STATE SPACE MODELING OF 2-D RECURSIVE DIGITAL FILTERS

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In this paper, a novel technique is introduced for the design of recursive digital filters in the state space form. The Roesser's state space model has been used for the spatial domain representation of the 2-D causal recursive digital filters. This model divides the local state into a horizontal and a vertical state which are propagated in horizontal and vertical directions respectively. The design approach for the 2-D recursive filter is realized as approximation problem in time domain. This problem is solved by standard gradient type optimization procedure. The effectiveness of this method has been demonstrated by modeling of the first quadrant Gaussian 2-D IIR filter.

Keywords: 2-D IIR digital filter, state space identification methods

1. INTRODUCTION

There are numerous techniques developed for digital filter design, both in frequency domain and in time domain. Most of these methods are analytical techniques. Typical approximation methods for 2-D FIR and IIR filters are based on windowing, spectral transformations, error criterion optimization, direct least square or minimax procedures [1-7]. 2-D IIR digital filters are usually designed either by spectral transformation of 1-D filter, linear programming, or nonlinear optimization approaches. A design technique based on nonlinear Newton's optimization for 2-D FIR and IIR digital filters is proposed in [3]. One direct method for approximately linear phase 2-D IIR digital filters is presented in [4]. Some of the 2-D IIR digital filter design approaches are realized by solving the weighting lest square optimization problem, where the objective functions are defined as errors between frequency responses and desired target functions [2], [5], [6]. The main idea of the other methods is to formulate the 2-D digital filters approximation in frequency domain as a minimax optimization problem [1], [7].

Various design techniques have been proposed for 2-D recursive digital filters, either in frequency domain or in spatial domain [8] -[11]. Most of those techniques are for a special class of 2-D filters called as separable-in-denominator digital filters [8], [11]. This is due to the fact that these spatial 2-D filters share some important properties of 1-D digital filters such as stability and minimality conditions. Therefore, many 2-D spatial design techniques have been developed using separable-in-denominator digital filters as the extensions of corresponding 1-D techniques [8], [11]. There are relatively few techniques developed on identification and design of general 2-D recursive digital filters. However, the problem of general 2-D identifications using an analytical solution has not been addressed due to its mathematically complex nature.

An IIR filter can be represented by either difference equation or state space form. The state space form in general involves more numbers of coefficients than a transfer function unless it is represented as one of the canonical forms. However, there are many benefits from using a state space model in the analysis, design, and implementation of digital filters. First, the state space model is more robust than a transfer function representation. In other words, it exhibits less coefficient sensitivity. Second, various forms of state space models possess distinctive properties that are desirable in different applications. Third, the major part of modern control theory is based on the state space model. Furthermore, the difference function representation could be uniquely determined by the state space form representation.

In this paper, the state space model will be utilized for the 2-D IIR digital filter design. The filter's input and resulting output could be selected arbitrarily by the designer in spatial domain. In other words, the proposed technique can be uniformly applied for identification of a 2-D filter with an impulse response, a step response or a response to a random 2-D input signal.

2. 2-D STATE SPACE MODEL OF DIGITAL FILTERS

The 2-D state space models have been mainly used for the spatial domain representation of the 2-D causal recursive digital filters. Kung et al. [13] have shown that the Roesser's model [12] is the most general form and the other representations can be imbedded in the Roesser's model. Roesser's state space model divides the local state into a horizontal and a vertical state which are propagated in horizontal and vertical directions respectively.

Consider the transfer function of a stable 2-D digital filter of order N_1 in z_1^{-1} and N_2 in z_2^{-1}:

$$H(z_1,z_2)=\frac{N(z_1,z_2)}{D(z_1,z_2)} \tag{1}$$

The input-output relation of the filter is represented by the following Roesser's state-space equations [12]:

$$\begin{bmatrix} \mathbf{x}_h(n_1+1,n_2) \\ \mathbf{x}_v(n_1,n_2+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} \begin{bmatrix} \mathbf{x}_h(n_1,n_2) \\ \mathbf{x}_v(n_1,n_2) \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} u(n_1,n_2) \tag{2}$$

$$y(n_1,n_2) = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_h(n_1,n_2) \\ \mathbf{x}_v(n_1,n_2) \end{bmatrix} + \mathbf{d}u(n_1,n_2)$$

where

n_1 is an integer-valued horizontal coordinate;

n_2 is an integer-valued vertical coordinate;

$u(n_1,n_2) \in R^p$ is the scalar input of the filter;

$y(n_1,n_2) \in R^q$ is the scalar output of the filter;

$\mathbf{x}_h(n_1,n_2) \in R^{N_1}$ is the $N_1 x1$ horizontal state vector;

$\mathbf{x}_v(n_1,n_2) \in R^{N_2}$ is the N_2 x1 vertical state vector.

Matrices $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4, \mathbf{b}_1, \mathbf{b}_2$, $\mathbf{c}_1, \mathbf{c}_2$ and $\mathbf{d}$ are real coefficient matrices with appropriate dimension.

This means that the state-space model has the dimension (N_1, N_2).

The block diagram of the 2-D state-space digital filter is given in Figure 1.

The transfer function $H(z_1,z_2)$ of the 2-D digital filter is given in terms of the coefficient matrices as:

$$H(z_1,z_2) = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{bmatrix} \left[\begin{array}{c|c} z_1\mathbf{I}_{N_1} - \mathbf{A}_1 & -\mathbf{A}_2 \\ \hline -\mathbf{A}_3 & z_2\mathbf{I}_{N_2} - \mathbf{A}_4 \end{array} \right]^{-1} \cdot \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} + \mathbf{d} \tag{3}$$

where $\mathbf{I}_{N_1}$ and $\mathbf{I}_{N_2}$ are the N_1 x N_1 and N_2 x N_2 identity matrices, respectively.

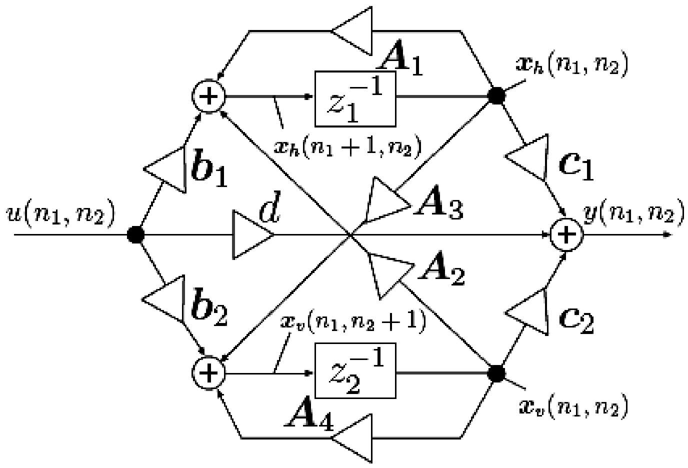

Fig. 1. Block diagram of a 2-D state-space digital filter

3. DESIGN TECHNIQUE

The design approach for the 2-D recursive filter is realized as approximation problem in time domain. This problem is solved by standard gradient type optimization procedure. The following L_2. norm [8]-[10] error criteria are defined:

$$\varepsilon_2 = \left[\sum_{(n_1,n_2 \in Z)} (h(n_1,n_2) - h_1(n_1,n_2))^2 \right]^{1/2} \Bigg/ \left[\sum_{(n_1,n_2 \in Z)} (h(n_1,n_2))^2 \right]^{1/2}$$

where $h(n_1,n_2)$ and $h_1(n_1,n_2)$ are the impulse responses of the original system and identified system, respectively, $Z = \{(n_1,n_2) \,|\, 0 < n_1 < N_1, 0 < n_2 < N_2\}$ is the given region where the error norms are calculated.

4. RESULTS

Example: First Quarter Gaussian Filter

The prototype model used by Aly and Fahmy in [9] for designing a 2-D causal recursive filter is presented in the work. It is a first quadrant Gaussian 2-D scalar filter described by the following impulse response:

$$h(n_1,n_2) = 0.256322 \exp\{-0,103203\,[(n_1-4)^2+(n_2-4)^2]\}$$

The selected region for identification consists of

$$Z = \{(n_1,n_2)\,|\,0 < n_1 < 10,\, 0 < n_2 < 10\}.$$

This impulse response is illustrated with the surface shown in Fig. 2. The identified 2-D filter in Roesser's model of order (3, 3) is given as:

$$\mathbf{A} = \begin{bmatrix} 3.0055 & -1.8843 & 2.2310 & 0.8824 & -0.3118 & -0.6487 \\ 2.0826 & -0.9692 & 2.0174 & 1.4865 & -0.5229 & -1.0908 \\ -0.9142 & 0.6924 & -0.1314 & 0.3518 & -0.1214 & -0.2584 \\ \\ -0.1757 & 0.1225 & -0.1563 & 0.4883 & -0.2750 & -0.1673 \\ 0.4458 & -0.2816 & 0.3202 & 0.8414 & 1.2680 & 0.4763 \\ -0.4550 & 0.3104 & -0.3876 & -0.8693 & -0.8770 & 0.1436 \end{bmatrix};$$

$\mathbf{b}_1 = [-6.6533e-3, -1.1209e-2, -2.6681e-3]^T$;

$\mathbf{b}_2 = [-1.0466e-1,\ 6.0182e-2,\ -1.4947e-1]^T$;

$\mathbf{c}_1$ = [3.8730e1 -2.9240e1 1.8920e1] ;

$\mathbf{c}_2$ = [-1.2433 4.3620e-1 9.1150e-1] ;

$\mathbf{d}$ = 9.4010e-3.

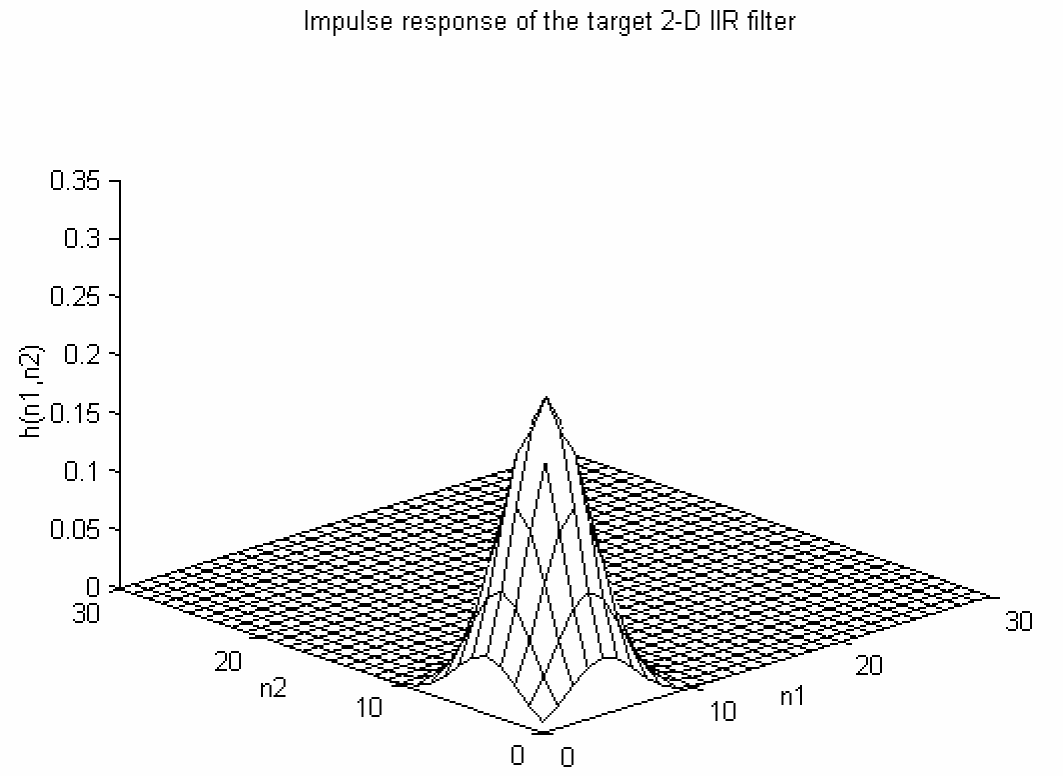

Fig. 2. Impulse response of the 2-D Gaussian Filter

The impulse response of the identified 2-D IIR filter is shown in Fig. 3.

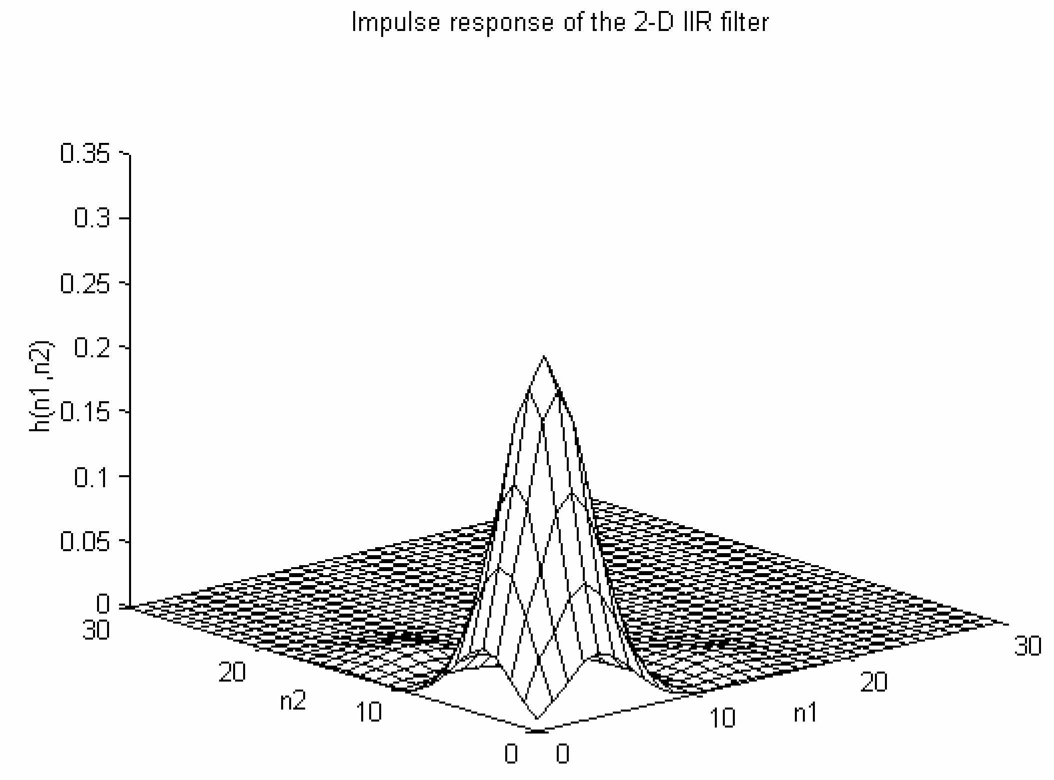

Fig. 3. Impulse response of the identified 2-D Filter

5. CONCLUSIONS

In this paper, a novel technique is introduced for the design of recursive digital filters in the state space form.

The proposed method has no limitation on the type of response to be approximated. Any type of responses with sufficient data points could be used as a sample for filter identification.

The effectiveness as well as robustness of this method has been demonstrated by modeling of the first quadrant Gaussian 2-D IIR filter.

6. REFERENCES

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