# ALGORITHM FOR CALCULATING OF THE NONLINEAR PRODUCTS' AMPLITUDES FROM ORDER *n*

**Oleg Borisov Panagiev**

Department of Radiocommunications Technical University of Sofia, Bulgaria
8 "Kl. Ohridski" Blvd, 1000 Sofia, tel. 02 965 2284, e-mail: olcomol@yahoo.com

*Below we describe an algorithm for computation of the amplitude of each nonlinear intermodulation product in the presence of arbitrarily large number of CW signals at the input. Our treatment of the relation between each intermodulation product and the parameters of the input signals for the system (as well as its transfer characteristics, given by the Volterra kernels of the related order) is based upon the use of binomial coefficients.*

**Keywords:** RF amplifiers, Intermodulation products, Volterra kernels

## 1. INTRODUCTION

Large amounts of data can be transmitted by radiocommunication systems using sophisticated modulation techniques, but these unfortunately lead to wide, dynamic signals that require linear amplification. It is possible to achieve linear amplification, but unfortunately at the expense of efficiency. RF amplifiers such as television's, radio's, GSM's or cable television amplifiers and optical transmitters usually works at extremely conditions, which in order of cases can bring up rising of nonlinear products (NP). Creating of new methods for suppressing and limiting of NP is accompanied with creating of methods and algorithms for analyze of nonlinear distortions and calculating of the amplitudes of NP, especially from ($3^{rd}$, $4^{th}$, $5^{th}$, $6^{th}$, etc.) order [1, 2, 3].

A system or device is considered to be nonlinear if the output is a nonlinear function of the input. There are a number of ways of modeling the nonlinearity of a system, one of the most useful of which is polynomial modeling, because regenerated spectral components can be calculated analytically based on polynomial coefficients, provided that the input signal is a sine wave or sum of sine waves. This is a unique property of polynomial modeling and greatly eases the computational complexity.

There is a one important drawback with polynomial input-output modeling, however, which is that the nonlinearity introduced by the system is only affected by the input amplitude and not by the frequency or bandwidth of the signal. The output of an amplifier modeled with a polynomial can be written as [4]:

$$y(t) = \sum_{n=0}^{\infty} \left|H_n(f)\right| . x^n(t) , \qquad (1)$$

where $H_n$ is Volterra kernel of $n$ – th order for the corresponding NP;

$x(t)$ - system/device input unmodulated CW signal;

$f$ - the combination frequency.

Below we describe an algorithm for computation of the amplitude of each nonlinear intermodulation product in the presence of arbitrarily large number of CW signals at the input. Our treatment of the relation between each intermodulation product and the parameters of the input signals for the system (as well as its transfer characteristics, given by the Volterra kernels of the related order) is based upon the use of binomial coefficients.

At the engineering practice the measurement devices depict in number or graphic type the respective quantities in logarithmic units. In this connection is more conveniently the amplitude of every nonlinear product to be calculated in a dBμV:

$$A_{IM} = A_{\Sigma} + S_{dB} + |H_n| - 6(n-1),\ [dB\mu V], \tag{2}$$

where $A_{\Sigma}$ is the sum from the amplitudes of the input signals in NP, in dBμV;

$S_{dB}$ – logarithmical value of the NP number with same frequency in the output of the system/device;

$|H_n| \equiv |H_n(f)|$ in dB. Volterra kernels up to third order including, are determinate by the methods described in [5], and that from upper order (4[th], 5[th], 6[th], etc.) will be an object for future researches (development). Exist some methods for identifying of the Volterra kernel, but in this case they are difficult applicable and not much accurate [6, 7, 8].

## 2. ALGORITHM DESCRIPTION

Suggested algorithm give an opportunity for calculating amplitudes of the nonlinear products from $n^{\text{th}}$ order at arbitrary number of carried channels (frequencies). Results for them are given in a logarithmic units, because of the up given considerations. Block circuit of the algorithm is presented in fig.1, and the steps, which is necessary to be fallowed are this:

**1.) Leading in values of:**

- $d_i$ - is the coefficient of *i*-th input frequency in the nonlinear product, where $d_i = |r_i|$, a $f = \sum_{i=1}^{N} r_i f_i$ is the frequency of nonlinearity product at the output system/devise; $f_i$ are the frequencies of the input signals; $r_i$ are arbitrary integers, possibly equal to zero. If the transmission characteristic of the system is of *n*-th order, then the coefficients $r_1, r_2, r_3 \ldots$, need to satisfy the inequality $|r_1| + |r_2| + |r_3| + \ldots \le n$; *i* - taking values from 1 to N, labels the carrier frequencies;
- $A_i$ is the signal amplitude of the *i*-th input frequency, in dBμV.
- $N$, $n$ and $|H_n|$.

**2.) Sum calculated from input signals' amplitudes in the NP, depending of $d_i$.**

$$A_{\Sigma} = \sum_{i=1}^{N} d_i A_i\ ,\ [dB\mu V]. \tag{3}$$

**3.) Defining of the indexes $p_i$ of binomial coefficients $C_{p_i}^{d_i}$.**

$$p_i = n - \sum_{i=1}^{i} d_{i-1}\ . \tag{4}$$

**4.) Calculating of the binomial coefficients $C_{p_i}^{d_i}$.**

$$C_{p_i}^{d_i} = \frac{p_i(p_i-1)(p_i-2)\ldots[p_i-(d_i-1)]}{d_i!}. \qquad (5)$$

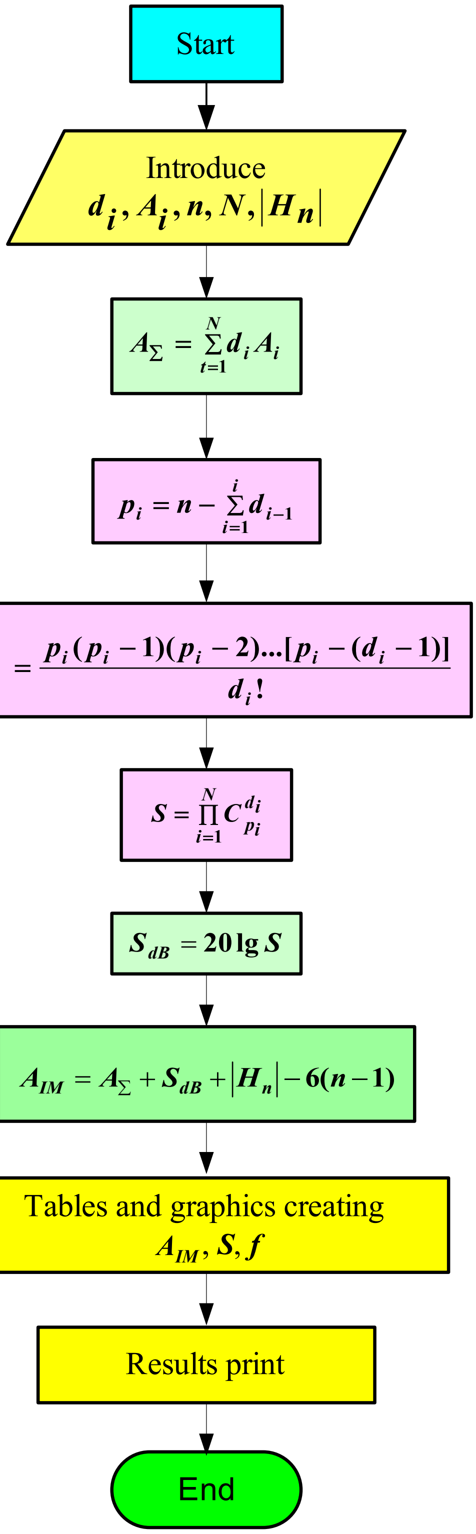


Fig.1. Block diagram of the algorithm

**Table 1**

**Composite intermodulation nonlinear products**

| order | $f$ | $A_{IM}$, dBμV | $S$ | order | $f$ | $A_{IM}$, dBμV | $S$ |
|---|---|---|---|---|---|---|---|
| **3** | $f_1 \pm 2f_2$ | $A_1 + 2A_2 + \lvert H_3 \rvert - 2,5$ | 3 | **5** | $f_1 \pm f_2 \pm 3f_3$ | $A_1 + A_2 + 3A_3 + \lvert H_5 \rvert + 2,02$ | 20 |
| | $f_1 \pm 2f_3$ | $A_1 + 2A_3 + \lvert H_3 \rvert - 2,5$ | 3 | | $f_1 \pm 3f_2 \pm f_3$ | $A_1 + 3A_2 + A_3 + \lvert H_5 \rvert + 2,02$ | 20 |
| | $f_2 \pm 2f_1$ | $A_2 + 2A_1 + \lvert H_3 \rvert - 2,5$ | 3 | | $3f_1 \pm f_2 \pm f_3$ | $3A_1 + A_2 + A_3 + \lvert H_5 \rvert + 2,02$ | 20 |
| | $f_2 \pm 2f_3$ | $A_2 + 2A_3 + \lvert H_3 \rvert - 2,5$ | 3 | | $f_1 \pm 2f_2 \pm 2f_3$ | $A_1 + 2A_2 + 2A_3 + \lvert H_5 \rvert + 5,54$ | 30 |
| | $f_3 \pm 2f_1$ | $A_3 + 2A_1 + \lvert H_3 \rvert - 2,5$ | 3 | | $2f_1 \pm f_2 \pm 2f_3$ | $2A_1 + A_2 + 2A_3 + \lvert H_5 \rvert + 5,54$ | 30 |
| | $f_3 \pm 2f_2$ | $A_3 + 2A_2 + \lvert H_3 \rvert - 2,5$ | 3 | | $2f_1 \pm 2f_2 \pm f_3$ | $2A_1 + 2A_2 + A_3 + \lvert H_5 \rvert + 5,54$ | 30 |
| | $f_1 \pm f_2 \pm f_3$ | $A_1 + A_2 + A_3 + \lvert H_3 \rvert + 3,5$ | 6 | | | | |
| **4** | $f_1 \pm 3f_2$ | $A_1 + 3A_2 + \lvert H_4 \rvert - 6,02$ | 4 | **6** | $f_1 \pm 5f_2$ | $A_1 + 5A_2 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $f_1 \pm 3f_3$ | $A_1 + 3A_3 + \lvert H_4 \rvert - 6,02$ | 4 | | $f_1 \pm 5f_3$ | $A_1 + 5A_3 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $f_2 \pm 3f_1$ | $A_2 + 3A_1 + \lvert H_4 \rvert - 6,02$ | 4 | | $f_2 \pm 5f_1$ | $A_2 + 5A_1 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $f_2 \pm 3f_3$ | $A_2 + 3A_3 + \lvert H_4 \rvert - 6,02$ | 4 | | $f_2 \pm 5f_3$ | $A_2 + 5A_3 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $f_3 \pm 3f_1$ | $A_3 + 3A_1 + \lvert H_4 \rvert - 6,02$ | 4 | | $f_3 \pm 5f_1$ | $A_3 + 5A_1 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $f_3 \pm 3f_2$ | $A_3 + 3A_2 + \lvert H_4 \rvert - 6,02$ | 4 | | $f_3 \pm 5f_2$ | $A_3 + 5A_2 + \lvert H_6 \rvert - 14,44$ | 6 |
| | $2f_1 \pm 2f_2$ | $2A_1 + 2A_2 + \lvert H_4 \rvert - 2,5$ | 6 | | $2f_1 \pm 3f_2$ | $2A_1 + 4A_2 + \lvert H_6 \rvert - 6,48$ | 15 |
| | $2f_1 \pm 2f_3$ | $2A_1 + 2A_3 + \lvert H_4 \rvert - 2,5$ | 6 | | $2f_1 \pm 4f_3$ | $2A_1 + 4A_3 + \lvert H_6 \rvert - 6,48$ | 15 |
| | $2f_2 \pm 2f_3$ | $2A_2 + 2A_3 + \lvert H_4 \rvert - 2,5$ | 6 | | $2f_2 \pm 4f_1$ | $2A_2 + 4A_1 + \lvert H_6 \rvert - 6,48$ | 15 |
| | $f_1 \pm f_2 \pm 2f_3$ | $A_1 + A_2 + 2A_3 + \lvert H_4 \rvert + 3,52$ | 12 | | $2f_2 \pm 4f_3$ | $2A_2 + 4A_3 + \lvert H_6 \rvert - 6,48$ | 15 |
| | | $A_1 + 2A_2 + A_3 + \lvert H_4 \rvert + 3,52$ | 12 | | $2f_3 \pm 4f_1$ | $2A_3 + 4A_1 + \lvert H_6 \rvert - 6,48$ | 15 |
| | $f_1 \pm 2f_2 \pm f_3$ | $2A_1 + A_2 + A_3 + \lvert H_4 \rvert + 3,52$ | 12 | | $2f_3 \pm 4f_2$ | $2A_3 + 4A_2 + \lvert H_6 \rvert - 6,48$ | 15 |
| | $2f_1 \pm f_2 \pm f_3$ | | | | $3f_1 \pm 3f_2$ | $3A_1 + 3A_2 + \lvert H_6 \rvert - 3,98$ | 20 |
| **5** | $f_1 \pm 4f_2$ | $A_1 + 4A_2 + \lvert H_5 \rvert - 10,02$ | 5 | | $3f_1 \pm 3f_3$ | $3A_1 + 3A_3 + \lvert H_6 \rvert - 3,98$ | 20 |
| | $f_1 \pm 4f_3$ | $A_1 + 4A_3 + \lvert H_5 \rvert - 10,02$ | 5 | | $3f_2 \pm 3f_3$ | $3A_2 + 3A_3 + \lvert H_6 \rvert - 3,98$ | 20 |
| | $f_2 \pm 4f_1$ | $A_2 + 4A_1 + \lvert H_5 \rvert - 10,02$ | 5 | | $f_1 \pm f_2 \pm 4f_3$ | $A_1 + A_2 + 4A_3 + \lvert H_6 \rvert - 0,46$ | 30 |
| | $f_2 \pm 4f_3$ | $A_2 + 4A_3 + \lvert H_5 \rvert - 10,02$ | 5 | | $f_1 \pm 4f_2 \pm f_3$ | $A_1 + 4A_2 + A_3 + \lvert H_6 \rvert - 0,46$ | 30 |
| | $f_3 \pm 4f_1$ | $A_3 + 4A_1 + \lvert H_5 \rvert - 10,02$ | 5 | | $4f_1 \pm f_2 \pm f_3$ | $4A_1 + A_2 + A_3 + \lvert H_6 \rvert - 0,46$ | 30 |
| | $f_3 \pm 4f_2$ | $A_3 + 4A_2 + \lvert H_5 \rvert - 10,02$ | 5 | | $f_1 \pm 2f_2 \pm 3f_3$ | $A_1 + 2A_2 + 3A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_1 \pm 3f_2$ | $2A_1 + 3A_2 + \lvert H_5 \rvert - 4$ | 10 | | $f_1 \pm 3f_2 \pm 2f_3$ | $A_1 + 3A_2 + 2A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_1 \pm 3f_3$ | $2A_1 + 3A_3 + \lvert H_5 \rvert - 4$ | 10 | | $2f_1 \pm f_2 \pm 3f_3$ | $2A_1 + A_2 + 3A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_2 \pm 3f_1$ | $2A_2 + 3A_1 + \lvert H_5 \rvert - 4$ | 10 | | $2f_1 \pm 3f_2 \pm f_3$ | $2A_1 + 3A_2 + A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_2 \pm 3f_3$ | $2A_2 + 3A_3 + \lvert H_5 \rvert - 4$ | 10 | | $3f_1 \pm f_2 \pm 2f_3$ | $3A_1 + A_2 + 2A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_3 \pm 3f_1$ | $2A_3 + 3A_1 + \lvert H_5 \rvert - 4$ | 10 | | $3f_1 \pm 2f_2 \pm f_3$ | $3A_1 + 2A_2 + A_3 + \lvert H_6 \rvert + 5,56$ | 60 |
| | $2f_3 \pm 3f_2$ | $2A_3 + 3A_2 + \lvert H_5 \rvert - 4$ | 10 | | $2f_1 \pm 2f_2 \pm 2f_3$ | $2A_1 + 2A_2 + 2A_3 + \lvert H_6 \rvert + 9,08$ | 90 |

**5.) Defining numbers *S* of nonlinearity products from respective order and group.**

$$S = \prod_{i=1}^{N} C_{p_i}^{d_i} . \qquad (6)$$

**6.) Calculating of the logarithmical coefficient $S_{dB}$.**

$$S_{dB} = 20 \lg S, \ [dB] . \qquad (7)$$

**7.) Calculating of amplitude $A_{IM}$ of the nonlinear product, in dBµV.**

Using expression (2), as the value for *n* shows the order of NP, and it doesn't depend from its group.

**8.) Visualization of the results for *f*, $A_{IM}$ and *S* in table and graphic type.**

**9.) Tables and graphics printing.**

Results of applying of the described algorithm for measuring of the amplitudes of composite nonlinear products from 3rd, 4th, 5th, and 6th order at N=3, are given in Table 1.

## 3. Example

Made is a calculating of the intermodulation products' amplitudes from 3rd order at tree frequencies for linear amplifier VX23A. Accepted is that frequencies of nonlinear products coincide with carrier frequencies of the image in accordance with the European standard CENELEC EN 50083 (Annex C). Frequency spectrum of the system, in which works the amplifier, is up to 520 MHz, and the distribution of the channels is on standard D/K. Values of the Volterra kernels are in accordance with [5]. Examined are tree cases for different values of input signals: 70 dBµV, 77 dBµV, 84 dBµV (see Table 2). On fig.2 is shown alteration of the amplitudes of composite nonlinear products from intermodulation in whole work frequency band of amplifier. Amplitudes of input signals are even: $A_1=A_2=A_3=A_{in}$.

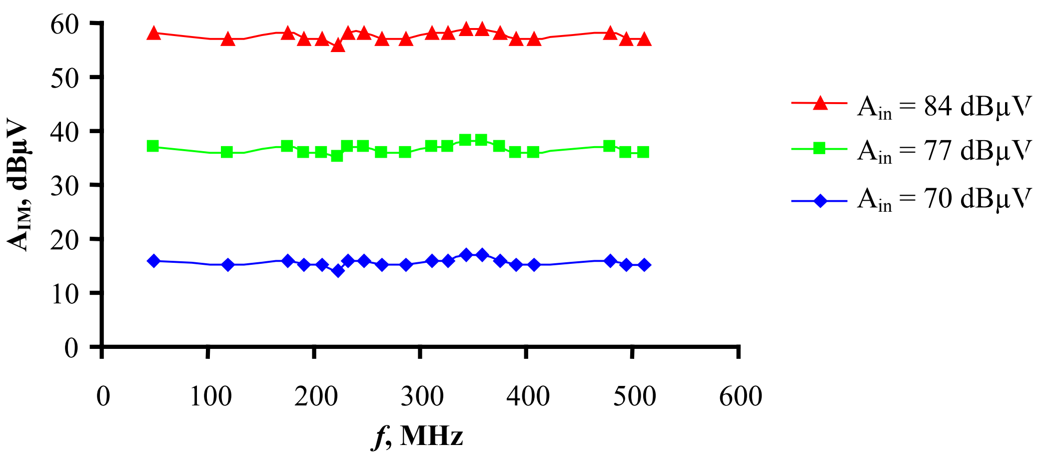


Fig.2. Alteration of the amplitudes of composite nonlinear products for linear amplifier VX23A

**Table 2**

| $f$ MHz | Composite intermodulation nonlinear products | | |
|---|---|---|---|
| | by $A_{in}$ = 70 dBμV | by $A_{in}$ = 77 dBμV | by $A_{in}$ = 84 dBμV |
| | $A_{IM,3}$, dBμV | $A_{IM,3}$, dBμV | $A_{IM,3}$, dBμV |
| 49,75 | 16 | 37 | 58 |
| 119,25 | 15 | 36 | 57 |
| 175,25 | 16 | 37 | 58 |
| 191,25 | 15 | 36 | 57 |
| 207,25 | 15 | 36 | 57 |
| 223,25 | 14 | 35 | 56 |
| 231,25 | 16 | 37 | 58 |
| 247,25 | 16 | 37 | 58 |
| 263,25 | 15 | 36 | 57 |
| 287,25 | 15 | 36 | 57 |
| 311,25 | 16 | 37 | 58 |
| 327,25 | 16 | 37 | 58 |
| 343,25 | 17 | 38 | 59 |
| 359,25 | 17 | 38 | 59 |
| 375,25 | 16 | 37 | 58 |
| 391,25 | 15 | 36 | 57 |
| 407,25 | 15 | 36 | 57 |
| 479,25 | 16 | 37 | 58 |
| 495,25 | 15 | 36 | 57 |
| 511,25 | 15 | 36 | 57 |

## 4. CONCLUSION

Logarithmical values of the coefficient $S$, can be estimated as first of all we logarithmate the binomial coefficients singly, and then we summarize them. In that case error at the last value for $S_{dB}$ will depend from the accuracy at make round. Increasing/decreasing of the error values with increasing of nonlinearity product's order, can be an object for research in future work.

## 5. REFERENCES

[1] Rahkonen T., T. Kankaala, *Using analog predistortion for linearizing class A - C power amplifiers*. Kluwer Acad. journal on Analog IC and Signal Processing, pp. 31-40, 2000.

[2] Cripps S., *RF power amplifiers for wireless communications*. Artech House, 1999.

[3] Vuolevi J., T. Rahkonen, *Analysis of amplitude dependent memory effects in RF power amplifiers*, Proc. Europ. Conf. on Circuit Theory and Design, Espoo, Finland, pp. 37-40, 2001.

[4] Panagiev O. B., *Determinating the amplitudes of intermodulation products of higher order by means Volterra kernels*, ICEST, Proc. of Papers, Bitola, vol.1, pp. 215-216, 16-19 June 2004.

[5] Panagiev O., K. Dimitrov, *CATV systems - Volterra kernels identification*, ICEST, Proc. of Papers, Ohrid, vol.2, pp.757-760, 24-27 June 2007.

[6] Levenec S. V., *Nonlinear dynamic objects identification*, OGPU, Odessa, 1995.

[7] Silva W.A., *Reduced-order models based on linear and nonlinear aerodynamic impulse responses*, AIAA Paper No.99-1262, 2000.

[8] Boyd S., Y. S. Tang and L. Chua, *Measuring Volterra kernels*, IEEE Trans. on Circuits and Systems, vol. CAS-30, No. 8, pp. 571-577, Aug 1983.