed in [3, 4].

Paper [5] elaborates a modification of the subtraction method in the case of PL frequency deviation over its nominal value F, witch is multiple to the sampling rate Φ. PL frequency variations are considered as temporal non-multiplicity between Φ and PL frequency. Experiments with odd multiplicity (n=2m+1) show that the modification successfully compensates the PL frequency variations, except for the cases of abrupt changes. Later experiments show that the procedure for interference extrapolation in non-linear segments of ECG is prone to autoexcitation in some specific cases of even multiplicity (n=2m) and in [6] a new formulae for those cases are derived.

2. THEORY, METHOD AND ALGORITHM

The aim of this study is to elaborate the subtraction method in case of PL frequency deviation around rated value and non-multiple sampling rate, i.e. when Φ/F is a real number. The round value n^* is used, which is the greatest integer less than or equal to Φ/F. Two sub-cases could be pointed, $n^*=2m+1$ (odd non-multiplicity) and $n^*=2m$ (even non-multiplicity). The applying of the subtraction method in case of non-multiplicity requires an adaptation of the three above mentioned main stages of the algorithm [4].

2.1 Modification of the linearity criterion

The Eq.(1) is a digital filter (called **D**-filter), having a frequency response

$$D(f) = -4\sin^2 \frac{n\pi f}{\Phi}. \tag{5}$$

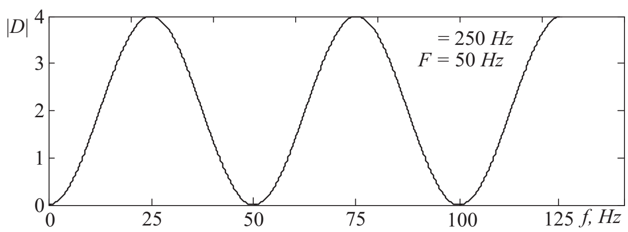


Fig.1. Frequency response of **D**-filter.

Experiments in [5] show that the **D**-filter does not need to be dynamically corrected during the procedure of PL frequency variation compensation, because of the very small influence of the PL frequency deviation on the value of $D(f)$ at frequency F. This is the reason we use a permanent **D**-filter.

According [3, 4] the **D**-filter modification for non-multiplicity is presented by

$$D_i^* = D_i + A_i \frac{D_F}{A_F}, \tag{6}$$

where A_i is an auxiliary FIR filter with frequency response with zero in $f = 0$. D_F and A_F are transfer coefficients of the corresponding filters for the PL frequency F. Using simple **D**- and **A**-filters, additional zeroes (except for $f = 0$ and $f = F$) are included. The frequency response of the **D***-filter becomes different (asymmetrical) for both sides in the vicinity of F and is not suitable for linear segment detection with variable PL frequency (see Fig.2).

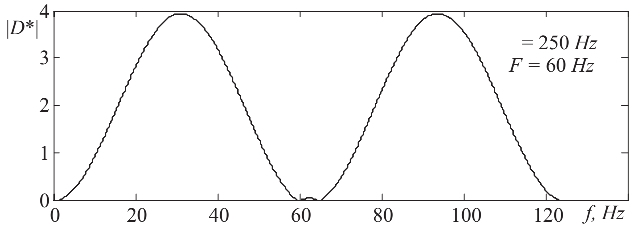


Fig.2. Simple modification of **D***-filter.

A more complex **D**- and **A**-filters must be used for synthesis an appropriate **D***-filter according Eq.(6). In case of non-multiplicity the basic **D**-filter by Eq.(1) includes virtual samples $X_{i-\Phi/F}$ and $X_{i+\Phi/F}$ and $D_i = X_{i-\Phi/F} - 2X_i + X_{i+\Phi/F}$. By applying a linear interpolation between the two surrounding real samples we may substitute $\left| \begin{array}{l} X_{i+\Phi/F} = X_{i+n*}(1-\Phi/F+n*) + X_{i+n*+1}(\Phi/F-n*) \\ X_{i-\Phi/F} = X_{i-n*}(1-\Phi/F+n*) + X_{i-n*-1}(\Phi/F-n*) \end{array} \right.$, thus resulting in

$$D_i = \left(X_{i-n*} + X_{i+n*}\right)\left(1 - \frac{\Phi}{F} + n*\right) + \left(X_{i-n*-1} + X_{i+n*+1}\right)\left(\frac{\Phi}{F} - n*\right) - 2X_i, \quad (7)$$

with a frequency response

$$D(f) = -4\sin^2\frac{n*\pi f}{\Phi}\left(1 - \frac{\Phi}{F} + n*\right) - 4\sin^2\frac{(n*+1)\pi f}{\Phi}\left(\frac{\Phi}{F} - n*\right). \quad (8)$$

The frequency response is shown in Fig.3 (curve *a*). It retains a zero for $f = 0$, but for $f = F$ it has just a local minimum.

A three point filter [4] is used as an auxiliary $A_i = \frac{-X_{i-\Phi/2F} + 2X_i - X_{i-\Phi/2F}}{4}$. The filter equation includes virtual samples $X_{i-\Phi/2F}$ and $X_{i+\Phi/2F}$, which could be substituted $\left| \begin{array}{l} X_{i+\Phi/2F} = X_{i+m*}(1-\Phi/2F+m*) + X_{i+m*+1}(\Phi/2F-m*) \\ X_{i-\Phi/2F} = X_{i-m*}(1-\Phi/2F+m*) + X_{i-m*-1}(\Phi/2F-m*) \end{array} \right.$, using a similar linear interpolation (the round value $m*$ is the greatest integer less than or equal to $\Phi/2F$). The auxiliary filter equation becomes

$$A_i = -\left(X_{i+m*} + X_{i-m*}\right)\left(1 - \frac{\Phi}{2F} + m*\right)\frac{1}{4} - \left(X_{i+m*+1} + X_{i-m*-1}\right)\left(\frac{\Phi}{2F} - m*\right)\frac{1}{4} + \frac{X_i}{2}, \quad (9)$$

having a frequency response

$$A(f) = -\sin^2\frac{m*\pi f}{\Phi}\left(1 - \frac{\Phi}{2F} + m*\right) - \sin^2\frac{(m*+1)\pi f}{\Phi}\left(\frac{\Phi}{2F} - m*\right), \quad (10)$$

which is shown in Fig.3 (curve *b*). The resulted **D***-filter (curve *c*) has zeroes for $f = 0$ and for $f = F$. More over, it retains a close to zero response almost symmetrical around the vicinity of F, where the PL interference deviation is disposed.

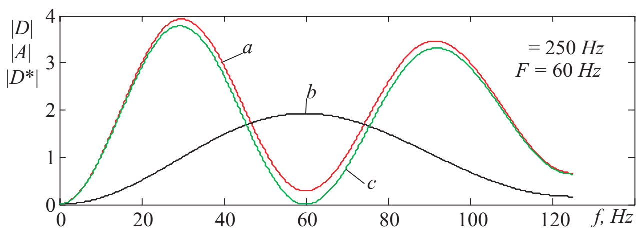


Fig.3. Complex modification of **D***-filter.

In the experiments the complex criterion $|D_i| \vee |D_{i-1}| \leq M$ is used [2].

2.2 Modification of the filter applied to linear segments

The filter used in linear segments Eq.(2) was called **K**-filter. Its frequency response is

$$K(f)=\frac{1}{n}\sin\frac{n\pi f}{\Phi}\Big/\sin\frac{\pi f}{\Phi},\quad n=2m+1;\quad K(f)=\frac{1}{n}\sin\frac{n\pi f}{\Phi}\Big/\tan\frac{\pi f}{\Phi},\quad n=2m. \quad (11)$$

The modified **K**-filter is given by:

$$Y^*_i=Y_i-(X_i-Y_i)\frac{K_F}{1-K_F}=X_i-\frac{X_i-Y_i}{1-K_F}=X_i-\frac{B_i}{1-K_F}; \quad (12)$$

where K_F is the transfer coefficient of the **K**-filter for the PL frequency F Eq.(4). The frequency response of the modified **K***-filter is

$$K^*(f)=K(f)/(1-K_F)-K_F/(1-K_F). \quad (13)$$

The value of the current interference sample B^*_i, which is stored in a temporal buffer, is given by

$$B^*_i=X_i-Y^*_i=\frac{B_i}{1-K_F}. \quad (14)$$

2.3 Modification of the extrapolation procedure restoring the interference in the non-linear segments

The extrapolated value of the B^*_i is estimated by the buffer content, which is processed by additional filter of 'moving average' type (marked as $\mathbf{K_B}$-filter) having a transfer coefficient K_{BF} for $f=F$. The applcation of the $\mathbf{K_B}$-filter on the content of the temporal buffer results $\frac{1}{n^*}\sum_{j=-n^*+1}^{0}B^*_{i+j}=B^*_{i-(n-1)/2}K_{BF}$ where $B^*_{i-(n-1)/2}$ is a virtual sample in the middle of the averaged interval. B^*_i can be calculated from this equation $B^*_i=B_{i-(n-1)/2}n^*K_{BF}-\sum_{j=-n^*+1}^{-1}B^*_{i+j}$. A much more simple equation can be obtained if we express by the same way $B^*_{i-n^*}=B^*_{i-(n^*-1)/2-1}n^*K_{BF}-\sum_{j=-n^*+1}^{-1}B^*_{i+j}$.

Subtracting two equations results in

$$B^*_i=B^*_{i-n^*}+n^*K_{BF}\left(B^*_{i-(n^*-1)/2}-B^*_{i-(n^*-3)/2}\right),\quad n^*=2m+1. \quad (15)$$

where $K_{BF}=K_F$.

If n^* is even, the virtual $B^*_{i-(n-1)/2}$ does not coincide with a real sample and in [6] it is proposed an approximation by $B^*_{i-(n^*-1)/2}=\left(B^*_{i-n^*/2}+B^*_{i-n^*/2+1}\right)\Big/2S_C$, where $S_C=\cos\frac{\pi F}{\Phi}$. The same way $B^*_{i-(n^*-1)/2-1}=\left(B^*_{i-n^*/2-1}+B^*_{i-n^*/2}\right)\Big/2S_C$, which gives

$$B^*_i=B^*_{i-n^*}+\left(B^*_{i-n^*/2+1}-B^*_{i-n^*/2-1}\right)\frac{n^*K_{BF}}{2S_C},\quad n^*=2m, \quad (16)$$

where $K_{BF} = \frac{1}{n^*} \cdot \frac{\sin \frac{n^* \pi F}{\Phi}}{\sin \frac{\pi F}{\Phi}} = \frac{1}{n^*} \cdot \frac{\sin \frac{n^* \pi F}{\Phi}}{\tan \frac{\pi F}{\Phi}} \cdot \frac{1}{S_C} = \frac{K_F}{S_C}$.

It can be seen, that in case of even non-multiplicity $K_F = K_{BF} S_C$.

In case of odd non-multiplicity the B_{mid} coincides with a real buffer sample and Eq.(18) results in

$$B_i^* = B_{i-n^*}^* + n^* K_{BF} \left(B_{i-(n^*-1)/2}^* - B_{i-(n^*-1)/2-1}^* \right), \quad n^* = 2m+1, \qquad (17)$$

where $K_{BF} = K_F$.

2.4 Modification of the algorithm

The algorithm offers coefficient K_F to be dynamically recalculated during processes of the interference removing from linear segments. According to Eqs.(16, 17), coefficients K_{BFnew} and K_{Fnew} have to be computed by:

$$K_{BFnew} = \frac{\left(B_i^* - B_{i-n^*}^* \right)}{n^* \left(B_{i-(n^*-1)/2}^* - B_{i-(n^*-3)/2}^* \right)}; \quad K_{Fnew} = K_{BFnew}, \quad n^* = 2m+1$$

$$K_{BFnew} = \frac{2 S_C \left(B_i^* - B_{i-n^*}^* \right)}{n^* \left(B_{i-n^*/2+1}^* - B_{i-n^*/2-1}^* \right)}; \quad K_{Fnew} = K_{BFnew} S_C, \quad n^* = 2m \qquad (18)$$

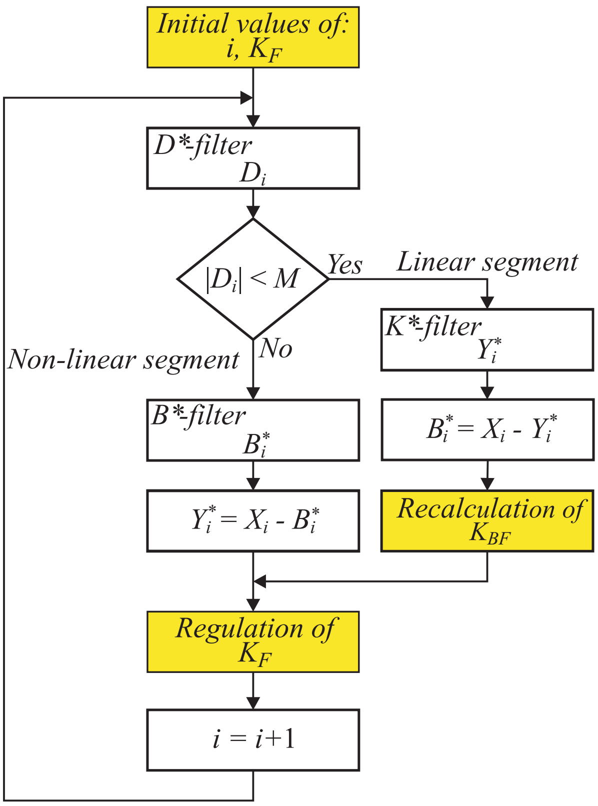


Fig.4. Flow chart of the algorithm.

2.5 Evaluation of the PL frequency deviation

The calculated coefficient K_{BFnew} could be used to evaluate the deviation dF of the PL frequency

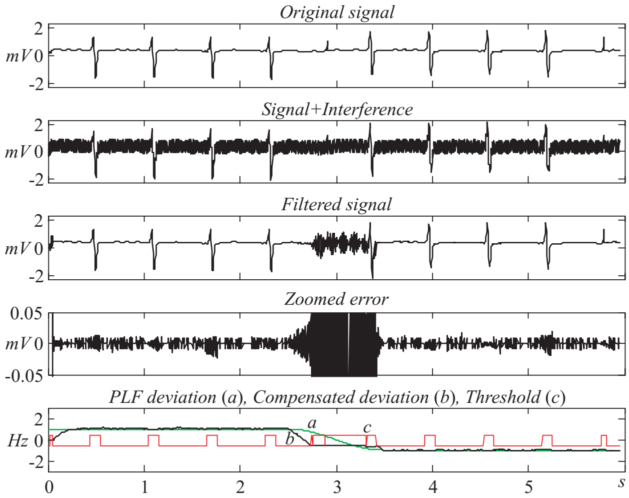
$$dF \approx \frac{\Phi \left(K_{BF} - K_{BFnew} \right)}{\pi \left(1 - K_{BF} \right)} \sin \frac{\pi F}{\Phi}. \qquad (19)$$

The flow chart of the procedure for compensating the PL frequency variations is shown in Fig.1. Introduced restrictions are defined in [5]: protection against division by zero in Eq.(18); limitation of the maximal speed of K_{Fnew}; keeping within the allowed range of PL frequency variation; proportionally integral rule of feedback control

$$K_F = \frac{2n-1}{2n} K_F + \frac{1}{2n} K_{Fnew}.$$

3. EXPERIMENTAL RESULTS

Fig.3 shows results of testing the algorithm for removal of changeable PL interference over a non-multiplied rated value $F = 60$ *Hz* with sampling rates $\Phi = 250$ *Hz* (left plot) and $\Phi = 500$ *Hz* (right plot). A smooth PL frequency change within 1 *s* from 59 to 61 *Hz* is simulated in the middle of the epoch. The last trace in fig. 2a shows the PL frequency (curve *a*), the estimated diversion of the PL frequency (curve *b*) and the criterion for linearity (curve *c*). The error committed is less than ±25 μV.



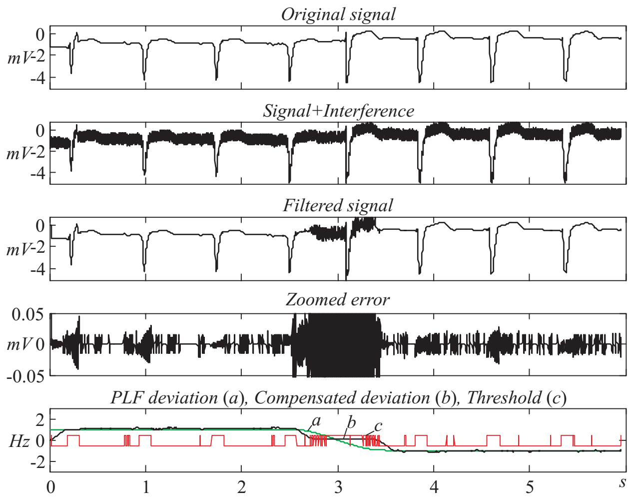


Fig.3. Automatic compensation of the PL frequency variation around a non-multiplied rated value.

4. CONCLUSIONS

The article develops the subtraction method for removing a PL interference from ECG in case of non-multiple sampling and power line frequency deviation. A new modification of the linear criterion is introduced that retains all needed features for non-multiplied sampling. Simple equations for interference extrapolating in non-linear segments of ECG and for dynamically calculation of filter's coefficients are worked out too. Experiments with even non-multiplicity show that the method successfully compensates the PL frequency variations.

5. REFERENCES

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