THEORETICAL AND EXPERIMENTAL ANALYSIS OF THE RECTIFIER WITH CAPACITIVE FILTER

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This article presents a simple and rather precise method for analysis and design of a transformer-coupled input rectifier with a filter capacitor, which is used as a power supply for electronic circuits. It is based on an original logarithmic equation that determines the conduction angle of the rectifier diodes as a function of the transformer equivalent resistance. This equation can replace the classical design procedure that is based on coefficients taken from nonlinear graphical diagrams. Some experimental results that verify the proposed theory are also presented.

Keywords: Rectifier Analysis and Design, Filter Capacitor, Rectifier Model.

1. INTRODUCTION

The classical power supply for the electronic circuits consists on a transformer supplied from the ac line voltage, a full-wave bridge rectifier with a capacitor filter and a voltage regulator (usually an integrated circuit).

The ripple at the rectifier output depends mainly on the capacitor value and the load current. The smoothing capacitor value can be computed based on the diode conduction angle, by utilizing few coefficients that can be found from tables or nonlinear diagram [1].

2. THEORETICAL ANALYSIS

The classical diode-bridge rectifier is presented in Fig.1. The load of the circuit, R is considered to be resistive. Firstly, the circuit components will be considered to be ideal (with C finite) and then an infinite capacitor and ideal diodes will be considered, with a non-zero transformer resistance [2].

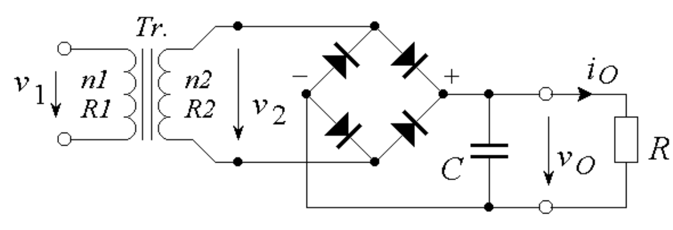

Fig. 1 – The diode-bridge rectifier with capacitor filter.

For a sine input, the transformer output (same with the rectifier input voltage) is:

$$v_2 = v_i = V_p \sin \omega t . \qquad (1)$$

2.1 Ideal rectifier with finite Capacitor

The rectifier waveforms for a time constant much greater than the output signal period, $RC = 5(T/2)$ in this case, are presented in Fig.2.

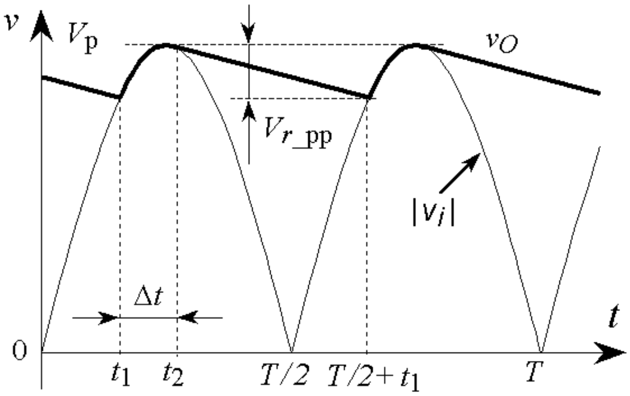

Fig. 2 – Rectifier waveforms for finite C (ideal Tr. and diodes).

For $RC >> T/2$ the minimum output voltage, an important value for designing the linear voltage regulator is:

$$v_{O\min} \cong V_p \exp\left(-\frac{T}{2RC}\right). \quad (2)$$

The ripple voltage (peak-to-peak and rms values) and the average output voltage are determined by the circuit time constant RC, [2] and [3]:

$$V_{r\_\text{pp}} \cong V_\text{p} \frac{T}{2RC}, \quad V_r \cong \frac{V_{r\_pp}}{2\sqrt{3}} \cong V_\text{p} \frac{T}{4\sqrt{3}RC}, \quad (3)$$

$$V_O = V_\text{p} - \frac{V_{r\_\text{pp}}}{2} = V_\text{p}\left(1 - \frac{T}{2RC}\right). \quad (4)$$

The ripple factor is:

$$r \cong \frac{V_r}{V_O} \cong \frac{T}{\sqrt{3}(4RC - T)} \cong \frac{T}{4\sqrt{3}RC}. \quad (5)$$

These results can be used for very low internal resistance of the transformer (much lower than R), but this is not the case for most of the practical circuits.

2.2 Ideal Rectifier with Finite Transformer Resistances

In the second analysis, an infinite capacitor and ideal diodes will be considered. The waveforms for these assumptions are presented in Fig.3.

The output waveform is a pure dc voltage, the ripple being zero (3), because of the infinite capacitor. With 2θ being the conduction angle (as in Fig.3), the output voltage and current can be computed:

$$V_O = V_\text{p} \cos\theta, \quad I_O = \frac{V_\text{p}\cos\theta}{R}. \quad (6)$$

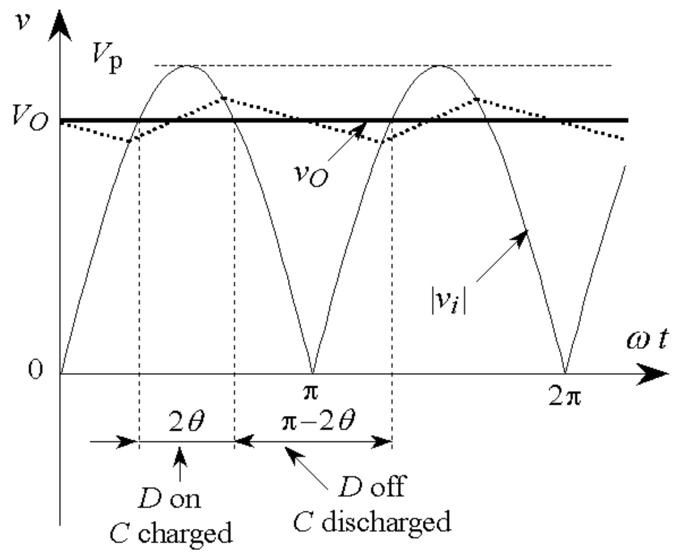

Fig. 3 – Rectifier waveforms for ideal diodes, infinite capacitor and finite transformer equivalent resistance.

Fig.4 presents the equivalent circuit while C is charging, R_i being the transformer equivalent resistance from the secondary point of view:

$$R_i \cong R_{\text{tr}} = R_2 + R_1 \left(\frac{n_2}{n_1} \right)^2 , \tag{7}$$

where R_2, n_2, R_1, n_1 are the resistance and the number of turns of the transformer secondary and primary windings, respectively.

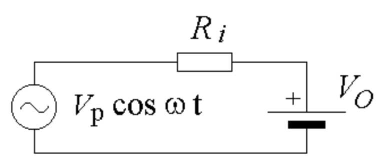

Fig. 4 – The equivalent circuit while the capacitor is charging.

The charge conservation law can be used to find the conduction angle. The average current received by the equivalent voltage source V_O during one period (of the output signal) should be equal with the average current supplied by the equivalent voltage source during the same period:

$$Q_{\text{ch}} = I_{\text{ch}} T = Q_{\text{dsc}} = I_O T . \tag{8}$$

The average current that charges the capacitor can be calculated by integration (in the schematics in Fig.3):

$$I_{\text{ch}} = \frac{2}{\pi} \int_0^{\theta} \frac{V_{\text{p}} \cos \omega t - V_{\text{p}} \cos \theta}{R_i} d\omega t . \tag{9}$$

By replacing (9) and (6) in (8):

$$\frac{V_{\text{p}}}{\pi R_i} \left(\sin \theta - \theta \cos \theta \right) = \frac{V_{\text{p}} \cos \theta}{R} , \tag{10}$$

the ratio between the internal resistance R_i and the load resistance R is expressed as a function of the conduction angle, $k_i(\theta)$:

$$\frac{R_i}{R} = \frac{tg\theta - \theta}{\pi} = k_i. \quad (11)$$

In order to compute the average output voltage, equation (6), one need to know the conduction angle. The function $\theta(k_i)$ can be found utilizing approximate analysis. The function (11) appears approximately linear when a logarithmic scale for k_i is used and a linear approximation of it can be outlined for the usual domain of k_i, as in Fig.5. The approximate function can be found (in radians or in degrees) considering two points of the line ($\theta = 0$, $k_i = 0.0031$ and $k_i = 1$, $\theta = 1.2 = 69°$):

$$\theta \cong 0.48 \lg k_i + 1.2\,[\text{rad}], \qquad \theta \cong 27.5 \lg k_i + 69\,[°]. \quad (12)$$

In Fig.5 the error (between the initial function and its approximation, expressed in %) is represented also. The approximate function can be used for $k_i = 0.05\ldots1$ with an error less than 1.5%.

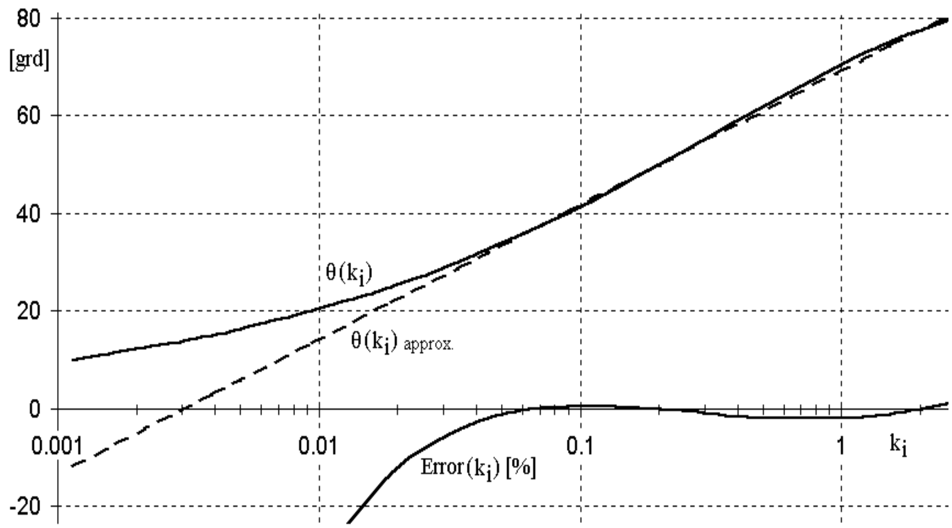

Fig. 5 – The graph of $\theta(k_i)$, its approximation (dashed line) and the error between these (in %).

The effect of the diodes voltage drop can be taken in account by subtracting a constant voltage from the input peak voltage (V_D=0.7…0.9V, for every diode connected in the current path). The analysis of the circuit with a finite capacitor and a non-zero transformer resistance leads to equations that can not be solved by hand. An analysis based on nonlinear graph [1] or a computer simulation can be made.

2.3 The Ripple Limits

Another possible method is to determine the ripple limits (the minimum and maximum values for a given situation). The ripple voltage and factor computed for an ideal rectifier (with $R_i = 0$) and a finite capacitor, equations (3)…(5), represents the maximum ripple. Two reason for that can be stressed: the capacitor discharging time is the maximum possible (10 ms) and for a non-zero R_i, the R_i-C group introduces a supplementary filtering effect that reduces the ripple.

To compute the minimum ripple value, the conduction angle determined for the actual R_i and for an infinite capacitor can be considered. The fact that this is the minimum ripple results from Fig.3 graph and will be experimentally verified. In Fig.3, where the waveform for a finite capacitor is sketched (with a dotted line), one can see that the diode off-time is the shortest one for an infinite capacitor (the thick horizontal line is shorter than the dotted one). The ripple voltage and factor is proportional with time and can be computed (for θ expressed in radians) with:

$$V_{r\_\min} \cong V_{\mathrm{p}} \frac{T}{4\sqrt{3}RC}\left(\frac{\pi-\theta}{\pi}\right), \quad r_{\min} \cong \frac{T(\pi-\theta)}{4\pi\sqrt{3}RC} \cong r_{\max}\left(\frac{\pi-\theta}{\pi}\right). \qquad (13)$$

3. EXPERIMENTAL RESULTS

Some experiments in the laboratory were made to prove the formulas derived in the theoretical analysis section of this article.

A full-wave bridge (1PM1) rectifier with a (220/12V, 0,2A) transformer, two different capacitors (220 μF and 1000 μF) and a variable load (50…160 Ω) were considered. The measurements, the theoretic calculations and the errors between these results are presented in table 1.

The theoretical results are computed as follows:

- R with Ohm's law at the rectifier output,
- k_i from equation (11),
- θ (in °) from equation (12),
- V_O (V) from equation (6) and
- The ripple factor: case "max", from (5) and case "min" from (13).

For the ripple factor, the differences between theory and experiment are high; for the "max" case the theoretical r-s are greater than the experimental ones with 50 to 93 %. The minimum r-s are lower than the experimental ones (with 17 to 23 %), and closer to the experiment than the maximum approximations.

If one is interested in finding a precise ripple factor, he is recommended to use equation (13), but the result is somehow lower than in the practical circuit. Another observation is concerning the tolerance of the filter capacitor that is rather high, usually -10…+50 %. The high dispersion of the capacitor value does not justify very precise formulas when this capacitor is involved, such that typical results can be found with equation (13), because the typical positive capacitor tolerance compensates the typical negative error given by formula (13).

On the other hand, the ripple factor can be computed by equation (5), the formula is much simpler, the result is covering all the cases, but it is much greater than the actual one. Equation (5) is appropriate for the worst case analysis.

A special remark should be made about the precision of the average output voltage (V_O) computing. The error is lower than few percents and more important, it doesn't depend significantly on the capacitor value. For computing V_O, the experiments indicates that the condition $RC \gg T/2$ is not a critical one; for example, the minimum time constant, for C = 220μF and R = 53Ω, is almost equal with the period of the signal at the rectifier output, that is $T/2 = 10$ms.

Table 1. The measurements, the theoretical calculations and the errors between these.

	I_O (mA)		80	100	180
C = 220μF	V_O (V dc)		12.8	11.9	9.5
	V_o (V ac)		0.69	0.83	1.22
	$r1$		0.054	0.069	0.128
C = 1000μF	V_O (V dc)		12.8	12	9.6
	V_o (V ac)		0.15	0.18	0.27
	$r2$		0.0117	0.015	0.028
Theory	R (Ω)		160	120	53.3
	Ki		0.098	0.131	0.295
	θ (°)		41.3	44.7	54.4
	V_O (V dc)		12.5	11.8	9.66
	$r1$	max	0.082	0.109	0.245
		min	0.045	0.055	0.098
	$r2$	max	0.018	0.024	0.054
		min	0.001	0.012	0.022
Errors	V_O (%)		-2.3	-1.6	+0.6
	$r1$ (%)	max	+52	+58	+91
		min	-17	-20	-23
	$r2$ (%)	max	+50	+60	+93
		min	-18	-20	-21

4. CONCLUSIONS

In this paper, the transformer-input, capacitor-filter, diode-bridge rectifier was analyzed and some original formulas were presented and justified. These formulas can be used to compute the diode conduction angle, the ripple factor and the average voltage at the rectifier output.

The proposed formulas were theoretically justified and experimentally verified. The differences between theory and experiment were stressed and practical recommendations were made in the previous section.

5. REFERENCES

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