# LOW SENSITIVITY DESIGN OF MULTIPLIERLESS ELLIPTIC IIR DIGITAL FILTERS

**Valentina Ilieva Anzova**

Department of Telecommunications, Technical University of Sofia
Technical University, Department of Telecommunications, 1756 Sofia, Bulgaria,
e-mail: via@tu-sofia.bg

*In this paper an improved method of design of low sensitivity multiplierless elliptic IIR digital filters is proposed. The design technique is based on the sensitivity analysis and a selection of proper allpass sections. The allpass sections used to realize the allpass structures are with minimized sensitivities. Using the symbolic analysis, the expressions for phase sensitivities to multiplier coefficients for the selected allpass sections have been derived. The amplitude sensitivities of the obtained realizations have been investigated. The comparison between the resulted digital multiplierless IIR filters has been made for different coefficients wordlength. The results are confirmed experimentally.*

**Keywords:** IIR digital filters, allpass sections, low sensitivity, multiplierless

## 1. INTRODUCTION

In VLSI implementations a multiplier is very costly. Therefore it is very attractive to represent the multiplications as series of shift and add operations. In this way in multiplierless realizations the overall filter complexity is significantly reduced.

The design of multiplierless IIR digital filters is very difficult because of the high sensitivity of their frequency response to the multiplier coefficients. Therefore, it is very important to minimize the frequency responses sensitivity. When the filter sensitivity is low it is possible to shorten coefficients wordlength without violating the filter specifications. It is well known that the filter structure consisting of two allpass subfilters in parallel is having very low coefficient sensitivity (especially in the passband). This structure is also very economical – it is realizing two $n^{th}$ - order transfer functions using only n multipliers.

In [1][2][3], it has been shown that the parallel-allpass-filter-structure may have a small number of shift and add operations if the transfer function (TF) is derived by the bilinear transformation from an elliptic minimal Q-factors (EMQF) analog prototype. Because of the properties of EMQF filters [4], (n+1)/2 multiplier coefficients can be implemented without quantization. Then the remaining (n-1)/2 less sensitive coefficients are quantized using canonic sign digit (CSD) code. This class of filters has all coefficients implemented with small quantization error.

The purpose of this contribution is to modify this approach and to apply it to design of narrowband lowpass low sensitivity multiplierless IIR filters. The allpass sections used to realized the allpass structures are with minimized sensitivities for transfer function coefficients values corresponding to poles near $z = 1$ [5][6]. The frequency responses of the multiplierless realization so obtained and the phase sensitivity to multiplier coefficients are compared with those of multiplierless IIR filter implemented using allpass sections selected by Milić and Lutovac [1][2].

## 2. IIR DIGITAL FILTERS DESIGN USING AN EMQF ANALOG PROTOTYPE

The design method suggested by Milić and Lutovac [1][2] starts with phase response sensitivity analysis of the allpass sections that implements the desired digital IIR filter to its multiplier coefficients. The (n+1)/2 most sensitive coefficients can be implemented as exact values with a minimal number of shift and add operations by an appropriate design of EMQF transfer function. Then the remaining (n-1)/2 less sensitive coefficients are quantized using canonic sign digit (CSD) code.

It is shown in [1][2] that an IIR filter, if derived from an EMQF analog prototype, can be designed to have in all second order sections of the allpass networks one common coefficient whose value depends only on the frequency at which the filter attenuation is 3dB and may be adjusted according to the predetermined number of shift and add operations. In this way, the direct control is established over the values of (n-1)/2 multiplication constants and they can be implemented as exact values, eliminating thus the influence of their high sensitivities.

In this contribution, multiplierless elliptic filter design is based on the TF $H(z)$ formed by the bilinear transformation from an EMQF analog prototype [1][3][4]. An important property of EMQF filter is that its poles are placed on a circle with the center at the origin of the $s$ plane. The radius of the circle is $\sqrt{\Omega_a}$, where the normalized stopband edge frequency is $\Omega_a$ and for $s = j\sqrt{\Omega_a}$ the filter has the attenuation of 3dB. The relation between $f_p$ (passband edge frequency of the digital filter) and $f_a$ (stopband edge frequency of the digital filter) and the stopband edge frequency of the analog prototype $\Omega_a$ has been established in [3]. For given $f_p$ and $f_a$, $\Omega_a$ is estimated from the expression

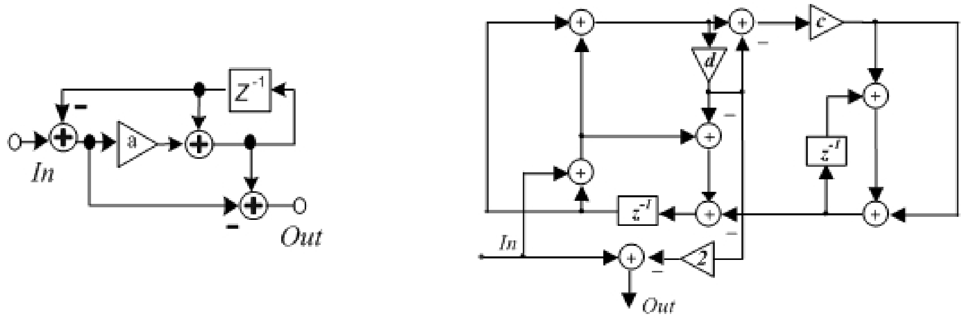
$$\Omega_a = \frac{tg\pi f_a}{tg\pi f_p}. \tag{1}$$

The frequency where a digital filter has the attenuation of 3dB corresponds in the analog filter domain to the frequency $\sqrt{\Omega_a}$, and can be determined by the relation

$$tg^2 \pi f_{3dB} = tg\pi f_p tg\pi f_a. \tag{2}$$

The transfer function of an odd order IIR filter $H(z)$ can be presented as a sum of two allpass functions $H_a(z)$ and $H_b(z)$. The allpass subfilters $H_a(z)$ and $H_b(z)$ are usually realized as a cascade connection of first and second order sections. In order to obtain a narrowband lowpass low sensitivity multiplierless IIR filter realizations, we need allpass sections having low sensitivities when realizing poles near $z = 1$. Such sections have been proposed and investigated in [5][6]. We shall call them ST1 (Fig. 1a) (the first order) and ST2A (Fig. 1b) (the second order section) and they realize the following transfer functions:

$$H_{ST1}(z) = \frac{-(1-a) + z^{-1}}{1-(1-a)z^{-1}}, \qquad H_{ST2A}(z) = \frac{1-2d-2(1-d)(1-2c)z^{-1} + z^{-2}}{1-2(1-d)(1-2c)z^{-1} + (1-2d)z^{-2}}. \tag{3}$$




a) b)

Fig. 1 Low sensitivity allpass sections: a) first order (ST1); b) second order (ST2A)

For comparison the same design procedure will be applied for the popular lattice wave sections selected by Milić and Lutovac in [1]. The TFs of these sections (called GM-sections) are:

$$H_{GM1}(z)=\frac{b+z^{-1}}{1+bz^{-1}}, \qquad H_{GM2}(z)=\frac{-a_1-a_2(1-a_1)z^{-1}+z^{-2}}{1-a_2(1-a_1)z^{-1}-a_1z^{-2}}. \tag{4}$$

The coefficients of the digital allpass sections have been derived from the corresponding first and second order analog prototypes Eq.(5) by bilinear transformation

$$S(s)=\frac{-s+B_0}{s+B_0}, \qquad S(s)=\frac{s^2-A_is+B_i}{s^2+A_is+B_i}. \tag{5}$$

If both the relation between the poles position and the coefficients of the TF and Eq.(1), Eq.(2) are used, the following expressions for coefficients in allpass sections can be obtained:

$$b=-\frac{1-B_0}{1+B_0} \quad \rightarrow \quad b=-\frac{1-tg\pi f_{3dB}}{1+tg\pi f_{3dB}}. \tag{6}$$

$$a_1=\frac{A_i-B_i-1}{A_i+B_i+1} \quad \rightarrow \quad a_1=-r_i^2. \tag{7}$$

$$a_2=\frac{1-B_i}{1+B_i} \quad \rightarrow \quad a_2=\frac{1-tg^2\pi f_{3dB}}{1+tg^2\pi f_{3dB}}. \tag{8}$$

$$a=2\frac{B_0}{1+B_0} \quad \rightarrow \quad a=2\frac{tg\pi f_{3dB}}{1+tg\pi f_{3dB}}. \tag{9}$$

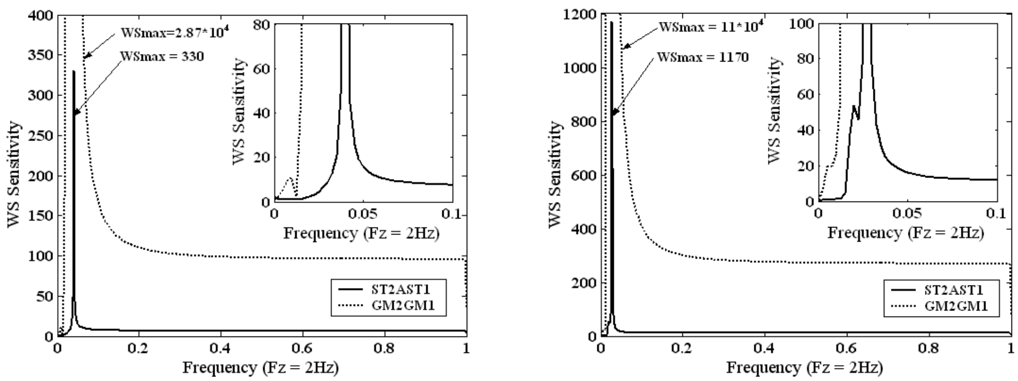
$$c=\frac{B_i}{1+B_i} \quad \rightarrow \quad c=\frac{tg^2\pi f_{3dB}}{1+tg^2\pi f_{3dB}}. \tag{10}$$

$$d=\frac{A_i}{A_i+B_i+1} \quad \rightarrow \quad d=\frac{1-r_i^2}{2}, \tag{11}$$

where $r_i$ is the complex pole radius.

## 3. SENSITIVITIES INVESTIGATIONS

We have investigated the worst-case (WS) amplitude response sensitivities of the obtained elliptic IIR filters (ST2AST1, GM1GM2) for two different TFs orders (3 and 5) using the package PANDA [7]. The results for the WS sensitivities vs. frequency are given in Fig. 2a (third order TF) and Fig. 2b (fifth order TF). The IIR filters obtained using allpass sections ST1 and ST2A are marked as ST2AST1. Respectively, the filter realizations with GM-sections are marked as GM1GM2. It is clearly seen that IIR filters ST2AST1 have many times lower sensitivities within the entire frequency range, but the difference is considerably stronger in the stopband and especially at maximal points. The WS sensitivity analysis in Fig. 2 demonstrates that difference between sensitivities of both realizations (ST2AST1 and GM1GM2) for fifth-order TF becomes even more significant.




a) b)

Fig. 2 Worst-case sensitivities of different IIR filter realizations: a) third order; b) fifth order

## 4. EXPERIMENTS

We designed and investigated two sets of filters with specifications as given in Examples 1, 2. In both cases first, the design procedure is applied for allpass section ST1 and ST2A (with TFs given in Eq.(3)). Then, for comparison, the same procedure is used for GM-sections (with TFs given in Eq.(4)). Using the symbolic analysis, the expressions for phase sensitivities to multiplier coefficients for the selected sections have been derived. Figs. 3, 4, 5 and 6 display the phase response sensitivities to coefficients of the corresponding first or second-order sections of the both examples.

*Example1*

The filter specifications are as follows:

$F_p = 0.006$, $F_a = 0.02$, $A_p = 0.2\,\text{dB}$, $A_a = 20\,\text{dB}$.

The filter coefficients can be divided into two groups that will be considered separately. The first group includes the most sensitivity coefficients (that is $a_2$ - GM1GM2 and $c$ - ST2AST1) that can be controlled by TFs, and the coefficients $a_1$ (GM1GM2) or $d$ (ST2AST1) representing the pole pair closest to the unit circle. The second group includes the remaining coefficients b (GM1GM2) and a (ST2AST1). For given specifications, the range of permitted values of $a_2$ or $c$ is determined from $F_p$ and $F_a$ according to [1]. Then, all values in the whole range are quantized in CSD-code. The values of $a_2$ or $c$ with the smallest quantization error is chosen. From Eq.(8) and Eq.(10) the frequency $f_{3dB}$ is evaluated. Keeping the values of $f_{3dB}$ unchanged, the transition bandwidth is to be adjusted to provide the suitable value for $a_1$ or $d$ according to [1][3]. At the end all remaining coefficients are quantized with different coefficients wordlength. Table I gives the results for all coefficients.

*Example2*

The filter specifications are: $F_p = 0.006$, $F_a = 0.01$, $A_p = 0.1$ dB, $A_a = 20$ dB.

The specifications are fulfilled with a fifth order TF filter. The second order sections have a common coefficient $a_2^{'} = a_2^{''}$ (GM1GM2) or $c' = c''$ (ST2AST1). The coefficients of the obtained multiplierless IIR filters are given in Table I. We have investigated the frequency responses of the multiplierless realizations so obtained. It is clearly seen from Figs. 7 and 8 that the shapes of the attenuation characteristics of the multiplierless filters obtained according to our approach (ST2AST1) do not differ from the ideal even though coefficients of the second group are quantized to 3bits.

TABLE I
MULTIPLIERLESS REPRESENTATION OF THE FILTER COEFFICIENTS

| ST2AST1 | | GM2GM1 | |
|---|---|---|---|
| $d$ | $2^{-5} - 2^{-7}$ | $a_1$ | $-1 + 2^{-4} - 2^{-6}$ |
| $c$ | $2^{-10}$ | $a_2$ | $1 - 2^{-9}$ |
| $a$ | $2^{-4} + 2^{-8} + 2^{-12}$ | $b$ | $-1 + 2^{-3}$ |

| ST2AST1 | | GM2GM1 | |
|---|---|---|---|
| $d'$ | $2^{-8} + 2^{-10}$ | $a_1'$ | $-1 + 2^{-7} + 2^{-9}$ |
| $c', c''$ | $2^{-11}$ | $a_2', a_2''$ | $1 - 2^{-10}$ |
| $d''$ | $2^{-6} + 2^{-7}$ | $a_1''$ | $-1 + 2^{-4} - 2^{-6}$ |
| $a$ | $2^{-5} + 2^{-6}$ | $b$ | $-1 + 2^{-4}$ |

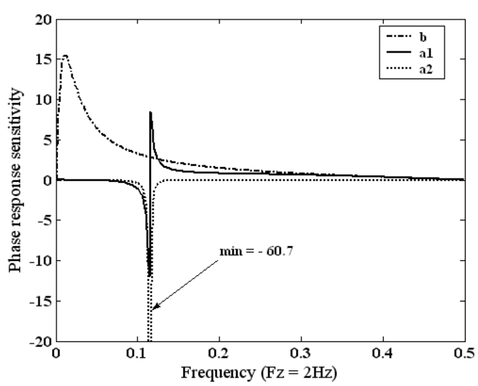



Fig. 3 Phase response sensitivity of 3th order GM2GM1 TF

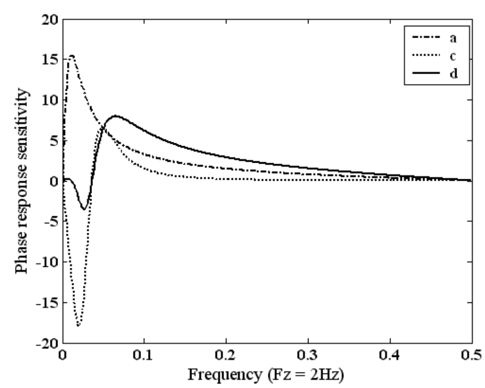



Fig. 4 Phase response sensitivity of 3th order ST2AST1 TF

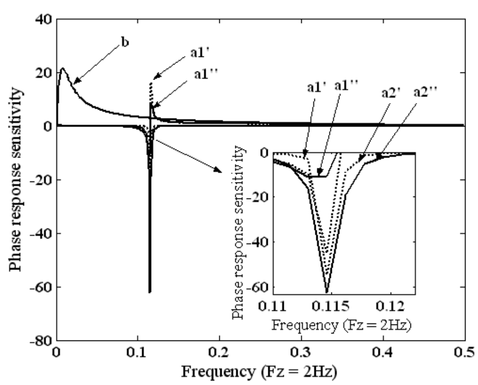



Fig. 5 Phase response sensitivity of 5th order GM2GM1 TF

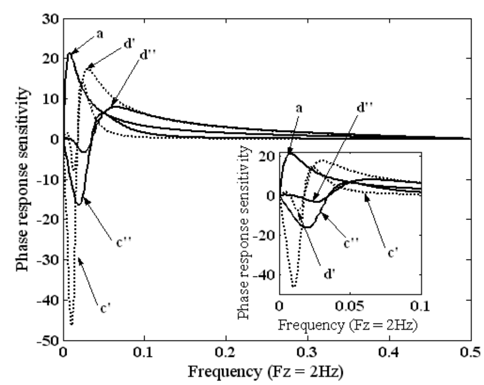



Fig. 6 Phase response sensitivity of 5th order ST2AST1 TF

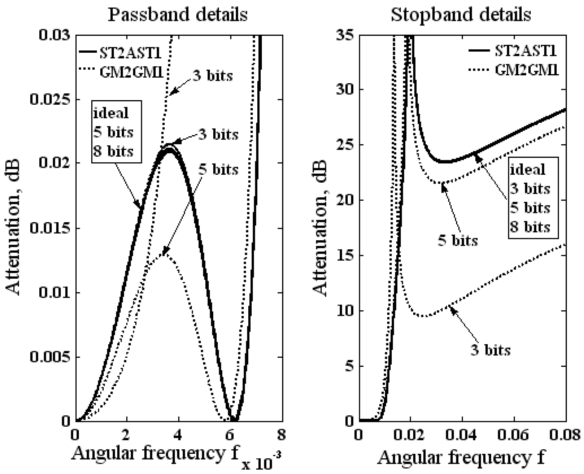



Fig. 7 Attenuation characteristics of 3$^{th}$ order TFs

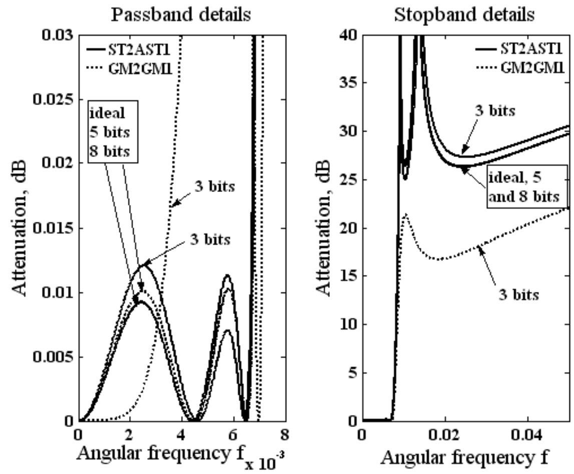



Fig. 8 Attenuation characteristics of 5$^{th}$ order TFs

## 5. CONCLUSIONS

In this paper a modification of the design method initially suggested by Milić and Lutovac has been proposed. The phase sensitivities to multiplier coefficients for the selected allpass sections have been derived. The half most sensitive coefficients have been represented with a small number of shifters and adders. The remaining coefficients are quantized with different coefficients wordlength. Consequently, the obtained multiplierless IIR realizations are with small quantization error and low complexity. It is shown that the sensitivities of the obtained filters can be considerably decreased if allpass sections used are with minimized sensitivities. It is firmly demonstrated (through experiments) that filters designed according to our approach are staying within the given specifications even after very severe quantization of the multiplier coefficients while the magnitudes of the reference filters (as design in [1][2]) are totally destroyed.

## 6. REFERENCES

[1] Milic L, M. Lutovac, *Design of multiplierless elliptic IIR filters with a small quantization error,* IEEE Trans. Signal Processing, vol.47, pp.469-479, no.2, Feb.1999

[2] Lutovac M, L. Milic, *Design of computationally efficient elliptic IIR filters with a reducednumber of shift-and-add operations in multipliers,* IEEE Trans. Signal Processing, vol.45, pp.2422-2430, Oct.1997

[3] Milic L, M. Lutovac, *Low-complexity lattice wave digital filters,* Proceedings of 13$^{th}$ Euro Conf. ECCTD, Budapest, Hungary, pp.1141-1146, Aug.1999

[4] Rabrenovic D, M. Lutovac, *Elliptic filters with minimal Q-factors,* Electron. Lett, vol 30, no.3, pp. 206-207, Feb.1994.

[5] Stoyanov G, H. Clausert, *A comparative study of first order digital all-pass structures*, Frequenz, vol. 48, No 9/10, Sept./Oct. 1994 pp. 221-226

[6] Stoyanov, G., V. Dimitrova, *Sensitivity and noise analysis of second-order digital all-pass sections* (in Bulg.). Proc. 23-th Sc. Conf. "Communication, electronics and computer systems'93", Sofia, Bulgaria, May 1993, vol. 1, pp. 104-115.

[7] Sugino H, A. Nishihara, *Frequency-domain simulator of digital networks from the structural description*, Trans.of the IEICE of Japan, Vol. E73, No.11, pp.1804-1806, Nov.1990.