MULTIUSER DETECTION ALGORITHMS

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This paper presents several multiuser detector algorithms: the conventional detector, the linear MMSE detector and the linear adaptive MMSE one. After a brief theoretical introduction, these algorithms are presented, emphasizing their advantages and disadvantages. Monte Carlo simulations have been performed for the linear MMSE and linear adaptive MMSE detectors, using a 4 user system with equal and different amplitudes, and using orthogonal and nonorthogonal codes. The results and several intereting conclusions are presented in section 4.

Keywords: multiuser detection, MMSE, adaptive, error probability

1. THE SYSTEM MODEL. THE CONVENTIONAL DETECTOR

The basic CDMA N user system model assumes that all users transmits binary data and antipodally synchronous signature waveforms. The channel is affected only by additive white Gaussian noise (AWGN), The received signal is therefore,

$$y(t) = \sum_{k=1}^{N} A_k b_k s_k(t) + \sigma n(t) \quad t \in [0, T] \tag{1}$$

where T is the bit period, b_k {-1,1}is the information bit transmitted by user k during time interval T, A_k is the amplitude of data received from user k , $n(t)$ is the AWGN with unit power spectral density (which models the thermal noise and all other noise sources unrelated to the transmitted signals) and σ is the standard deviation of the noise. The code sequences are normalized such that

$$\int_0^T s_k^2(t)dt = 1, \ (\forall)k \tag{2}$$

The cross-correlation between the i^{th} user sequence and the j^{th} one is defined by

$$\rho_{ij} = \int_0^T s_i(t) s_j(t) dt < 1 \tag{3}$$

and the cross-correlation matrix is given by

$$\mathbf{R} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{N1} & \rho_{N2} & \cdots & \rho_{NN} \end{pmatrix} \tag{4}$$

The simplest strategy to demodulate CDMA signals is the use of a bank of matched filters that operates simultaneously, each of them matched to one user signature signal $h_k(t) = s_k^*(T-t), \ k = 1, \ N$. It represents the simplest linear detection approach, and maximized the signal to noise ratio at each matched filter output, assuming that the channel noise and interference may be assimilated with AWGN. The block diagram of the conventional detector is shown in figure 1.

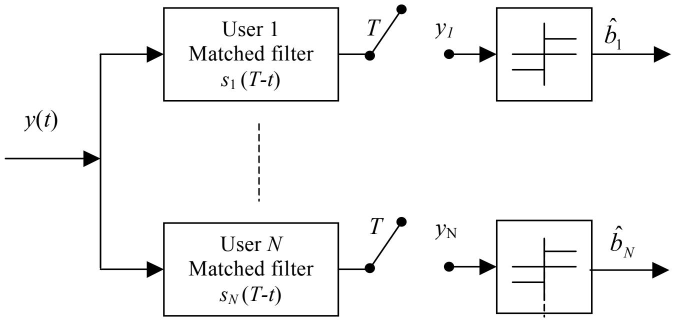

Fig. 1. Conventional Detector

For the received signal given in (1), the k^{th} user matched filter output, sampled at the end of the bit period is

$$y_k = \int_0^T y(t)s_k(t)dt = A_k b_k + \sum_{j\neq k} A_j b_j \rho_{jk} + n_k,\ k = 1, N \tag{5}$$

where
$$n_k = \sigma\int_0^T n(t)s_k(t)dt \quad k = \overline{1,N} \tag{6}$$

is a gaussian random variable with zero mean and variance σ^2. Using a matrix representation, (5) becomes,

$$\mathbf{Y} = \mathbf{RAb} + \mathbf{N} \tag{7}$$

where $\mathbf{Y} = [y_1, y_2, \ldots, y_N]^T$ is a column vector that includes the outputs of the matched filters, $\mathbf{R}$ is given in (4), $\mathbf{A} = diag\{A_1, A_2, \ldots, A_N\}$ is the matrix of the amplitudes of the received bits, $\mathbf{b} = [b_1, b_2, \ldots, b_N]^T$ is a column vector that contains the bits received from all users, and $\mathbf{N} = [n_1, n_2, \ldots, n_N]^T$ is the sampled noise vector., such that

$$E[\mathbf{N}\cdot\mathbf{N}^T] = \sigma^2\mathbf{R} \tag{8}$$

The estimated bit, after the threshold comparison, is

$$\hat{b}_k = \operatorname{sgn}(y_k) = \operatorname{sgn}(A_k b_k + n_k) \tag{9}$$

2. Linear Minimum Mean Square (MMSE) Multiuser Detector

The linear mean square multiuser (MMSE) detector uses a linear combination of the matched filter outputs that minimizes the mean square error between these outputs and the correspondent transmitted data, maximizing thus the bit error probability. For each user k, the detector has to determine a vector of N weighting coefficients m_k that maximize

$$E\left[\left(b_k - m_k^T y\right)^2\right] \tag{10}$$

Taking into account all users and using the matrix representation (7), this leads to

$$\min_{M \in R^{N\times N}} E\left[\left\|\mathbf{b}-\mathbf{MY}\right\|^2\right] \tag{11}$$

where **M** is a $N \times N$ matrix whose k column is equal to m_k. Algebraic caculations leads to the solution

$$M^* = \mathbf{A}^{-1}\left[\mathbf{R}+\sigma^2\mathbf{A}^{-2}\right]^{-1} \tag{12}$$

where it has been assumed that **A** is a non-singular matrix (i.e. only those user that are active are taken into consideration). Extracting the column corresponding to each user, the estimated bit associated to user k is

$$\hat{b}_k = \mathrm{sgn}\left(\frac{1}{A_k}\left(\left[R+\sigma^2A^{-2}\right]^{-1}y\right)_k\right) = \mathrm{sgn}\left(\left(\left[R+\sigma^2A^{-2}\right]y\right)_k\right) \tag{13}$$

The linear MMSE detector is represented in figure 2, where the linear transformation applied to the matched filter outputs is $\left[R+\sigma^2A^{-2}\right]$, where $\sigma^2A^{-2} = diag\left\{\frac{\sigma^2}{A_1^2},\dots,\frac{\sigma^2}{A_2^2}\right\}$. The decision is affected by the amplitude of the transmitted bits by means of the ratios A_k/σ

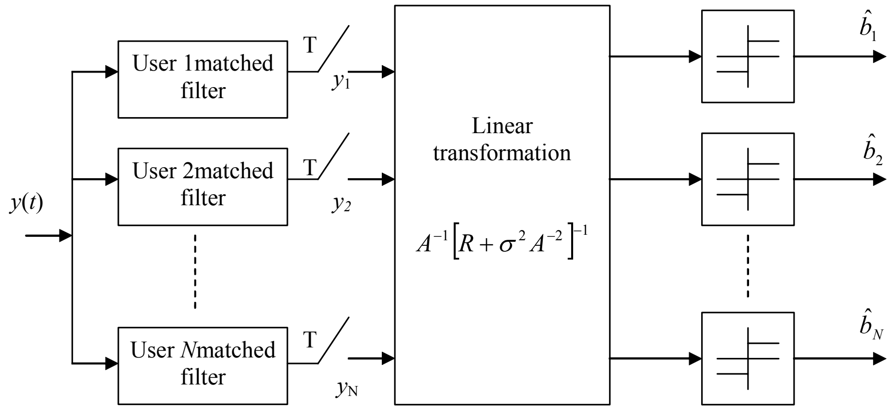

Fig.2. Linear MMSE detector

3. Linear Adaptive Minimum Mean Square (MMSE) Multiuser Detector

The structures of linear detectors (like the conventional and the MMSE ones) are attractive because they can be implemented in a decentralised fashion. However, those implementations needs the apriori knowledge of the signature waveforms and of the amplitudes of each user. For asynchronous channels, where cross-correlations are time-varying or for channels with time-varying receiver powers it might be desirable to implement a multiuser detector able to eliminate the need of on-line computation of the impulse response and of the cross-correlations between users. This goal can be accomplished by using an adaptive implementation of the MMSE detector, which "learns" the desired impulse responses from the received waveform, *provided that the data of the desired user are apriori known to the receiver.* In practice, this requires the transmission of a *training sequence* prior to the transmission of the actual data string. The receiver uses an adaptive law to adjust its linear transformation during the transmission of the training sequence. If cross-correlations and / or amplitudes vary over time, the training sequences has to be send periodically to readjust the receiver.

Considering the convex cost penalty function

$$\Psi(\mathbf{v}) = E\left[\left(u - \mathbf{v}^T\mathbf{w}\right)^2\right] \tag{14}$$

where u is a random scalar and $\mathbf{w}$ is an L-dimensional vector, then, starting from a given initial condition, the minimization of (14) can be made using the steepest descent algorithm [2]

$$\mathbf{v}[n] = \mathbf{v}[n-1] - \mu\left(\mathbf{v}^T[n-1]\mathbf{w}[n] - u[n]\right)\mathbf{w}[n] \tag{15}$$

where $[n]$ is the step index and μ is the step size (fixed or dependent on the step; tipically $\mu_n = \frac{1}{n}$). The argument that minimize (14) is [2,3]

$$\mathbf{v}^* = E\left[\mathbf{w}\mathbf{w}^T\right]^{-1} E[u\mathbf{w}] \tag{16}$$

Using the above algorithm and choosing $u[n] = b_1[n], \mathbf{w}[n] = \mathbf{r}[n]$, where $\mathbf{b}_1$ is either the training sequence or the data step n from user 1 and $\mathbf{r}$ is the received sequence, the MMSE weighting coefficients are

$$\mathbf{v}^* = E\left[\mathbf{r}\mathbf{r}^T\right]^{-1} E[b_1\mathbf{r}] \tag{17}$$

A block diagram that implements the above algorithm is shown in figure 3.

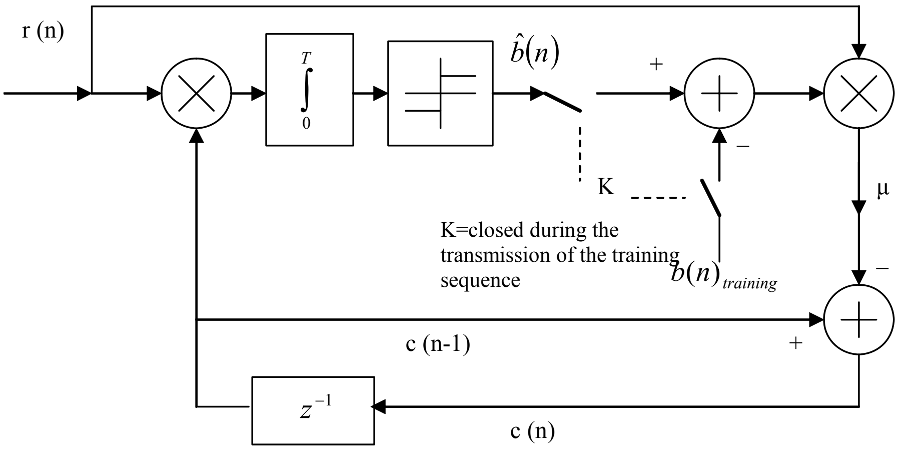

Fig.3. Implementation of the linear adaptive MMSE detector

The convergence towards the solution that minimize the MMSE is achieved only if the amplitudes and the inter-correlation coefficients are constant; If those parameters varies slowly (relative to the algorithm convergence speed) it is possible that the algorithm will be able to track those variances. However, if there is a sudden change in the CDMA channel parameters (such as a strong interferer user that starts transmiting) the algorithm will start using unreliable decisions that will be used instead of true data, so it will no longer converge. There become necessary for the receiver to be able to detect those sudden changes in the channel and to require retransmission of the training sequence.

4. SIMULATION RESULTS AND CONCLUSIONS

Monte Carlo simulations have been performed for the linear MMSE and linear adaptive MMSE detectors, using a 4 user system with equal and different amplitudes, and using orthogonal and non-orthogonal codes of length 64. In both cases 10000 bits have been transmitted and for each vaue of BER the average have been made over 100 trials, In the case of the adaptive MMSE detector, a 100 bits training sequence has been transmitted prior to the effective data. The crosscorrelation matrix in the case of non-orthogonal users is

$$\mathbf{R} = \begin{bmatrix} 1 & -0{,}5 & 0{,}25 & 0{,}25 \\ -0{,}5 & 1 & 0{,}25 & 0{,}25 \\ 0{,}25 & 0{,}25 & 1 & -0{,}5 \\ 0{,}25 & 0{,}25 & -0{,}5 & 1 \end{bmatrix} \qquad (18)$$

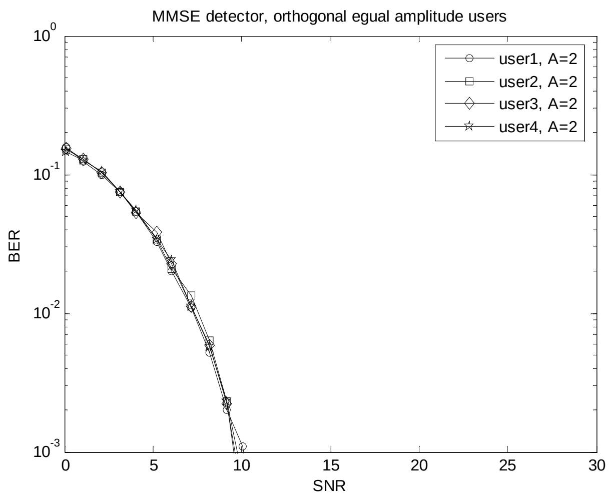

Fig. 4. Error probability vs SNR for MMSE detector, equal amplitude orthogonal users

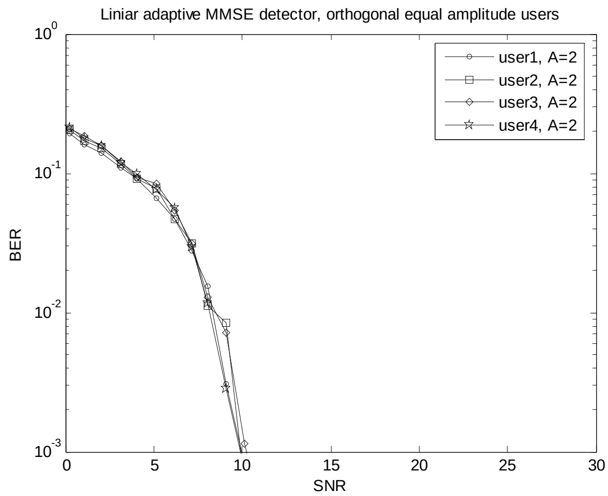

Fig. 5. Error probability vs SNR for adaptive MMSE detector, equal amplitude orthogonal users

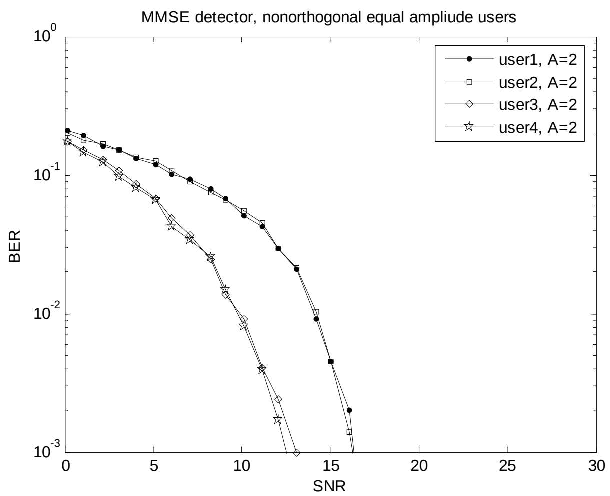

Fig. 6. Error probability vs SNR for MMSE detector, equal amplitude non-orthogonal users

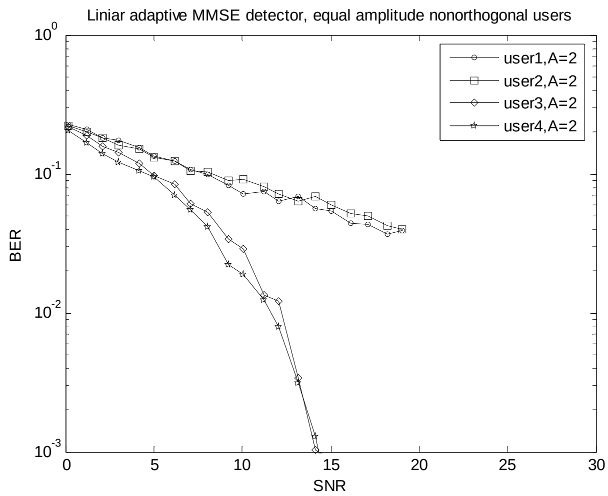

Fig.7. Error probability vs SNR for adaptive MMSE detector, equal amplitude non- orthogonal users

Based on those results several conclusions can be highlighted:

- in the case of orthogonal, equal amplitude users, the difference between users is negligible; moreover, the difference between the results obtained in the case of orthogonal MMSE receiver are slightly better (less then 3dB) then the ones obtained by the orthogonal adaptive MMSE receiver;
- in the case of non-orthogonal, equal amplitude users, the two less-correlated users achieves better results then the closer-correlated ones, similar to those obtained in the orthogonal case, but the results obtained by the closer-correlated users are worse; however, the results obtained for the closer-correlated users are slightly better in the in the case of orthogonal MMSE receiver then the the ones obtained by the orthogonal adaptive MMSE receiver when the SNR is small (less then 3dB difference for SNR<10dB); when the SNR increases, the difference between the two structures increases (larger the 10dB for SNR>15dB);

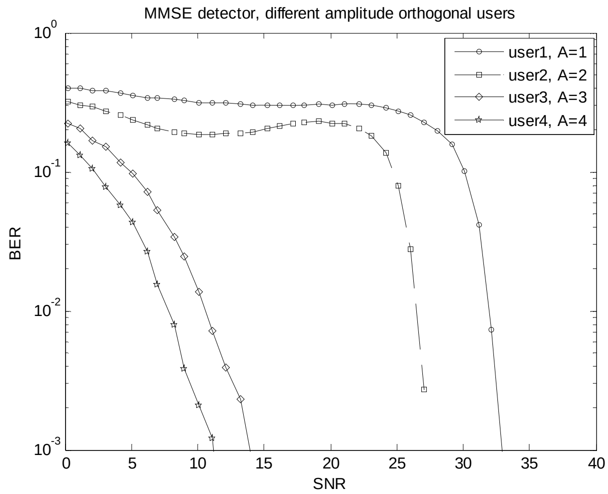

Fig. 8. Error probability vs SNR for MMSE detector, different amplitude orthogonal users

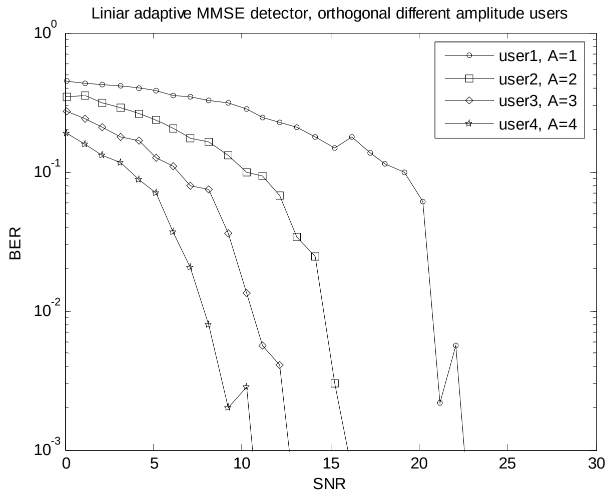

Fig.9. Error probability vs SNR for adaptive MMSE detector, different amplitude orthogonal users

- in the case of different amplitude orthogonal users, the results obtained by the users with large amplitudes are better, but this is achieved be decreasing the performances of the users with smaller amplitudes; the orthogonal MMSE receiver obtain better results for the large amplitude users (BER>0.1 for SNR>25dB) while in the case of orthogonal adaptive MMSE receiver the results are a little bit worse (BER>0.1 only for SNR>15dB); for small amplitude users the difference between the two structures is less the 5dB;

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