ANALYSIS OF THE ACCURACY OF DIGITAL SINEWAVE GENERATOR WITH STEP-LINEAR APPROXIMATION

Katya Krasteva Stefanova  
Department of electronics, TU – Plovdiv, 61 Sankt Petrburg bvd, 4000 Plovdiv, Bulgaria,  
+359 32 67 00 25, k_stefanova@yahoo.com

Iliya Eduardov Petrov  
Department of electronics, TU – Plovdiv, 61 Sankt Petrburg bvd, 4000 Plovdiv, Bulgaria.

Beka Henrih Koen  
Department of electronics, TU – Plovdiv, 61 Sankt Petrburg bvd, 4000 Plovdiv, Bulgaria.

The paper presents the analysis of the absolute error of a digital sinewave generator with step linear approximation. The initial data for the synthesis of the sinewave (number of linear sections and number of steps in a section) are chosen according to a specified value of the amplitude error. Calculations are made for the first linear section, the angle coefficients of the other linear sections and the amplitudes of the end points of the sections. In this way is synthesized only the first quarter of the sinewave. The symmetry of the sinewave is used for the synthesis of the full cycle wave. The investigations show that the magnitude of the absolute error depends strongly on the number of linear sections and the number of steps in a section.

Keywords: error, sinewave generator, linear approximation, quantization.

Digital generators are used to generate frequency-, amplitude- and phase- stable sinewave signals. With improvement of technology they fast become alternative to traditional analog sinewave generators. They are applied in precise measuring devices for measuring of R, L and C values, and as master oscillators [1]. They are widely spread in medicine measuring appliances and telecommunication systems.

The main advantage of these generators is the stability of the parameters of the output sinewave. Besides they have high tuning resolution of the output frequency in the range from micro-Hertz to hundreds MHz. The initial phase can also be easily tuned which facilitates quadrature synthesizers design [2].

The methods for the synthesis of the sinewave are step approximation and line-step approximation. The line-step approximation is considered in the paper.

Sometimes the memory available for storing values is limited. That is why for the approximation of the output signal is used the symmetry of the sine function in order to reduce the necessary memory resources. The sine function has mirror symmetry in one half of the period (1) and central symmetry according to the point on the time axis, corresponding to the middle of the period (2):

\[
\sin\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} - x\right); \quad (1)
\]
This symmetry allows all investigations to be done only in the first quarter of the sinewave. The regarded part is divided in $k$ linear sections, each with $m$ steps and each step is represented by $n$ points (Fig.1).

\[
\sin(\pi + x) = -\sin(\pi - x). 
\] (2)

The values of the end points of each linear section ($k+1$) are calculated and stored in the memory. These points are used for computation of the angle coefficients of each linear section ($k$). These coefficients are also stored in the memory. Then all points of the first linear section ($m$ values) are calculated and also stored in the memory. The values of the next linear sections are calculated from the points of the first section by multiplying each of them by the corresponding coefficient for the regarded section.

The angular coefficients are calculated by the following formula:

\[
a_i = \frac{A_{i+1} - A_i}{\Delta t},
\] (3)

where $A_i$ is the value of the end point of $i$-th linear section, $\Delta t$ is the duration of one linear section.

When using step approximation, in the memory are stored the values of the steps for
the whole sinewave. Then the number of the memory cells is $4.k.m$. With linear-step approximation using the symmetry of the sine function the number of cells is reduced to $m+2.k+1$.

The error of the synthesized by the described method sinewave compared with the ideal sinewave is investigated in order to prove that it can be used for designing of precise devices. The absolute error is calculated for the points of the quarter sinewave (Fig.2).

![Fig.2. Absolute error of the step-linear approximation of the sinewave](image)

It can be seen from Fig.2 that the absolute maximum error appears in the second step of the first linear section. It is not surprising because of the property of the sine function that it has the greatest slope around the zero values. Another peculiarity of the error is that at every $m$-th step it becomes zero. That is so because at this point the sinusoidal function coincides with the approximating curve.

Comparison of the absolute errors using the two methods of approximation is insignificant [3]. The results are shown in Table 1.

<table>
<thead>
<tr>
<th>Number of steps</th>
<th>Absolute maximum error, [ V ]</th>
<th>Difference, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.77676095</td>
<td>0.94</td>
</tr>
<tr>
<td>24</td>
<td>0.64750018</td>
<td>0.65</td>
</tr>
<tr>
<td>28</td>
<td>0.55510322</td>
<td>0.48</td>
</tr>
<tr>
<td>32</td>
<td>0.48577386</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The investigations show that the magnitude of the absolute error depends strongly on the number of linear sections $k$ and the number of steps in a section $m$. This dependence can be seen on Fig. 3. The X-axis is the number of steps in a linear section, the Y-axis is
the maximum value of the absolute error. The number of linear sections is a constant value for each curve of the shown family.

When step-linear approximation is applied in a device, the memory and the D/A converter of the sinewave generator can work simultaneously. This allows the maximum frequency of the generated sinewaves to be increased several times.

**Fig.3. Family curves of the maximum errors for step-linear approximation of the sinewave**

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**REFERENCES**