PERFORMANCES OF RAKE RECEPTION OF DS-CDMA SIGNALS WITH MAXIMAL RATIO AND EQUAL GAIN COMBINING IN RAYLEIGH FADING

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The RAKE correlator is a digital receiver for DS-CDMA signals that takes advantage of the autocorrelation properties of the spreading/scrambling sequences in order to resolve and efficiently combine the multipath received components in order to increase the average received Signal to Noise Ratio (SNR). Assuming slow, selective, uncorrelated Rayleigh fading with an equally spaced exponential power delay profile, a decision aided (DA) scheme is proposed in order to estimate the complex channel coefficients needed for coherent combining. Monte Carlo simulations are carried out to comparatively assess the performance of Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC) RAKE reception. Channel estimation using the UMTS downlink Primary Common Pilot Channel (P-CPICH) is discussed. Thus, the rough estimates resulted after descrambling the corresponding signal are further refined by way of the above mentioned DA scheme.

Keywords: RAKE receiver, Bit Error Rate, Rayleigh fading channel

1. INTRODUCTION

Transmissions in indoor and congested urban areas suffer from frequency selective fading, a phenomena which may severely degrade the Bit Error Rate (BER). Diversity combining is a method of mitigating the effects of multipath propagation, time and space diversity being the most common practical implemented solutions. It assumes that a set of replicas of the same transmitted signal (more or less independently affected by fading) is present at the receiver end. The desiderate is to *combine* the identified replicas in order to obtain a signal whose SNR is larger than that of any received multipath component. In order to achieve this in the context of DS-CDMA systems one can take advantage of the implicit time diversity (induced by multipath fading) and the autocorrelation properties of the codes using a RAKE receiver (or correlator).

2. MULTIPATH CHANNEL AND RECEIVER MODEL

2.1. Transmitted signal model

Let us consider a binary DS-CDMA system employing BPSK modulation with K active and independent users. The composite baseband downlink transmitted signal at the input of the channel is given by

$$z(t) = \sum_{p=0}^{K} \sqrt{2S}\, z^{(p)}(t) \tag{2.1}$$

where S is the power of the transmitted signal at the common carrier frequency f_0 and $z^{(p)}(t) = c^{(p)}(t)s^{(p)}(t)$, $p = \overline{1,K}$ are the complex envelopes of the signals transmitted for each of the K users. Each pth user is assigned a unique code sequence $c_i^{(p)} \in \{\pm 1\}$

with the corresponding code waveform $c^{(p)}(t)$ given by (2.2) where T_{ch} defines the chip durration and $P_{T_{ch}}(\cdot)$ is the unit rectangular pulse of duration T_{ch}. Similarly the data

$$c^{(p)}(t) = \sum_i c_i^{(p)} P_{T_{ch}}(t - iT_{ch}) \qquad (2.2) \qquad\qquad s^{(p)}(t) = \sum_i s_i^{(p)} P_{T_b}(t - iT_b) \qquad (2.3)$$

signal waveform is given in (2.3). The ratio between T_{ch} and the bit durration T_b defines the processing gain G_P.

2.2. Multipath channel and received signal model

We assume a frequency selective, slow Rayleigh fading channel which can be modeled by a linear filter with the impulse response (as seen by the p_0th user)

$$h^{(p_0)}(t) = \sum_{l=1}^{L} a_l^{(p_0)} \exp\left(-j\theta_l^{(p_0)}\right)\delta\left(t - \tau_l^{(p_0)}\right) = \sum_{l=1}^{L} h_l^{(p_0)}\delta\left(t - \tau_l^{(p_0)}\right),\ 0 < p_0 \le K \qquad (2.4)$$

where L is the number of received multipath components, $h_l^{(p_0)} = a_l^{(p_0)} \exp\left(-j\theta_l^{(p_0)}\right)$ are the complex channel coefficients (fading amplitudes and phases, respectively), $\tau_l^{(p_0)}$ are the corresponding multipath delays and $\delta(t)$ is the Dirac pulse. Considering that $\Omega_l^{(p_0)} = E\left\{ a_l^{(p_0)2} \right\}$ and defining the average SNR of the lth multipath component as $\bar{\gamma}_l^{(p_0)} = \Omega_l^{(p_0)} \frac{E_b}{N_0}$ the channel model may be further extended considering an equally spaced exponential power delay profile with respect to the first received replica i.e.

$$\bar{\gamma}_l^{(p_0)} = \bar{\gamma}_1^{(p_0)} \exp\left[-(l-1)\delta\right],\ \delta \ge 0,\ l = \overline{1, L} \qquad (2.5)$$

where the quantity δ is called the power decay factor. The signal recieved by the p_0th user may be written as

$$r^{(p_0)}(t) = \sum_{p=0}^{K} \sum_{l=1}^{L^{(p_0)}} \sqrt{2S^{(p)}} a_l^{(p_0)} \exp(-j\theta_l^{(p_0)}) z^{(p)}(t - \tau_l^{(p_0)}) + n^{(p_0)}(t) \qquad (2.6)$$

where $n^{(p_0)}(t)$ is the additive white Gaussian noise (AWGN) modeled as a stochastic process with zero mean and two sided power spectral density $\frac{N_0}{2}$. Note that although there are K active users in the system there is an extra term in the (2.1) and (2.6) sums denoted by p=0 indicating the presence of a pilot channel. In the case of the downlink UMTS this identifies with the P-CPICH. This channel is characterised by the fact that the transmitted simbols of 1+j are scrambled with the cell specific primary scrambling code. The function of the P-CPICH is to aid the channel estimation (coefficients and/or multipath delays) at the terminal for the dedicated channel, to provide channel estimation reference for the common channels when they are not associated with the dedicated channels and is also used for handover and cell selection/reselection measurements. The P-CPICH is transmitted with a fixed rate of 30kbps which corresponds to G_P=256.

2.3. Receiver model

We shall consider a $L_0 \le L^{(p_0)}$ fingers matched filter RAKE receiver as shown in figure 1, where the order of diversity L_0 is an implementation defined parameter.

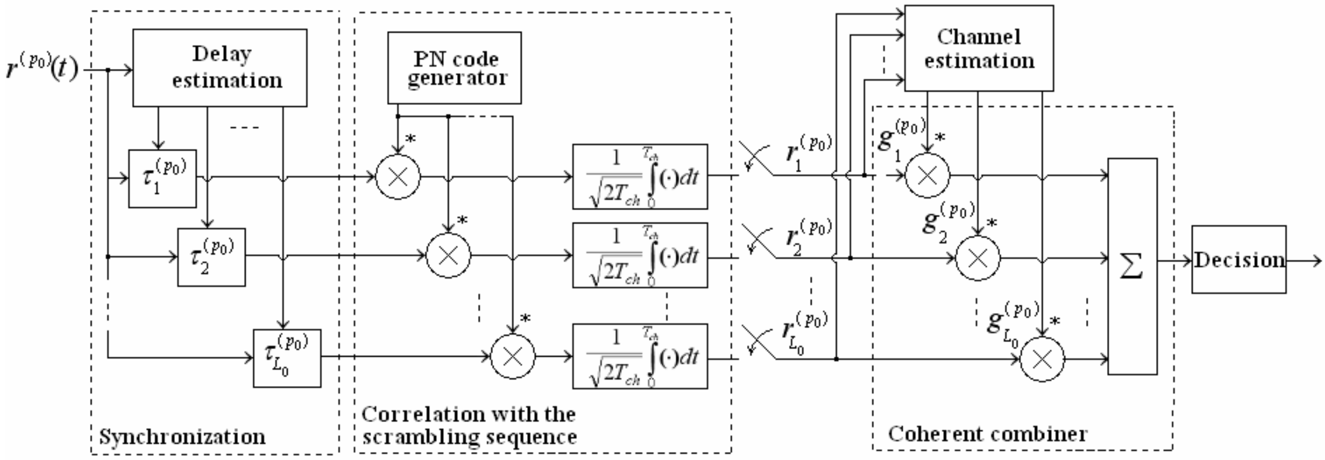


Figure 1. - RAKE receiver functional block diagram with L_0 fingers

Assuming perfect knowledge of the multipath delays and perfect phase synchronization, let $\{\boldsymbol{r}_l^{(p_0)}\}$ denote the $\boldsymbol{l} = \overline{1, \boldsymbol{L}_0}$ matched filter outputs. These quantities are linearly and individually weighted and then coherently combined to form the *p*th user's decision variable given by

$$\Lambda^{(p_0)} = \sum_{l=1}^{L_0} g_l^{(p_0)} r_l^{(p_0)} \tag{2.6}$$

where $\boldsymbol{g}_l^{(p_0)}$ is the weight of the *l*th branch. If MRC is assumed then the weighting coefficients are given in (2.7a). Also, it can be shown that the MRC-RAKE receiver with perfect fading amplitudes estimates (ideal MRC) is optimal in the Maximum Likelihood (ML) sense yielding the best possible output average SNR. The weights for

$$g_l^{(p_0)} = a_l^{(p_0)},\ l = \overline{1, L_0} \quad (2.7a) \qquad\qquad \boldsymbol{g}_l^{(p_0)} = 1 \quad (2.7b)$$

the much simpler but suboptimal EGC alternative are all equal to unity as shown in (2.7b). Thus the decision variable for the *p*th user may be rewritten as

$$\Lambda^{(p_0)} = \pm\left(\sum_{l=1}^{L_0} g_l^{(p_0)} a_l^{(p_0)}\right)\sqrt{E_b} + \sum_{l=1}^{L_0} g_l^{(p_0)} (I_S + I_M + n) \tag{2.8}$$

where $E_b = ST_b$ is the bit energy I_S, I_M are self interference and multiple access interference terms, respectively, and n is the zero mean AWGN component with $\frac{N_0}{2}$ variance. The interference terms may be also modeled as zero mean gaussian random random variables (RVs)

$$I_S \sim N\left(0, \frac{\Omega_T^{(p_0)} - 1}{2G_p} \Omega_1^{(p_0)} E_b\right) \quad (2.9) \qquad I_M \sim N\left(0, \frac{(K-1)\Omega_T^{(p_0)}}{3G_p} \Omega_1^{(p_0)} E_b\right) \quad (2.10)$$

where $\Omega_T^{(p_0)} = \frac{\sum_{l=1}^{L_0} \Omega_l^{(p_0)}}{\Omega_1^{(p_0)}}$. Thus we can define the equivalent interference/noise two sided spectral density as

$$\frac{N_e^{(p_0)}}{2} = \sigma_{N_e}^{(p_0)2} = \left[\frac{(2K+1)\Omega_T^{(p_0)} - 3}{6G_p}\right] E_b + \frac{N_0}{2} \tag{2.11}$$

and reevaluate the average SNR of the lth multipath component as $\bar{\gamma}_l^{(p_0)} = \Omega_l^{(p_0)} \frac{E_b}{N_e}$.

As stated before, the RAKE receiver must estimate the delays of the multipath components (on a per symbol basis, as slow fading is assumed) to be combined and the channel fading amplitudes and/or phases for each finger. For the UMTS downlink, these can be achived using the pilot channel or pilot symbols in the Downlink Dedicated Physical Channel (DPCH). The first approach, although computational intensive, is simpler as no interpolation between adjacent slots is needed to obtain the channel coefficients and this is to be our method of choice in the following. The signal corresponding to the pilot channel is sampled with a period equal to T_s in order to estimate the multipath delays and channel coefficients. Assuming perfect estimates for the multipath delays, the identified replicas of the received signal are multiplied with the conjugate of the known scrambling code to yield a set of $P_0 = G_P \frac{T_{ch}}{T_S}$ samples that are each to be viewed as a rough estimate for the channel coefficient (eq. 2.10). To reduce the computational effort, the final estimate obtained

$$\boldsymbol{h}_l^{(\boldsymbol{p})} = [h_l^{(p_0)} + n_1 \quad h_l^{(p_0)} + n_2 \quad \ldots \quad h_l^{(p_0)} + n_{P_0}] \tag{2.12}$$

via the DA scheme shall be computed using only $M_0 = \frac{P_0}{N_0}$ out of the available $\boldsymbol{P}_0$ samples. In equation 2.10 n_i are noise samples with variance given by 2.8. The DA estimate is given by

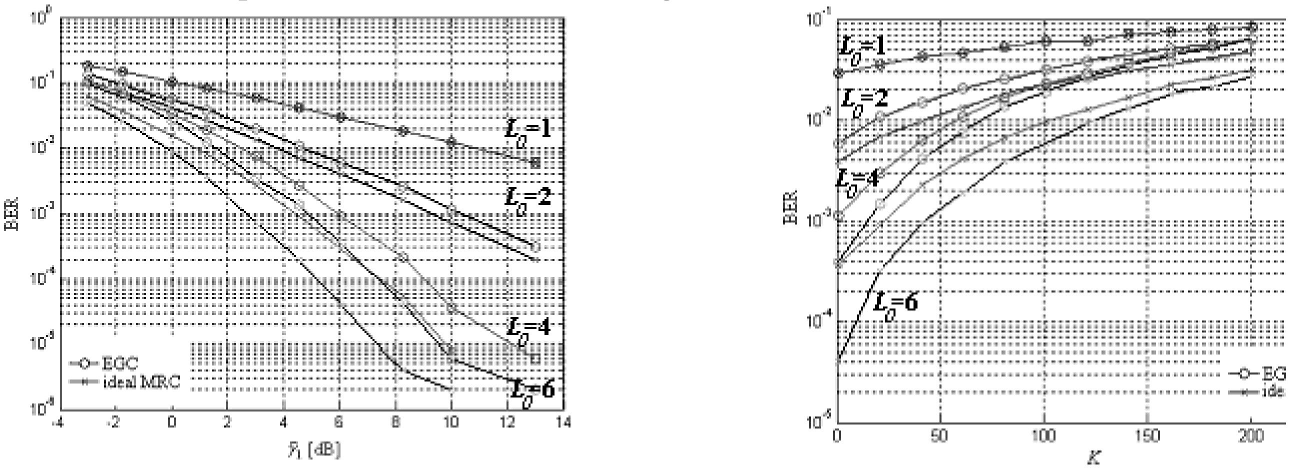
$$\hat{h}_l^{(p_0)} = \frac{1}{M_0} \sum_{k=0}^{M_0-1} h_l^{(p_0)}(k) d^*(k) \tag{2.13}$$

where $d(k)$ are the known symbols transmitted on the pilot channel. Therefore, taking into account that in the particular case of the P-CPICH we have $E\{\hat{h}_l^{(p_0)}\} = h_k$ and that $\sigma^2_{\hat{h}_k^{(p_0)}} = \frac{1}{M_0^2} \sum_{j=1}^{M_0} \sum_{\substack{i=1 \\ i \neq j}}^{M_0} E\{n_i n_j\} + \frac{1}{M_0^2} \sum_{j=1}^{M_0} E\{n_j^2\}$, it is obvious that $\hat{h}_l^{(p_0)}$ is a normal RV.

$$\hat{h}_l^{(p_0)} \sim N\left(h_l^{(p_0)}, \frac{\sigma_{N_e}^{(p_0)2}}{M_0} \right) \tag{2.14}$$

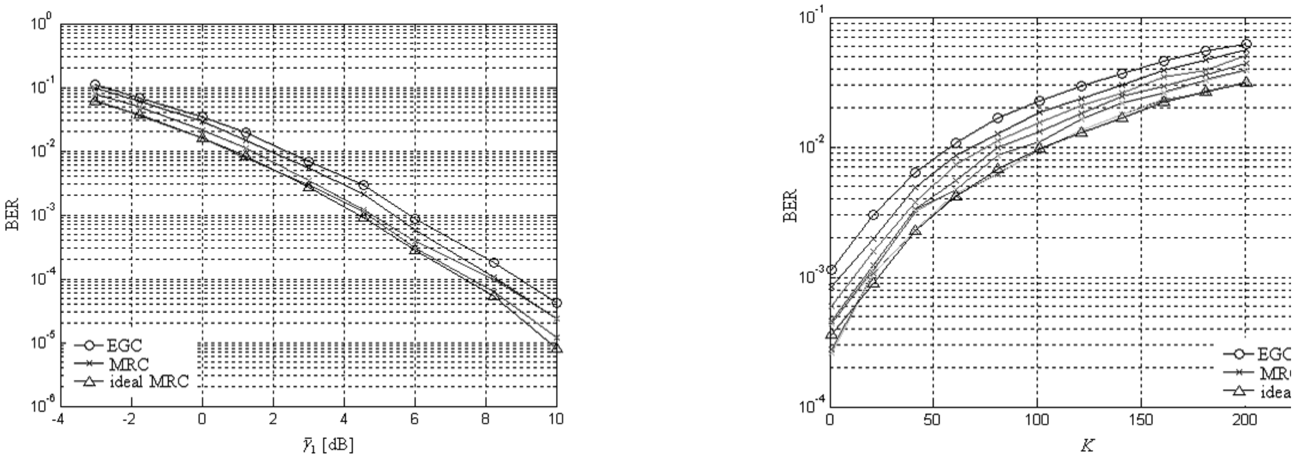
3. Performance Analysis

Although many of the formulas above have been derived assuming BPSK modulation, extensions to QPSK are immediate. Simulations were carried out choosing a spreading factor of 256 and a null power decay factor. The bit energy is considered to be equal to the unit and the second moment of $a_l^{(p_0)}$ equal to 2. A comparative analysis of MRC- and EGC-RAKE reception had been undertaken with respect to the average SNR of the replicas and the number of active users in the system. Simulations indicate that, in the hypothesis of perfect channel estimates (figures 2a and 2b) the performance of RAKE reception employing MRC is superior to that of RAKE-EGC, this aspect being more visible as the order of diversity L_0 increases. Note that as the average SNR increases (see fig. 2a), for a fixed value of L_0, the MRC improvement becomes more significant.



(a) (b)

Figure 2. RAKE-MRC vs. RAKE-EGC performance in Rayleigh fading channel for different orders of diversity ($L_0 \in \{1,2,4,6\}$) with respect to (a) the average SNR of the first received multipath component and (b) the number of active users



(a) (b)

Figure 3. BER degradation for the RAKE-MRC receiver with imperfect channel estimates. M_0 was chosen to be equal to (a) M_0=4,8,16 and (b) 4,8, 12,16, 64. (L_0=4)

In figures 3a and 3b we observe that if the number of samples M_0 on which the estimation is based is subject to poor choice, a significant performance degradation for RAKE-MRC reception is observed. However, as the number of samples considered for the DA estimation increases, the performance of imperfect RAKE-MRC approaches that of the ideal MRC, the resulting curves providing valuable guidelines for an adequate choice of this parameter.

4. CONCLUSIONS

We have compared the BER performance of DS-CDMA systems with MRC- and EGC-RAKE reception over slow, selective, uncorrelated Rayleigh fading. Channel estimation using the UMTS downlink P-CPICH has been discussed this being a simple but computational intensive approach. The rough estimates resulted after descrambling the corresponding signal are further refined by way of the DA scheme considering less samples than available in order to reduce the computing time. The resulting curves provide guidelines for choosing the number of samples on which the DA estimation is based in order to maintain acceptable performance for MRC-RAKE reception.

5. REFERENCES

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