# Electrical Design of Single-Phase Diode Rectifiers with Active-Capacitive Load by MATLAB 


#### Abstract

Evgeniy Ivanov Popov, Liliya Ivanova Pindeva Department of Power Electronics, Technical University of Sofia, Kl. Ohridski Bul. No.8, 1000 Sofia, Bulgaria, phone: +359 2 9652642, e-mail: epopov@tu-sofia.bg Single-phase diode rectifiers with active-capacitive load widely applied in different supplying sources pose specific problems to the engineers. The known methods ('eastern", based on the assumption that the voltage across the filtering capacitor is almost even and 'western") for their electrical design are graphic-analytical and suffer from limited accuracy and waste of time. Some quantities, namely the r.m.s. values of the transformer secondary and primary winding currents, are specified. The standard transformer power, the powers of its windings, the output (load) characteristic and the distortion coefficient of the primary current are also obtained. MATLAB programs for exact graphical representation of the functions, participating in the full electrical design of the rectifiers and for computer-aided design implementation are created. PSPICE simulations confirm the results for some practical examples.


Keywords: MATLAB design, single-phase, diode rectifiers, capacitive load.

## 1. InTRODUCTION

Often with power supplies for electronic equipment for not large values of the rectified power, there follows after the diode rectifier connection a filtering (smoothing) capacitor. Primarily such circuits filter the direct voltage, not the direct current. The capacitor participates in the rectifying action and influences the principle of operation that becomes completely different compared to the other types of loads [1], [2], [3]. The parameters of the rectifiers with active-capacitive load (Fig.1) depend on the load resistance $R_{d}$, on the capacitance of the filtering capacitor $C$ and the last but not the least on the internal resistance of the rectifier $R_{\text {rect }}$. The last includes the ohmic resistance of the a.c. line and the transformer windings, the resistance introduced by the diodes and the resistance of the protective resistor, which is used to prevent non-permitted surge current when the filtering capacitor is charged after switching-on the equipment.

There are generally two known methods for their electrical design that poses specific problems to the engineers. The first method is published in the Russian and Bulgarian language technical literature [1], [2], [4] and we can call it "eastern". It is based on the simplifying but close to the practice assumption that the rectifier load voltage (across the filtering capacitor) is almost even (rippling factor $0.05 \div 0.2$ ). Other assumptions include the negligence of the leakage inductances and the linear character of the V-A diode characteristics. The second method can be called "western" and it is published in books of the leading companies SEMIKRON and SIEMENS [3], [5]. In this case the value of the rippling factor is not restricted that
additionally complicates the analysis. Both methods are graphic-analytical. Naturally they suffer from limited accuracy and waste of time. It can be proved that the second method is less accurate than the first. It should be underlined that some graphics of the functions in various references applying one and the same method differ from one to another.


Fig.1. Single-phase diode rectifiers.
Hence, the aims of this paper is to further develop the first ("eastern") method assuming almost constant output voltage; to specify the analytical expressions of all the variables; to create two MATLAB programs: (a) for exact graphical representation of the functions, participating in the full electrical design of the rectifiers and (b) for precise design implementation on a computer; to prove the correctness of the design.

## 2. Problem Statement

In the method under consideration [1] first the coefficient $A$ is calculated

$$
\begin{equation*}
A=\frac{\pi R_{r e c t} I_{d}}{m V_{d}}=\frac{\pi R_{r e c t}}{m R_{d}}=\tan \theta-\theta \tag{1}
\end{equation*}
$$

where $I_{d}$ and $V_{d}$ are the average values of the rectified current and voltage, $m$ is the number of phases ( $m=1$ for Fig.1a, $m=2$ for Fig.1b and Fig.1c), $\theta$ is the cut-off angle (Fig.2). Equation (1) expresses the implicit function $\theta=f_{\theta}(A)$ (Fig.3). After its calculation the following parameters are determined from:

The r.m.s. value of the transformer secondary winding voltage $E_{2}$

$$
\begin{equation*}
\frac{E_{2}}{V_{d}}=\frac{E_{2 m}}{\sqrt{2} V_{d}}=B=\frac{1}{\sqrt{2} \cos \theta}=f_{B}(\theta) \text { (Fig.4) } \tag{2}
\end{equation*}
$$

The peak current through the diodes $I_{m}$

$$
\begin{equation*}
\frac{I_{m} \cdot m}{I_{d}}=F=\frac{\pi(1-\cos \theta)}{\sin \theta-\theta \cos \theta}=f_{F} \text { (A) (Fig.5) } \tag{3}
\end{equation*}
$$



Fig.2. Time diagrams of the main quantities.
The r.m.s. value of the transformer secondary winding current $I_{2}$ for the single and double phase half-wave rectifiers (Fig.1a and Fig.1b respectively)

$$
\begin{equation*}
\frac{I_{2} \cdot m}{I_{d}}=D=\frac{\sqrt{\pi[\theta(1+0.5 \cos 2 \theta)-0.75 \sin 2 \theta]}}{\sin \theta-\theta \cos \theta}=f_{D}(\text { A }) \text { (Fig.6) } \tag{4}
\end{equation*}
$$

and for the single phase bridge rectifier (Fig.1c), given correctly in [5]

$$
\begin{equation*}
D_{b r}=\sqrt{2} D \text { (Fig.6) } \tag{5}
\end{equation*}
$$

Expressions (4) and (5) are given differently in the references [1], [2], [4]. Simply they can be explained as follows: in the half-wave circuits (Fig.1a and Fig.1b) the current flows through the conductor of the secondary winding once a period without changing its direction, while in bridge connection (Fig.1c) the current flows through the conductor of the secondary winding twice a period changing the direction. Applying the rules for finding the average values leads to the result, expressed by (4) and (5).

The capacitance of the capacitor for a given rippling factor $S=(0.05 \div 0.2)$ is

$$
\begin{equation*}
C[\mu F] \geq H /\left(S . R_{\text {rect }}\right) \tag{6}
\end{equation*}
$$

where ( $\omega=2 \pi f$ is the mains frequency)

$$
\begin{gather*}
H_{m \neq 1}=\frac{10^{6} .2(\sin m \theta \cos \theta-m \cos m \theta \sin \theta)}{m \omega \pi\left(m^{2}-1\right) \cos \theta}=f_{H} \text { (A) (Fig.7) }  \tag{7}\\
H_{m=1}=\frac{10^{6} .(\theta-\sin \theta \cos \theta)}{\omega \pi \cos \theta}=f_{H} \text { (A) (Fig.7) } \tag{8}
\end{gather*}
$$

The transformer secondary winding power related to the rectified power $S_{2} / P_{d}$ is found from (2),(4),(5) and is drawn in Fig.8. The transformer primary winding current is $\left(N_{t r}=w_{2} / w_{1}\right.$, for Fig.2b $\left.w_{2 a}=w_{2 b}=w_{2}\right)$

$$
\begin{array}{ll}
I_{1}=N_{t r} \cdot D_{b r} I_{d} / m & \text { for Fig.1b } \\
I_{1}=N_{t r} \cdot \sqrt{I_{2}^{2}-I_{d}^{2}} & \text { for Fig.1a } \tag{10}
\end{array}
$$

The normalized output (load) characteristic is (Fig.9)

$$
\begin{equation*}
V_{d} / E_{2 m}=\cos \theta=f_{o}\left[\pi R_{r e c t} I_{d} /\left(m E_{2 m}\right)\right]=f_{o}(\sin \theta-\theta \cos \theta) \tag{11}
\end{equation*}
$$

The distortion coefficient of the primary current, equal to the power factor of the rectifier and connected to the effectiveness of the rectification process is (important nowadays) (Fig.10)

$$
\begin{align*}
& \kappa_{m \neq 1}=\frac{I_{1(1)}}{I_{1}}=\frac{m(\theta-0.5 \sin 2 \theta)}{\sqrt{2} D_{b r}(\sin \theta-\theta \cos \theta)}=f_{\kappa}(A) \quad \text { for Fig.1b and Fig.1c }  \tag{12}\\
& \kappa_{m=1}=\frac{I_{1(1)}}{I_{1}}=\frac{\theta-0.5 \sin 2 \theta}{\sqrt{2}(\sin \theta-\theta \cos \theta) \sqrt{D^{2}-1}}=f_{\kappa}(A) \quad \text { for Fig.1a } \tag{13}
\end{align*}
$$

## 3. Results

MATLAB programs for exact graphical representation of the main functions and for precise design implementation on a computer are created. The graphics are presented in Fig. 3 to Fig.10, as it has been already explained. A special mark shows the position of the parameters on the graphics for the concrete example (Table 3). The numerical results for one and the same example from the second MATLAB program and from the PSPICE simulations are compared in Table 1 for Fig1a, in Table 2 for Fig.1b and in Table 3 for Fig.1c. $S_{1}$ is the transformer primary winding power, $S_{t r}$ is the standard transformer power, $T_{1}$ is the power of distortions at the mains. Results, obtained from [3], [5] are approximately the same but not so precise.

## 4. Conclusions

The method for electrical design of single-phase diode rectifiers, assuming almost constant output voltage, is further developed. The analytical expressions of all the variables are specified. MATLAB programs for exact graphical representation of the functions, participating in the full electrical design of the rectifiers and for precise design implementation on a computer are created. They can be downloaded from: http://mail.dir.bg/~epopov/rectrc.m (for the graphics generation) or http://mail.dir.bg/~epopov/rectrc02.m (for the computer-aided design implementation). Simulations and other methods confirm the results.

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## 5. REFERENCES

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Table 1 (Fig.1a).

| $V_{d}$ | $I_{d}$ | $R_{r e c t}$ | $m$ | $S$ | $P_{d}$ | $\theta$ | $E_{2}$ | $I_{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | A | $\Omega$ | - | $\%$ | W | deg | V | A |
| 12 | 1 | 3.5 | 1 | 10 | 12 | 63.766 | 19.196 | 4.3277 |
| 11.975 | 0.9979 | 3.5 | 1 | 11.82 | 11.95 | 66.348 | 19.258 | 4.3161 |
| $I_{2}$ | $I_{1}$ | $S_{1}$ | $S_{2}$ | $S_{t r}$ | $C$ | $\kappa$ | $T_{1}$ | Method |
| A | A | W | W | VA | $\mu F$ | - | W |  |
| 1.8523 | 0.136 | 29.929 | 35.556 | 32.742 | 4691.8 | 0.8022 | 17.87 | MATLAB |
| 1.8444 | 0.135 | 29.78 | 35.52 | 32.64 | 4691.8 | 0.8039 | 17.71 | PSPICE |

Table 2 (Fig.1b).

| $V_{d}$ | $I_{d}$ | $R_{\text {rect }}$ | $m$ | $S$ | $P_{d}$ | $\theta$ | $E_{2}$ | $I_{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | A | $\Omega$ | - | $\%$ | W | deg | V | A |
| 12 | 1 | 3.5 | 2 | 10 | 12 | 54.714 | 14.689 | 2.5067 |
| 11.969 | 0.9974 | 3.5 | 2 | 10.48 | 11.938 | 57.969 | 14.791 | 2.5016 |
| $I_{2}$ | $I_{1}$ | $S_{1}$ | $S_{2}$ | $S_{t r}$ | $C$ | $\kappa$ | $T_{1}$ | Method |
| A | A | W | W | VA | $\mu F$ | - | W |  |
| 0.998 | 0.0942 | 20.733 | 29.321 | 25.027 | 1817.2 | 0.9151 | 8.36 | MATLAB |
| 0.993 | 0.0937 | 20.624 | 29.372 | 24.998 | 1817.2 | 0.9165 | 8.25 | PSPICE |

Table 3 (Fig.1c).

| $V_{d}$ | $I_{d}$ | $R_{r e c t}$ | $m$ | $S$ | $P_{d}$ | $\theta$ | $E_{2}$ | $I_{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | A | $\Omega$ | - | $\%$ | W | $\operatorname{deg}$ | V | A |
| 12 | 1 | 3.5 | 2 | 10 | 12 | 54.714 | 14.689 | 2.5067 |
| 11.968 | 0.9974 | 3.5 | 2 | 10.48 | 11.937 | 58.734 | 14.791 | 2.5016 |
| $I_{2}$ | $I_{1}$ | $S_{1}$ | $S_{2}$ | $S_{t r}$ | $C$ | $\kappa$ | $T_{1}$ | Method |
| A | A | W | W | VA | $\mu F$ | - | W |  |
| 1.4114 | 0.0942 | 20.733 | 20.733 | 20.733 | 1817.2 | 0.9151 | 8.36 | MATLAB |
| 1.4041 | 0.0937 | 20.624 | 20.768 | 20.696 | 1817.2 | 0.9165 | 8.25 | PSPICE |



Fig.3. $\theta=f_{\theta}(A)$.


Fig.4. $B=f_{B}(A)$.


Fig.5. $F=f_{F}(A)$.


Fig.7. $H=f_{H}(A)$.


Fig.9. $\kappa=f_{\kappa}(A)$.


Fig.6. $D=f_{D}(A)$.


Fig.8. $S_{2} / P_{d}=f_{S 2}(A)$.


Fig.10. $\cos \theta=f_{o}(\sin \theta-\theta \cos \theta)$.

