

## A DETAILED STUDY OF THE HIGH ORDER SERIAL RESONANT INVERTER FOR INDUCTION HEATING

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*The real high order power circuit of the serial resonant inverter without free-wheeling diodes widely applied for induction heating and melting is analyzed assuming a sine-wave shape of the load voltage. The load circuit is replaced by serially connected equivalent resistance and reactance. The parameters of the inverter circuit, especially the load voltage, the voltage across the power semiconductor devices and in the thyristor variant the circuit turn-off time are determined. A MATLAB program calculates and graphically displays the frequency controlling characteristics. A direct steady state simulation of the inverter operation confirms the results. The requirements towards the closed loop automatic system of frequency control can be easily defined.*

**Keywords:** serial inverter, resonant inverter, high-order circuit, characteristics

### 1. INTRODUCTION

The serial resonant inverter without free - wheeling diodes applies widely for induction heating and melting due to its convenience for high frequency performance. The inverter may be constructed with serially connected fully controllable devices (transistors or GTO thyristors) and diodes. For cheaper and not so high frequency applications when there is a negative voltage across the devices right away after their conduction interval the power circuit can be built with thyristors. The serial resonant inverter [1] has not been fully analyzed in the practical case when the inverter circuit is of high order.

That's why this article is aimed at investigation of the high order power circuit of the serial resonant inverter for induction heating and obtaining the frequency control characteristics of the main parameters. That will allow the power circuit to be correctly designed taking into account the large variations of the load and the restrictions of the power circuit. The requirements towards the controlling system can also be defined. Simulations and possibly experiments should confirm the results.

### 2. MAIN ASSUMPTIONS

A half bridge circuit (Fig. 1) is under study but it can be easily converted into the bridge one. The power losses in the inverter are neglected. The commutation of the power semiconductor devices is instantaneous. A parallel equivalent circuit represents the induction heater. The quality factor of the load circuit is sufficiently high that the voltage across the load has a close to the sine wave shape. A combination of the method of transitory values and the Fourier transformation is applied in the analysis.

### 3. REPRESENTATION OF THE LOAD CIRCUIT

A parallel equivalent circuit containing the active resistance  $R_l$  and the inductance  $L_l$  that are frequency dependent represents the inductor heater:

$$(1) \quad R_l = R_{lr} \left( \frac{\omega}{\omega_{lr}} \right)^{n_1}$$

$$(2) \quad L_l = L_{lr} \left( \frac{\omega}{\omega_{lr}} \right)^{n_2 - 1},$$

where  $\omega_{lr}$  is the resonant frequency of the load circuit  $R_l, L_l, C_l$

$$(3) \quad \omega_{lr} = \frac{1}{\sqrt{C_l L_{lr}}}$$

$R_{lr}, L_{lr}$  are the inductor parameters for the resonant frequency.  $\omega = 2\pi f$  is the controlling frequency. The values of  $n_1$  and  $n_2$  are given in [2].

If the quality factor of the load circuit  $R_l, L_l, C_l$  at resonance (index  $r$ )

$$(4) \quad Q_r = R_{lr} \sqrt{\frac{C_l}{L_{lr}}}$$

is high enough the load voltage is almost sine wave and the load circuit can be represented by a serial equivalent circuit containing the parameters  $R_{eq}, X_{eq}$  (Fig.2).

$$(5) \quad \frac{R_{eq}}{R_{lr}} = \frac{\left( \frac{\omega}{\omega_{lr}} \right)^{n_1}}{1 + Q_r^2 \left( \frac{\omega}{\omega_{lr}} \right)^{(2n_1 - n_2 + 1)} \left( \frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega} \right)^2}$$

$$(6) \quad \frac{X_{eq}}{R_{lr}} = \frac{Q_r \left( \frac{\omega}{\omega_{lr}} \right)^{\left( 2n_1 + \frac{1-n_2}{2} \right)} \left( \frac{\omega_{lr}}{\omega} - \frac{\omega}{\omega_{lr}} \right)}{1 + Q_r^2 \left( \frac{\omega}{\omega_{lr}} \right)^{(2n_1 - n_2 + 1)} \left( \frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega} \right)^2}$$

Below the resonance of the load circuit the equivalent reactance  $X_{eq}$  is in fact a serial equivalent inductance

$$(7) \quad \frac{L_{eq}}{L_{lr}} = \frac{Q_r^2 \left( \frac{\omega}{\omega_{lr}} \right)^{\left( 2n_1 + \frac{1-n_2}{2} \right)} \left( \frac{\omega_{lr}^2}{\omega} - 1 \right)}{1 + Q_r^2 \left( \frac{\omega}{\omega_{lr}} \right)^{(2n_1 - n_2 + 1)} \left( \frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega} \right)^2}$$

Therefore the serial inverter circuit consists of  $R = R_{eq}$ ,  $L = L_{eq} + L_S$ ,  $C = C_S$ . Above the resonance of the load circuit the equivalent reactance  $X_{eq}$  is a serial equivalent capacitance

$$(8) \quad \frac{C_{eq}}{C_l} = \frac{1 + Q_r^2 \left(\frac{\omega}{\omega_{lr}}\right)^{(2n_1 - n_2 + 1)} \left(\frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega}\right)^2}{Q_r^2 \left(\frac{\omega}{\omega_{lr}}\right)^{(2n_1 + \frac{1-n_2}{2})} \left(\frac{\omega^2}{\omega_{lr}^2} - 1\right)}$$

and the serial inverter circuit consists of  $R = R_{eq}$ ,  $L = L_S$ ,  $C = \frac{C_{eq}C_S}{C_{eq} + C_S}$ ,  $\alpha = C/C_{eq}$ . Then the voltage across the inverter capacitor  $C$  is divided between the capacitors  $C_S$  and  $C_{eq}$  in accordance with the theory of electrical engineering [3]. Graphical representations of the functions (5), (6), (7) and (8) are given in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 respectively.

#### 4. PARAMETERS OF THE SERIAL INVERTER CIRCUIT

The whole fourth order inverter circuit is reduced to a second order RLC resonant ( $R_{eq} < \sqrt{L/C}$ ) circuit. This inverter is analyzed in [4], [5], [1], [6].

#### 5. CALCULATION OF THE LOAD VOLTAGE

The RMS of the load voltage can be determined from its harmonic components as follows

$$(11) \quad V_l = \sqrt{\sum_{m=1,3,5,\dots} V_{l(m)}^2}$$

Each harmonic component of the load voltage is

$$(12) \quad V_{l(m)} = I_{norm} \cdot \frac{c(m)}{\sqrt{2}} \cdot \sqrt{R_{eq(m)}^2 + X_{eq(m)}^2}$$

where  $R_{eq(m)}$  and  $X_{eq(m)}$  are calculated from (5) and (6), exchanging  $\omega/\omega_{lr}$  with  $m\omega/\omega_{lr}$ .  $I_{norm} = V_d / R_{eq}$  for natural commutation ( $\omega \leq \omega_0$ ) and  $I_{norm} = V_d / (\omega_0 L)$  for forced commutation ( $\omega > \omega_0$ ) where  $\omega_0 = \sqrt{1/(LC) - R_{eq}^2/(4L^2)}$  is the inherent frequency of the serial inverter circuit and  $\omega = 2\pi f$  is the controlling frequency. The normalized peak value of the  $m$  – harmonic component ( $m=1,3,5,\dots$ ) of the inverter current  $c(m) = \sqrt{a(m)^2 + b(m)^2}$  is given in [1] for natural commutation and in [7] for forced commutation.

#### 6. VOLTAGE ACROSS THE POWER SEMICONDUCTOR DEVICES

At natural commutation during the interruption interval  $t_{X1} = 0$  to  $t_P = \pi/\omega - \pi/\omega_0$  the voltage of the turned-off devices is

$$(13) \quad V_{VS} = 0.5V_d \mp (k - 0.5)(1 - \alpha)V_d \pm \frac{c(1)V_d}{\cos \varphi_{eq}} \sin(t_{X1} - t_P + \varphi(1) + \varphi_{eq})$$

where the upper sign corresponds to the just turned-off devices,  $\varphi_{(1)} = \arctg(a_{(1)}/b_{(1)})$ ,  $\varphi_{eq} = \arctg(X_{eq}/R_{eq})$ .

When the other device conducts  $t_{X2} = 0$  to  $\pi/\omega_0$  the voltage of the turned-off device is

$$(14) \quad V_{VS} = 0.5V_d + (1 - \alpha)V_d \left[ 0.5 - ke^{-\delta t_{X2}} \left( \frac{\delta}{\omega_0} \sin \omega_0 t_{X2} + \cos \omega_0 t_{X2} \right) \right] + \\ + \frac{c_{(1)}V_d}{\cos \varphi_{eq}} \sin(t_{X2} + \varphi_{(1)} + \varphi_{eq})$$

where  $\delta = \frac{R_{eq}}{2L}$  is the damping coefficient and  $k = \frac{1}{1 - e^{-\delta \frac{\pi}{\omega_0}}}$  is the coefficient of

hesitation. Expressions (13) and (14) are applied for calculation of the maximal device voltage and in the thyristor variant the circuit turn-off time. For forced commutation the necessary expression is given in [6].

## 7. FREQUENCY CHARACTERISTICS

A MATLAB program processes all the mathematical information describing the steady-state operation of the inverter together with its load circuit in the allowed frequency range. The frequency characteristics of the inverter are obtained and graphically displayed in Fig. 6. These particular characteristics correspond to a practically implemented inverter with the following data:  $V_d=500$  V;  $L_S=45$   $\mu$ H;  $C_S=84$   $\mu$ F;  $R_{lr}=0.4739599$   $\Omega$ ,  $f_{lr}=2083$  Hz,  $L_{lr}=8.8717$   $\mu$ H,  $C_b=657.88$   $\mu$ F,  $n_1=0$ ,  $n_2=1$ ,  $f=1600-2600$  Hz. The first set of graphics shows: two times the average input current of the inverter  $2I_d$  [A] (solid line), two times the load voltage  $2V_l$  [V] (dashed line), ten times the circuit turn-off time of the thyristors  $10 t_{q.c.}$  [ $\mu$ S] (dotted line). The second set of graphics shows: the peak serial capacitor voltage  $V_{CSm}$  [V] (dash-dotted line), the maximal device voltage  $V_{VSm}$  (solid line). The same MATLAB program can calculate frequency characteristics for different input data. Other parameters of the circuit can also be calculated and displayed. The calculation is more precise if the shape of the load voltage is closer to the sine-wave form. That is checked for each particular frequency.

The stability and efficiency of the inverter can be studied from the characteristics. In general there is a minimum of the input current and power, load voltage, serial capacitor voltage and device voltage around the resonance of the load circuit. If the parameters of the inductor heater vary during the induction heating process it is advisable to maintain almost constant input current (power) in a slight capacitive derangement of the load circuit  $R_l$ ,  $L_l$ ,  $C_l$ , where the operation is stable, by exercising an influence on the controlling frequency. The calculated slope of the frequency characteristic of the input current is used for determining the parameters of the closed loop automatic control system [1].

## 8. DIRECT SIMULATION OF THE STEADY STATE

The steady state is simulated by a MATLAB program based on the method described in [8] that is experimentally confirmed. The results from the frequency characteristics and from the simulation are in good agreement according to Table 1. That confirms the correctness of the whole study. The detailed simulation results of the power inverter are graphically displayed (Fig. 7) for  $f=2195$  Hz. The first set of diagrams shows: the serial capacitor voltage  $v_{CS}$  [V] (solid line), inverter current  $i_{CS}$  [A] (dashed line) and twice the load voltage  $2v_l$  [V] (dotted line). The second set of diagrams shows: a quarter of the load inductance current  $i_{Ll}$  [A] (dotted line) and the voltage across a device  $v_{VSI}$  [V] (solid line). The simulation results for the same circuit at  $f=694,4$  Hz displaying an inefficient but possible operation in an inductive derangement of the load generally with the third harmonic component of the inverter current (a case that is difficult to be studied analytically) are given in Fig. 8.

**Table 1.**

f[Hz]	694,4	1823		2018		2083		Recom. 2195		2394	
Study	Sim.	Fr.ch.	Sim.	Fr.ch.	Sim.	Fr.ch.	Sim.	Fr.ch.	Sim.	Fr.ch.	Sim.
Load	Ind.	Ind.		Ind.		Res.		Cap.		Cap	
$I_d$ [A]	169,5	400,7	388,7	200	196,5	178,2	180,1	<b>199,8</b>	<b>208,2</b>	400,6	416,3
$V_l$ [V]	200,4	304,4	303,5	215,4	215,8	201,7	206,6	<b>211,9</b>	<b>222,1</b>	302,2	314,1
$t_{q.c.}$ [μS]	604	113,6	125,9	61	68,3	63,8	72,5	<b>82,9</b>	<b>89,9</b>	97,3	99,6
$V_{CSm}$ [V]	1456	1308	1270	590	580	509	515	<b>542</b>	<b>565</b>	996	1034
$V_{VSm}$ [V]	1927	1342	1342	866	875	863	886	<b>1003</b>	<b>1045</b>	1636	1689

## 9. CONCLUSION

A detailed study of the real serial resonant inverter without free - wheeling diodes for induction heating is performed. The steady state parameters and characteristics are determined. The requirements for the control system can be defined.

## 10. REFERENCES

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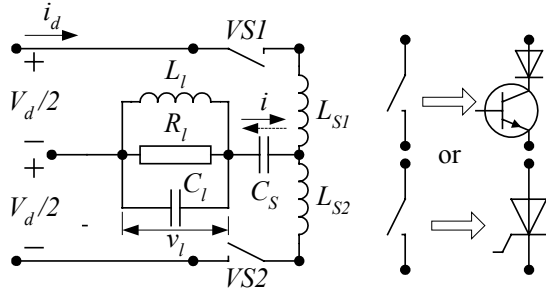


Fig. 1.

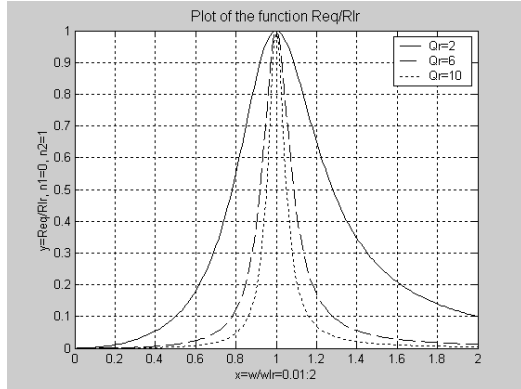


Fig. 2.

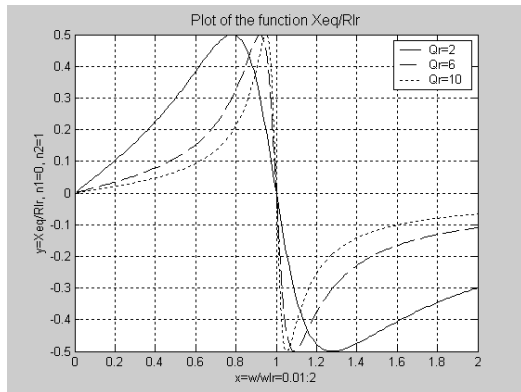


Fig. 3.

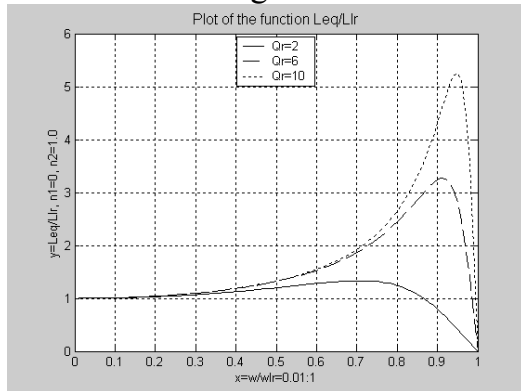


Fig. 4.

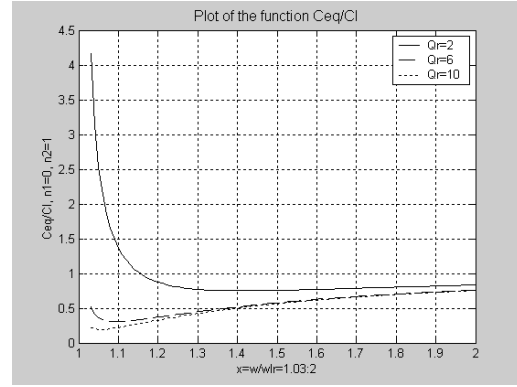


Fig. 5.

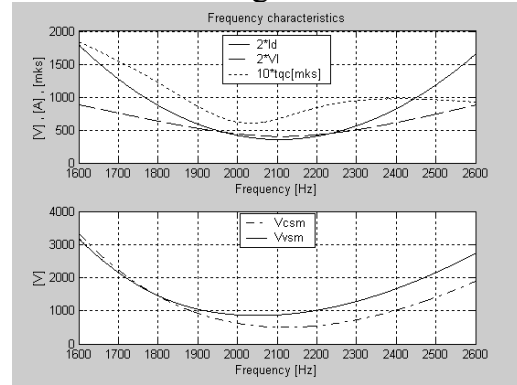


Fig. 6.

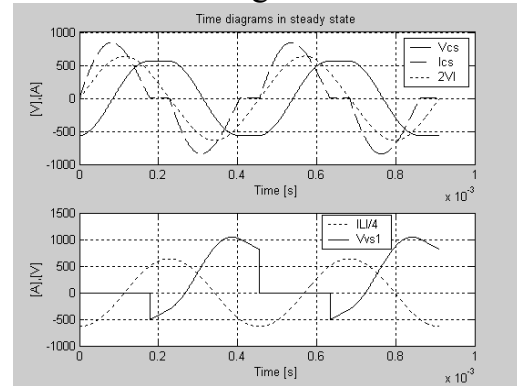


Fig. 7.

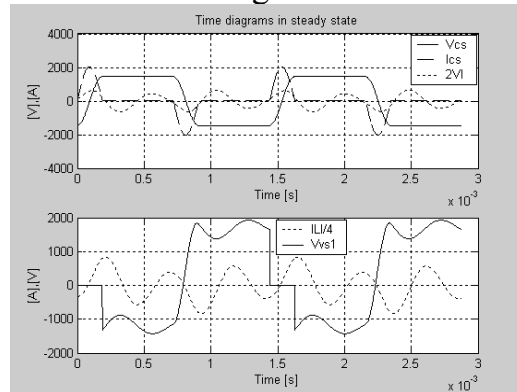


Fig. 8.