SOME METHODS TO IMPROVE THE EVALUATION OF AMPLITUDE SPECTRUM GIVEN BY FFT

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FFT gives satisfactory evaluation of amplitude spectrum only when the frequency of the fundamental is an integer (or very close to integer) multiple of frequency resolution. Usually we do not know precisely the frequency of the power converters feeding machines in electric drives. This is why we cannot adjust the frequency resolution during the measurements. The paper proposes three methods to improve the evaluation of the fundamental frequency and amplitude of a signal by FFT. In this way the advantages of FFT are preserved and the disadvantages are compensated by efficient techniques. The applications field of these techniques can be the power measurement in the electrical drives where the electrical machine is fed by frequency converters. We outline that two of the three proposed methods are insensitive to the noise content of the signal.

Keywords: Fast Fourier Transform, Leakage and grid effect, Optimisation

1. INTRODUCTION

The evaluation of the performance of an electrical load fed by industrial network claims the measure of voltages, currents and power. In this case the frequency of fundamentals (for voltages and currents) is known with a good precision, the very small variations of frequency have a reduce influence on the final results of measure. Usually a frequency resolution of 1Hz is good enough. The amplitude spectrum can be trusted because it is clean, without grid effect or leakage.

If the load is fed by a voltage or current converter, the frequency of fundamental is unknown, even in cases where we try to keep it constant. The best example is offered by a speed control system in an electric drive with a.c. machines. The imposed value is the speed and the frequency of feeding voltages is a secondary parameter, anyway variable with the load torque. Now the frequency resolution of the data acquisition system is constant and impossible to match with the fundamental frequency. The amplitude spectrum is not clean and we cannot trust it. The discontinuities introduced by a finite record of signal produce leakage of spectral information, resulting in a discrete-time spectrum that is a smeared version of the original continuous-time spectrum.

The paper proposes some methods to recover the true value of the signal parameters by using the same environment to acquire and post-process the data.

2. PRESENTATION OF THE PROBLEM

We consider a voltage or current signal that contains the fundamental harmonic

$$y_1 = A\sqrt{2}\sin(2\pi f_1 t + \varphi_1) \tag{1}$$

The signal is acquired with a sampling frequency f_s and N samples, that is with a frequency resolution $\Delta f = f_s / N$. If the frequency f_1 of the fundamental harmonic is not a multiple of frequency resolution Δf we can write

$$f_1 = (k+\alpha)\Delta f \text{ where } \alpha \in (0,1) \tag{2}$$

Let us evaluate the error given by harmonic analysis when α takes different values. The Fourier Transform of the signal y_1 with N samples is

$$F(\omega) = \int_0^{N\Delta t} A\sqrt{2}\sin(2\pi f_1 t)\exp(-j\omega t)dt \tag{3}$$

where Δt is the time pitch. The real and imaginary components of the integral (3) are given below:

$$\begin{aligned}\Re[F(\omega)] &= \frac{A}{\sqrt{2}}\left[\frac{1-\cos(\omega_1+\omega)N\Delta t}{\omega_1+\omega} + \frac{1-\cos(\omega_1-\omega)N\Delta t}{\omega_1-\omega}\right]\\ \Im[F(\omega)] &= \frac{A}{\sqrt{2}}\left[\frac{\sin(\omega_1-\omega)N\Delta t}{\omega_1-\omega} - \frac{\sin(\omega_1+\omega)N\Delta t}{\omega_1+\omega}\right]\end{aligned} \tag{4}$$

Let us put the discrete „exploring frequency" ω in the form (5) where m is an integer $m \in (0, N/2)$

$$\omega = 2\cdot\pi\cdot\Delta f\cdot m \tag{5}$$

With

$$\begin{aligned}\omega_1 + \omega &= 2\pi\Delta f(k+\alpha+m)\\ \omega_1 - \omega &= 2\pi\Delta f(k+\alpha-m)\end{aligned} \tag{6}$$

we can transform (4) in the form:

$$\begin{aligned}Ac_m &= A\left[\frac{1-\cos 2\pi(k+\alpha+m)}{2\pi(k+\alpha+m)} + \frac{1-\cos 2\pi(k+\alpha-m)}{2\pi(k+\alpha-m)}\right]\\ As_m &= -A\left[\frac{\sin 2\pi(k+\alpha-m)}{2\pi(k+\alpha-m)} - \frac{\sin 2\pi(k+\alpha+m)}{2\pi(k+\alpha+m)}\right]\end{aligned} \tag{7}$$

For $\alpha = 0$ both components of (7) have a zero limit except the case $m = k$ where for As_m we obtain:

$$\lim_{m\to k} As_m = -A\left[\frac{\sin 2\pi(k-m)}{2\pi(k-m)} - \frac{\sin 2\pi(k+m)}{2\pi(k+m)}\right] = -A \tag{8}$$

This means that at location $m = k$ we'll have the value A and everywhere $m \neq k$, we'll have zero. Of course, in this case the amplitude spectrum is very „clean" and the signal frequency $f_1 = k\cdot\Delta f$ is correctly evaluated. The phase (at the beginning point of the sampling) of the harmonic components $m\cdot\Delta f$ of the signal is wrong evaluated because $\arctan(As_m / Ac_m)$ gives an indetermination. Anyway LabView delivers in this case a continuous evolution of the wrong evaluated phase, perhaps conserving the idea that the phase „has to grow" anyway, as Fig.1 proves.

In order to supply examples for our statements we'll use as sample a sum of two sinusoidal signals with the same RMS value 1 and different frequencies: f_1 and $3f_1$. The variation will be the value of f_1 and sometimes a white noise. Let it be as example the frequency f_1 = (20+α) Hz.

$$y(t) = \sqrt{2}\sin(2\pi(20+\alpha)t) + \sqrt{2}\sin(2\pi\cdot 3\cdot(20+\alpha)t) \quad (9)$$

For $\alpha \neq 0$ the amplitude spectrum is no more „clean", the actual frequency is „somewhere" between two adjacent ($m = k$ and $m = k+1$) locations where the detected amplitudes are much greater then the values in the vicinity. The actual amplitude A cannot be evaluated. The phase angle evaluation is better because $Ac_m \neq 0$ and $As_m \neq 0$. It has a monotonous evolution till $m = k$ and a sudden jump of π radians to the location $m + 1$. This property can be used as identification criterion for the location $m = k$. Fig. 2 shows very clear this evolution for signal (9). In this case the frequency $f_1 = 20.2$ Hz.

Because here $\alpha = 0{,}2 < 0{,}5$ the amplitude a_k at the location $k+1 = 21$ (don't forget that at location 1 we have the mean value of the signal!) is greater than $a_k + 1$ at the location 22. For frequency $f_2 = 60.6$ Hz, $\alpha = 0{,}6 > 0{,}5$ and the amplitude a_k at the location 61 is smaller than the amplitude $a_k + 1$ at the location 62.

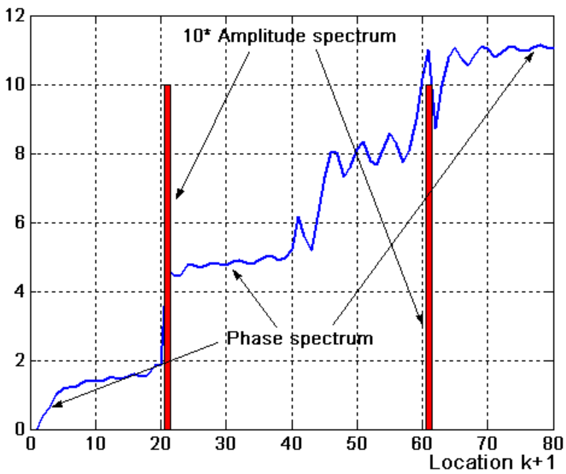


Fig. 1. FFT spectrum of amplitude and phase for $\alpha = 0$

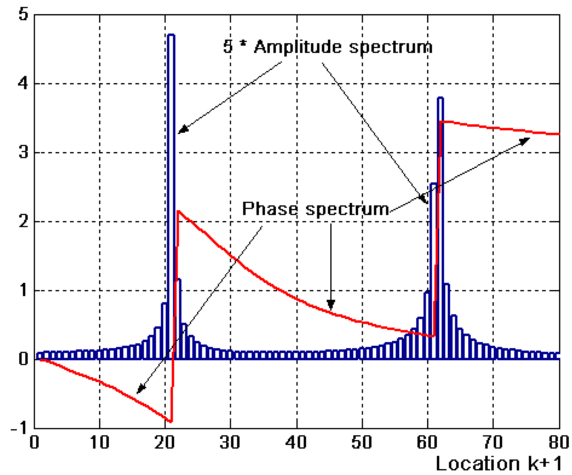


Fig. 2. FFT spectrum of amplitude and phase for $\alpha \neq 0$

It is worth to outline that in both cases the location of the phase jump is independent of the detected amplitude, and the phase jump is there where α would be zero.

To calculate the evaluation's error of the fundamental RMS A from (1) we built the RMS values a_k and $a_k + 1$ of the signal for $m = k$ and $m = k+1$ and α growing from 0 to 1. The two amplitudes are represented in Fig.3 for $k = 20$. The peak detector (a VI of *LabView*) will „find" the amplitude in the $k+1$ location as representative for the signal y_1 so long as $\alpha < 0.5$, and the amplitude in the location $k+2$ for $\alpha > 0.5$. (Do not forget that at location 1, where $k = 0$, we find the mean value of the signal if it exists!)

The maximum error of the evaluated amplitude appears when $\alpha \approx 0.5$ and it is great enough: 35%. The error and the evolution of the amplitudes a_k and $a_k + 1$ against α are practically independent of frequency resolution Δf.

The ratio ρ of the amplitudes at the location k and $k+1$ offers good information for the value of α. Despite the equation's form, the evolution of ratio ρ is remarkably smooth as Fig.4 shows.

A very close idea is presented in [1]. To obtain the value of α, the equation (10) can be solved by suitable methods. The calculated value of the ratio $\rho = a_k / a_{k+1}$ gives a vicinity where the iterative methods have a good convergence.

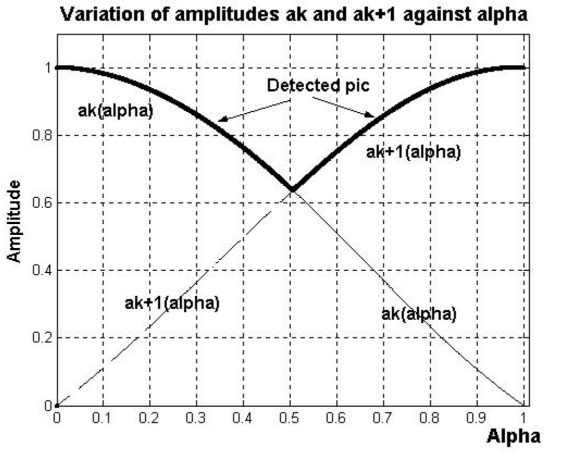


Fig. 3. The amplitude values given by a *LabView* peak detector for $0 < \alpha < 1$

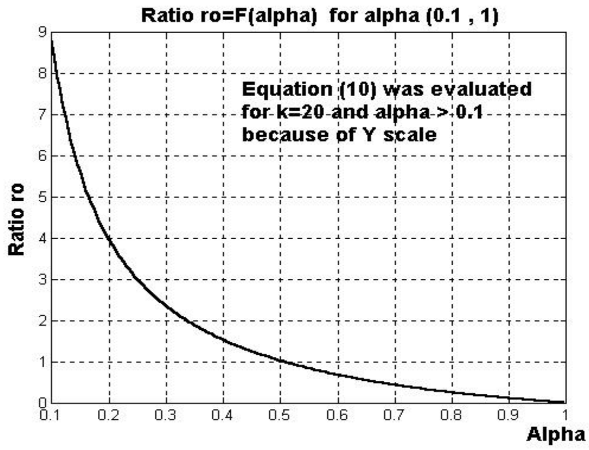


Fig. 4. Ratio $\rho = a_k / a_k+1$ against α

$$\rho = \frac{a_k}{a_{k+1}} = \sqrt{\frac{\dfrac{(1-\cos 2\pi\alpha)^2(k+\alpha)^2 + k^2\sin(2\pi\alpha)^2}{\alpha^2(2k+\alpha)^2}}{\dfrac{(1-\cos 2\pi\alpha)^2(k+\alpha)^2 + (k+1)^2\sin(2\pi\alpha)^2}{(\alpha-1)^2(2k+\alpha+1)^2}}} \tag{10}$$

Equation (10) will be rearranged in the form:

$$\rho = \frac{a_k}{a_{k+1}} = \frac{(\alpha-1)(2k+\alpha+1)}{\alpha(2k+\alpha)}\sqrt{\frac{k^2+(k+\alpha)^2\tan^2\pi\alpha}{(k+1)^2+(k+\alpha)^2\tan^2\pi\alpha}} \tag{11}$$

It is useful to consider the variation of the second factor (with sqrt) of the equation (11). The amplitude of the variation is reduced against the variable α as we can see in Fig.5. Considering that the mean value of the variation has a good approximation in the final value for $\alpha = 1$ we'll obtain a simple equation for the α estimation.

This equation, for $\alpha = 1$ in the "sqrt" factor becomes:

$$\rho \cong \frac{(1-\alpha)(2k+\alpha+1)}{\alpha(2k+\alpha)}\frac{k}{k+1};\ \frac{\rho}{C} = \frac{(1-\alpha)(2k+\alpha+1)}{\alpha(2k+\alpha)};C = \frac{k}{k+1}. \tag{12}$$

The solution of the second order algebraic equation is given below. The error for $k = 10$ is about 1.45 % against the value given by (10). For higher values of k the error is smaller.

$$\alpha = k\left[\sqrt{1+\frac{2k+1}{k^2(1+\dfrac{\rho}{C})}}-1\right] \tag{13}$$

The most important problem remains the identification of k. This is why the study of the phase evolution for $\alpha \neq 0$ is very important.

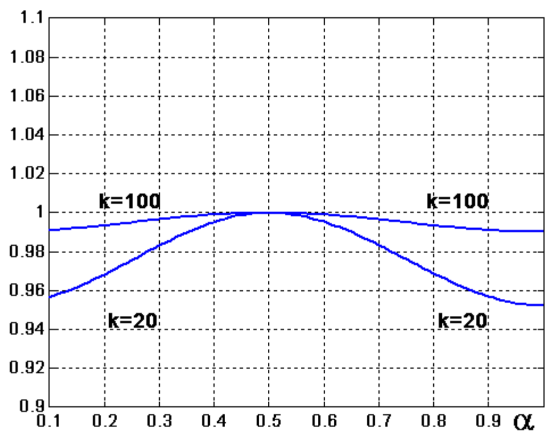


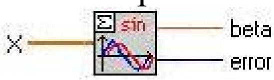
Fig. 5. The variation of the factor "sqrt" from (11)

The location k will appear before the phase jump!

3. PROPOSED CORRECTION METHODS

3.1. The use of Buneman frequency estimator

If the data acquisition is done with *LabView*, it seems logical to look for a correction method among the *LabView* instruments. The frequency f_1 of the fundamental y_1 is the most important parameter to be determined. With the right value of frequency f_1 we can use the well-known evaluation method of the signal's amplitude by integration over the period $T_1=1/f_1$. The Buneman estimator of frequency is a Virtual Instrument (VI) of the *LabView* library. It calculates the successive frequencies of the sinusoidal components by using the results of FFT.



The Buneman algorithm identifies two maximum values a_k and a_{k+1} of a vicinity and their locations k and $k + 1$. For a given Δf the unknown frequency f_{1B} is calculated as:

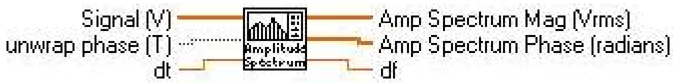
$$f_{1B} = k * \Delta f + \frac{N}{\pi} a\tan\left(\frac{\sin\frac{\pi}{N}}{\cos\frac{\pi}{N} + \frac{a_k}{a_{k+1}}} \right) = \beta * \Delta f \tag{14}$$

As we can see, the Buneman algorithm uses the information about a vicinity where the frequency f_1 is found. The data of Table 1 can be used to verify the Buneman's algorithm accuracy.

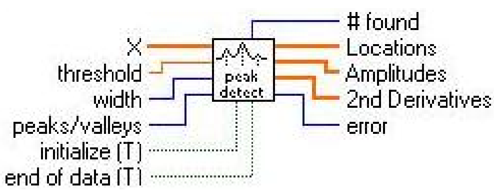
In order to calculate the amplitude components for the real signal y_1 we'll use the equation (15) because the Buneman frequency offers the value of k and α.

3.2. The use of pic detector and solving eq. (10)

The amplitude and phase spectrum is obtained by the *LabView* VI that executes a FFT of the input signal.



The amplitude spectrum is analysed by a peak detector, another VI of *LabView* that gives the locations and values of successive peaks of amplitudes. The first peak is supposed to belong to the fundamental. In order to verify the actual position of the $k \cdot \Delta f$ frequency for $\alpha = 0$ we'll examine the phase spectrum in the near of peak's location.



The phase jump will give the right value of the k location. Then we can calculate the ratio ρ and evaluate the right value for α with (13) and the unknown frequency $f_x = (k + \alpha) \cdot \Delta f$. The actual rms value of the unknown amplitude will be calculated using the equation:

$$A_x = \pi a_k \sqrt{\frac{\alpha^2 (2k+\alpha)^2}{(1-\cos 2\pi\alpha)^2 (k+\alpha)^2 + k^2 \sin(2\pi\alpha)^2}} \tag{15}$$

The procedure is repeated for all peaks that belong to the higher harmonics of the signal, or to another waves contained in the signal.

4. EXPERIMENTAL RESULTS

Three *LabView* programs were built, one for each proposed method. The same signal was analised by all the programs. The errors for recovered amplitudes and frequencies were compared in order to evaluate the efficiency of the proposed methods.

The use of Buneman frequency estimator

The precision of the method is very good. In the presence of noise the frequency recovery is very good and the error for amplitude recovery is satisfactory, around 1–1.5 %, dependent on the noise amplitude and type (Gaussian, white noise).

The use of pic detector and solving eq. (10)

The method gives practically the same precision for the frequency and amplitude recovery, but is sensitive to the noise because the corrupted phase spectrum by noise.

Repeated FFT with different number of samples N”

The precision of the method is good. The advantages of the method are simplicity and immunity to noise.

5. CONCLUSIONS

The paper proposes three methods to improve the evaluation of the fundamental frequency and amplitude of a signal by FFT. In this way the advantages of FFT are preserved and the disadvantages are compensated by efficient techniques.

The applications field of these techniques can be the power measurement in the electrical drives where the electrical machine is fed by frequency converters.

With some additional work the ideas can be applied in the measurement of active power of all voltage and current harmonics of the same order.

We outline that two of the three proposed methods are insensitive to the noise content of the signal.

6. REFERENCES

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