THE CAPACITY OF CHANNEL OF COMPLEX BPSK-LP SYSTEM

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In this paper the performances of BPSK (bipolar phase shift-key) communication system in two time moments in Gaussian channel are determined. It is given expressions for the error probability and capacity channel, and it is compared to BPSK communication system in one time moment. Also, the influence of noise correlated in two time moments is analyzed.

Keywords: Pdf, Capacity of Channel, BPSK, Correlation

1. INTRODUCTION

The performance of PSK communication systems in the presence of the noise, cochannel and intersymbol interference, as well as imperfect reference carrier signal recovery, has been analyzed in many papers. Beside error probability, one of necessary detail which gives real characteristic to design of communication system is capacity channel.

The information is transmited over binary states in PSK telecommunication systems that the state is determined over waveform phase that the phase is zero if the state is binary one and phase is π if the state is binary zero. PSK system contains input narrowband filtar and the reference carrier is extracted by the first order loop, which is assumed to be perfect. Model of complex PSK communication system is shown on the figure 1. the pdf (probability density function) depends by amplitude, phase and frequency, but in this moment it is considered influence of estimation the signal in two time moments when the noise is correlated. Threshold is obtained from likelihood ratio and the waveform is estimated in two time moments on output of receiver and this is main difference by system with one estimation.

2. SYSTEM ANALYSIS

The system is consisted from two parts. The first is block of bipolar phase shift – key where is detected the signal received. The signal contained additive white Gaussian noise (AWGN) which is described with $n_1(t)$ which has varience σ^2. Second part is lowpass filter (LP). It contains active components for recovery signal which is loss of power but it plays the role to eliminate presence of noises and to reconstruct the signal transmited.

The BPSK system is modeled on next way:

$$H_0: \quad s_0 = A\cos\omega t \tag{1}$$

$$H_1: \quad s_1 = A\cos(\omega t + \pi) \tag{2}$$

where s_0 and s_1 are input waveforms on first recever and A is amplitude of transmited signal.
On the output of first part the pdf is:

$$p_0(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A)^2}{2\sigma^2}} \tag{3}$$

$$p_1(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 + A)^2}{2\sigma^2}} \tag{4}$$

for case H_0 and second for the case H_1 which represent two hypothesis. This signal goes to the input of second part and on the output of this degree the pdf is:

$$p_0(z_2) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - z_1)^2}{2\sigma^2}} p_0(z_1) dz_1 \tag{5}$$

$$p_1(z_2) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - z_1)^2}{2\sigma^2}} p_1(z_1) dz_1 \tag{6}$$

for case H_0 and H_1, respectively.

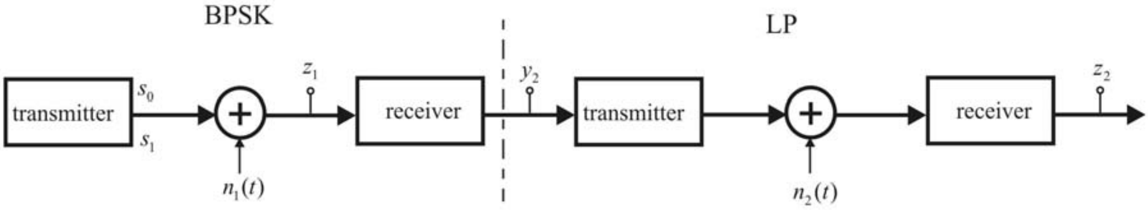

Fig. 1. Model of Complex System with BPSK and LP

It observes output of second degree and after averaging by all values z_1, it obtains:

$$p_0(z_2) = \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(z_2 - A)^2}{2\sigma^2}} \left[1 + erf\left(\frac{z_2 + A}{2\sigma}\right)\right] \tag{7}$$

$$p_1(z_2) = \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(z_2 + A)^2}{2\sigma^2}} \left[1 + erf\left(\frac{z_2 - A}{2\sigma}\right)\right] \tag{8}$$

3. Results

In preview consideration, the influence coeficient of correlation doesn't used. But, it is essencial condition for further researching and in the next consideration it is observed existing coefficient of correlation in the signal. It is consequence presence of noise correlated.

Using preview expressions, including noise correlated, it obtains total probability error for signal in two time moments.

There is:

$$P_e = \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{A-z_2} p_1(z_1, z_2) dz_1 \approx \frac{1}{2}\sqrt{\frac{1-R}{1+R}}\left(1 - erf\left(\sqrt{\frac{\gamma}{1-R}}\right)\right) \cdot \left(1 + erf\left(\frac{1}{\sigma\sqrt{2}}\sqrt{\frac{1+R}{1-R}}\right)\right) + \frac{1}{\pi} \cdot \frac{1-R}{1+R}\sqrt{\frac{1+R}{1-R}} \cdot e^{-\frac{\gamma}{1+R}} \tag{10}$$

where γ is signal to noise ratio and R is coeficient of correlation.

The simulations are realized in Wolfram Mathematica and for better simulation it is used next development for error function:

$$erf\,(a - bx) \approx erf\,(a) - \frac{2}{\sqrt{\pi}} b e^{-a^2} x \tag{11}$$

and

$$e^{-x^2} = 1 - \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k+2} \tag{12}$$

The capacity of channel is defined as the most average joint information between the input and output of the channel with the set of defined input probability. On the basis distributions of envelops and joint probability density functions, it is possible to estimate the capacity of channel. From definition [1] of capacity of channel:

$$C = \max_{p(x_i)} \sum_{i=0}^{q-1} \int_{-\infty}^{\infty} p(y/x_i) \cdot P(x_i) \cdot \log_2 \frac{p(y/x_i)}{p(y)} \tag{13}$$

where is [1]:

$$p(y) = \sum_{k=0}^{q-1} p(y/x_k) \cdot P(x_k) \tag{14}$$

In this case using expressions (7), (8) and (10) it obtains the capacity of channel:

$$\begin{aligned} C = {} & \frac{1}{2}\int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2\sqrt{1-R^2}} e^{-\frac{z_1^2 - 2R \cdot z_1 z_2 + z_2^2}{2\sigma^2(1-R^2)}} \times \\ & \times \log_2 \frac{2}{1 + e^{-\frac{2\gamma}{1-R^2}} e^{\frac{A(z_1+z_2)}{\sigma^2(1+R)}}} dz_1 + \\ & + \frac{1}{2}\int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2\sqrt{1-R^2}} e^{-\frac{(z_1-A)^2 - 2R \cdot (z_1-A)(z_2-A) + (z_2-A)^2}{2\sigma^2(1-R^2)}} \times \\ & \times \log_2 \frac{2}{1 + e^{\frac{2\gamma}{1-R^2}} e^{-\frac{A(z_1+z_2)}{\sigma^2(1+R)}}} dz_1 \end{aligned} \tag{15}$$

On the table 1. its are given values the capacity of channel for one (with index *OTM*) and two (with index *TTM*) time moments.

k=3		
γ	C_{OTM} (bit/symbols)	C_{TTM} (bit/symbols)
0	0.278652	1.
1	90.6875	$1.3\cdot 10^4$
2	$5.96\cdot 10^2$	$1\cdot 10^5$
3	$1.88\cdot 10^2$	$3.3\cdot 10^5$
4	$4.33\cdot 10^3$	$7.98\cdot 10^5$
5	$8.3\cdot 10^3$	$1.555\cdot 10^6$
6	$1.41\cdot 10^4$	$2.682\cdot 10^6$
7	$2.22\cdot 10^4$	$4.254\cdot 10^6$
8	$3.3\cdot 10^4$	$6.344\cdot 10^6$
9	$4.68\cdot 10^4$	$9.027\cdot 10^6$
10	$6.39\cdot 10^4$	$1.237\cdot 10^7$

Table 1. Values of Capacity Channel for *k=3*

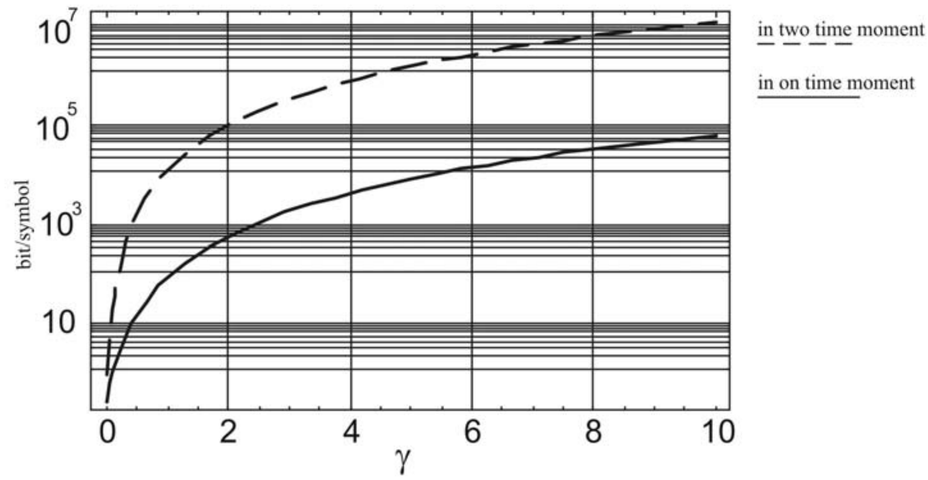

Fig. 2. The Characteristics of Channel Capacity of Complex System with BPSK and LP

On the table 2. it is given values the capacity of channel for one (*OTM*) and two (*TTM*) time moments for greater number of items for development serie of expression (15).

k=4		
γ	C_{OTM} (bit/symbols)	C_{TTM} (bit/symbols)
0	0.27	1.
1	$2.13\cdot10^{2}$	$1.68\cdot10^{5}$
2	$2.56\cdot10^{3}$	$2.59\cdot10^{6}$
3	$1.18\cdot10^{4}$	$1.29\cdot10^{7}$
4	$3.58\cdot10^{4}$	$4.06\cdot10^{7}$
5	$8.52\cdot10^{4}$	$9.88\cdot10^{7}$
6	$1.73\cdot10^{5}$	$2.04\cdot10^{8}$
7	$3.17\cdot10^{5}$	$3.77\cdot10^{8}$
8	$5.37\cdot10^{5}$	$6.43\cdot10^{8}$
9	$8.54\cdot10^{5}$	$1.03\cdot10^{9}$
10	$1.29\cdot10^{6}$	$1.56\cdot10^{9}$

Table 2. Values of Capacity Channel for $k=4$

From this results it can be to see that the contribution is attained increasing number of items and this conclusion is logical.

The results can be applied in the design of the wireless systems.

4. CONCLUSION

In this paper the performances of PSK communication are determined. Beside error probability, one of necessary detail which gives real characteristic to design of communication system is capacity channel. Such as probability density function, error probability depends by amplitude, phase and frequency, but in this moment it is considered only influence of noise correlated for error probality and capacity channel. It is given expressions for the error probability and capacity channel, and it is compared to PSK communication systems in one and two time moments.

On the basis of the results shown in Figure 1. the capacity channel of PSK system in two time moments is greater than capacity channel of PSK system with one estimation.

5. REFERENCES

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