EVALUATION OF THE ACCURACY IN INCLINOMETER TESTING

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In the article problems connected with the metrological securing of the measurements of the inclination angle with an inclinometer are analyzed. On the basis of the existing measuring devices, a lot of theoretical and scientific problems are pointed out, allowing the introduction of the system approach in the process of characteristics formation when building such devices. The parameters of the process $\psi(t)$ depend on the values of the series of characteristics of the system road-machine. Then for one and the same value of the period T, a satisfactory credibility will exist, for some values of the system characteristics, and will not exists for other.

Keywords: inclination angle, mathematical expectation, dispersion**.**

1. ANALYSIS OF THE PROBLEMS CONNECTED WITH THE RESEARCH AND DEVELOPMENT OF THE INCLINATION ANGLE MEASURING DEVICES.

The inclinometer is a device for determining the inclination angle and the displacement amplitude of the probe from the vector of gravity and for controlling amplitude's correct direction. In the scientific literature with the name "inclinometer" are also associated devices for determining the inclination angle from the vertical axis, which coincides with the vector of the acceleration of gravity, for a point of a material object in space during static and dynamic loading of the object. The problems which appear during the invention and realization of the inclinometers are with different origin. When reviewing the existing information, it can be seen that they are not satisfactory reflected in the literature. For example in the main part of the literature sources, devoted to inclination angle measuring devices, information about the technical data is translated from catalogues of the different producers and have mainly advertising character.

All this shows that there is a necessity not only to build such kind of measuring devices but also solving a series of theoretical and research problems, making possible the implementation of a systematic approach to the process of characteristic formation. This approach means determination of the nominal values and the boundary deviations and the functional parameters of the measuring devices on the basis of their connection with metrological and exploitation indexes.

Building of an inclinometer depends on the chosen measuring method, on the base on the real metrological process and on the characteristics of the result of the measurement. The measurement can be divided conditionally into two stages according to the development of the metrological algorithm in time. In the first stage a real measurement procedure is realized, with whose help is registered the existing realization of the $\psi(t)$ process. In the second stage, a functional transformation is made, according to the mathematical model of the measured quantity $\psi_s(t)$ and the result is shown on a display.

2. PROBABILITY CHARACTERISTICS OF THE DEFINING FUNCTION ψ(T). DETERMINATION OF THE OPTIMAL PERIOD FOR TAKING THE REALIZATIONS.

The function $\psi(t)$, which is the information carrier of the measured quantity ψ_s, is characterized by the following important properties:

- The function is uninterrupted chance process in time [5]. This means the main characteristics of this quantity are probability characteristics and for their determination a chance processes theory will be used.
- The chance process which defines this function is a stationary one [5]. In the stationary processes the momentum distributions are constant it time. Actually this definition is valid only for strictly stationary processes. In weakly stationary processes constant in time are only the first and the second momentums. But in both cases the mathematical expectation is a constant quantity. This means that the mathematical expectation (ME) of the process, defining the dynamical angle of the inclination is also a constant quantity. The static angle of inclination as a function of the process dynamic inclination angle can be accepted, in the range of one realization, to be a constant quantity. This circumstance as well as the essence of the physical process, characterising the dynamic fluctuations of the tested object around its equilibrium condition, makes it possible the static inclination angle to be evaluated using the ME of the function, which defines the dynamic fluctuations of the object.
- The stationary chance process, characterising the fluctuations of the inclination angle is ergodic [2,3,4]. This property means that taking the average of multiple realizations will give the same result as taking the average of an infinite realization. From here it follows that for ME evaluation of process an average arithmetic mean of one realization can be used.
- The process can be viewed as a normal [5,6]. For the normally distributed chance processes weak and the strong stationarity as well as the weak and the strong ergodicity coincide. Furthermore from the above condition the mathematical apparatus of the distribution becomes famous.

3. MATHEMATICAL EXPECTATION AND EVALUATION OF THE MATHEMATICAL EXPECTATION OF THE CHANCE PROCESS, DEFINING THE DYNAMICAL FLUCTUATIONS OF THE OBJECT ALONG ITS LONGITUDINAL AXIS.

In the common case, mathematical expectation $M[\psi(t)]$ of the dynamic inclination angle $\psi(t)$ and therefore the quantity inclination angle ψ_s, can be defined by the formula:

$$\psi_s = M[\psi(t)] = \int_D \psi(t,x).dF(x)\,, \tag{1}$$

where $F(x)$ – law for probability distribution and the integration is over the whole region D, where the law of distribution is defined. Since the examined dynamic process is a stationary and ergodic, it follows that ME can be determined from set of realizations, like in formula 1, as well as in the frames of one single infinitely long realization, e.g.:

$$\psi_s = M[\psi(t)] = \lim_{T\to\infty} .\frac{1}{T}.\int_0^T \psi(t).dt. \tag{2}$$

Formula (2) gives the opportunity for practical real time determination of the quantity ψ_s. because the averaging is made not on the set but on the time. Of course the practical realization of an infinitely long realization is impossible. That is why, in practice, we are working with the evaluation of the ME of the process, which is realized for a finite time, equal to the optimally chosen period T. Decreasing the time for the practically implementable realizations means that by the evaluation it can be found only the approximate value of the measuring quantity.

In general, the evaluation $\tilde{\psi}_s$ can be presented as a quantity, determined by the equation:

$$\tilde{\psi}_s = \int_0^T a(t).\psi(t).dt\,, \tag{3}$$

where: *a(t)* is the weight function.

The quality of the evaluation depends on the choice of the function *a(t)*. The function *a(t)* should be chosen to correspond to the following more important requirements:

1. Evaluation $\tilde{\psi}_s$ of the quantity ψ_s should be sustainable. This condition is satisfied if the dispersion of the evaluation ψ_s tends to zero with increasing the size of the realization, e.g. when satisfying the equation:

$$\lim_{T\to\infty} D[\tilde{\psi}_s] = 0\,. \tag{4}$$

The sustainable evaluation is a necessary condition for its applicability, because otherwise by increasing the length of the realization in time, the accuracy of the searched parameter is not increased.

2. The evaluation should be non-displaced, which means that the ME of the evaluation should be equal to the searched quantity, e.g.:

$$M[\tilde{\psi}_s] = \psi_s\,. \tag{5}$$

Non-displaceability is also one of the important practical qualities of the evaluation, because its existence frees the realization from the presence of systematic errors.

3. Efficiency. The efficiency condition for the evaluation is that it should have the smallest possible dispersion.

In the mathematical statistics [1] it is proved that non-displaced, sustainable and possibly the most effective evaluation is that one which weight is determined by:

$$a(t) = \begin{cases} \frac{1}{T}, when\, 0 \le t \le T \\ 0, when t < 0, t > T. \end{cases} \tag{6}$$

With such a choice of the weighted function it follows that:

$$\widetilde{\psi}_s = \frac{1}{T}.\int_0^T \psi(t).dt\,, \qquad (7)$$

e.g. the evaluation of the ME of the quantity $\psi(t)$, should be average integral value of the concrete realization. The evaluation (7) of ME of the stationary process $\psi(t)$, is defined for a continuous realization. In the real measurement process this realization is implemented as a continuous function of the angle between the physical pendulum and the inclinometer's body in time. Evaluation (7) can be realized, if the method of measuring and processing of the results does not contain any error. In practice, due to errors such as dynamic, instrumental, level discretization errors, errors due to friction and so on, the result from the measuring of the function $\psi(t)$ is different from the real value of this quantity.

4. DETERMINING THE OPTIMAL PERIOD OF THE REALIZATIONS

This is the main parameter of the averaging filter. Consequently the possibility to reach the theoretical maximum of the accuracy, corresponding to the optimal filter characteristics, depends on its choice. In this sense it is necessary to create systematic base for creating this parameter. Due to the dynamic characteristics of the parameters defining the measurement environment, an important property of the module, which determines the period of the realizations, is its adaptivity to the current values of the defining parameters.

The parameters of the process $\psi(t)$ depend on the values of the series of characteristics of the system road-machine. Then for one and the same value of the period T, a satisfactory credibility will exist, for some values of the system characteristics, and will not exists for other.

Therefore it is necessary to find to choose a criterion for effective choice of T, under which from one side quite good credibility will exists and from the other there will be no heap of unnecessary values in the operational memory. This choice should be based on the characteristics of the process itself and the boundary values of the parameters, taking part in its formation.

Since each evaluation is a function of the realizations of the chance process, it should also be a chance variable. That is why as a quality criteria of the evaluation can be chosen the probability to fall into given boundaries according to the real value of the measured quantity. If we take a second as a dispersion measure, e.g. the dispersion of the evaluation dispersion from the real value, then the dispersion with the smallest value can be taken as an optimal one.

In the mathematical statistics [1,4] a formula for the dispersion of the average integral evaluation of a chance process is derived, which can be adapted for the present case and have the following shape:

$$D[\widetilde{\psi}_s] = \frac{2}{T^2}.\int_0^T (T-\tau).K_\psi(\tau).d\tau\,, \qquad (8)$$

where $K_\psi(\tau)$ is the correlation function of the process, defining the object deviations in the longitudinal cross-section.

By using formulae for the correlation function [5] $K_{u_j}(\tau)$ of the chance processes $u_j(t)$ characterizing the different the different types of object fluctuations, can be approximated good enough by the following formula:

$$K_{u_j}(\tau)=\sigma_{u_j}^2.e^{-\mu_j.|\tau|}.\left(\cos\lambda_j.\tau+\frac{\mu_j}{\lambda_j}.\sin\lambda_j.|\tau|\right), \tag{9}$$

After that (8) is transformed in the following equality:

$$D[\tilde{\psi}_s]=\frac{2.D_\psi}{T^2}.\int_0^T (T-\tau).e^{-\mu_\psi^*.|\tau|}.(\cos\lambda_\psi^*.\tau+\frac{\mu_\psi^*}{\lambda_\psi^*}.\sin\lambda_\psi^*.|\tau|).d\tau\ . \tag{10}$$

From (8) it follows that the characteristics of the dispersion $D[\tilde{\psi}_s]$ depend on the correlation function of the process *ψ(t)* and on the period T. By the correlation function in (8) the influence of all basic parameter, which take part in the forming of the real physical process, is taken into account and the quality of the measuring process and the accuracy of the results depends on the size T. Thus using (8) it can be followed the deviation of the estimate from the real value of the measuring quantity as a function of the period T, for specific values of the parameters defining the corresponding correlation function. This means that (10) can be used as a base for the optimal choice of T. For this purpose it is necessary functions of the following type to be solved:

$$D[\tilde{\psi}_s]=f\left[K_\psi(\tau),T\right]. \tag{11}$$

The graphical solution of (11) for different types of objects and different deviation from planarity of the road is shown on fig. 1 and fig. 2:

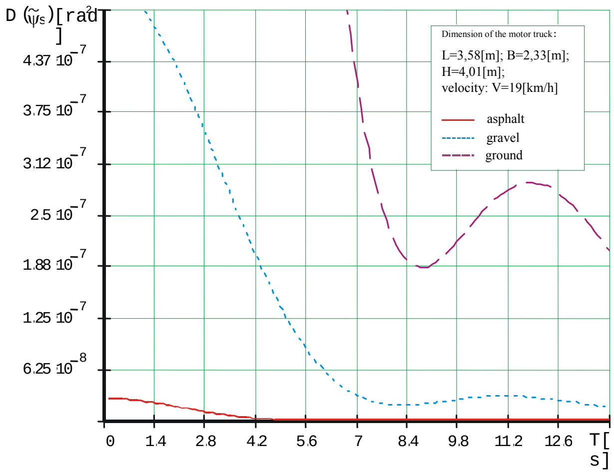

Fig. 1

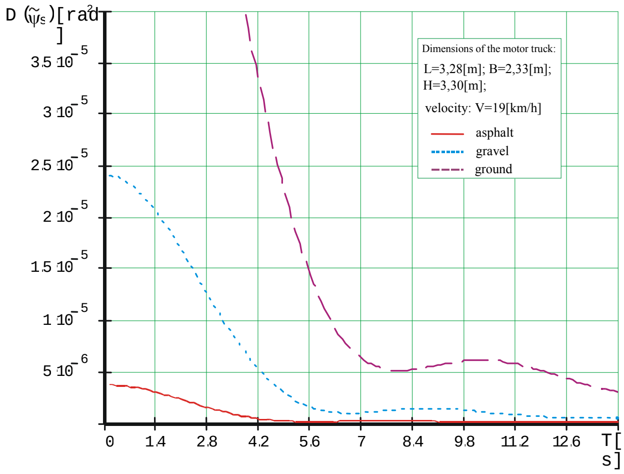

Fig. 2

From fig. 1 and fig 2 it can be seen that the estimate $\tilde{\psi}_s$ is independent: Moreover the period for which the curves tend to the abscissa is different for each different non-planarity. When the non-planarity is bigger, the curve approaches the zero values for greater values of the period T. Difference in the character of the function (11) is introduced by the object parameters, as well as by object's current kinematic characteristics like velocity and acceleration. Therefore every concrete case can be different according to values of the determining parameters. That is why in the program, which processes the data from the different realizations, it is necessary to have a module to enter the period T. If the inclinometer is connected to a complex navigation system, which is for example the system for optimization of the usage modes of the object, it is possible the optimal period to be automatically determined and the relevant change to be made in the processing software module according to the real value of T. In the case when it is not possible the value of T to be changed or corrected, only one value for the period is entered. This value is calculated on the basis of these values of the parameters that determine the real possible working modes, under which the function (11) will have the most extended convergence to the abscissa.

Determination of the maximum possible dispersion value of $D[\tilde{\psi}_s]$ as a criterion for optimal choice of the period T, is made taking into account the concrete requirements to the metrological process for determining the quantity ψ_s. From where on the basis of the formulae and the conditions, presented in this work, the choice of the values for the period T is made.

5. Conclusion.

The possible variants for the choice of weight function of ME are analysed and the choice itself is made on the basis of the criteria for sustainability, non-displaceability and effectivity.

A new method is created for optimal choice of the time length of the realizations based on the characteristics of the measured environment and the parameters of the averaging and the memory module of the measuring algorithm.

The mathematical model of the method for choosing the time length is built according to the adaptivity requirement of the measurement system to the concrete work environment, regarding the dynamic accuracy of the measurement.

6. References

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