STUDY OF THE IMPACT OF THE TIMING JITTER ON THE STATISTICAL INDICATORS OF THE SIGNAL SAMPLING BASED MEASUREMENTS

Ivo G. Aldimirov[1]

[1]ATSA, Communications Dept., 4023 Plovdiv International Airport, Bulgaria, e-mail: aldimirov@yahoo.com;

Keywords – timing jitter, digital signal processing, statistical indicators, test results distribution.

This paper presents the results of a study focused on the impact the timing jitter has on the statistical indicators of the signal sampling based tests.

1. INTRODUCTION

The statistical indicators of the quality tests in production have crucial importance for the overall quality and cost of the new produced integrated circuits (IC) [8, 9]. The process of qualification of the new developed IC goes through the detailed investigation of the statistical indicators - *Cp*, *Cpк* and standard deviation – σ *of all tests* in the product specification [6, 8, 9]. Lower values of *Cp* and *Cpк* might lead to prolonged industrialization process of the IC, quality loss and cost increase as well as longer *time-to-market* period.

This paper presents the results of a study focused on the impact the timing jitter has on the statistical indicators of the digital signal processing (DSP) based tests. It is presented the mathematical model of the process of signal sampling in the presence of timing jitter. It is given the analytical equation describing the relation between the standard deviation and the timing jitter quantity.

2. STATISTICS PROBLEMS OF THE DSP BASED MEASUREMENTS IN THE PRESENCE OF TIMING JITTER

In the base of the DSP based test methods is the amplitude measurement of a known spectrum component of interest defined in the test program - spectral bin, frequency of interest. The presence of timing jitter in the signal sampling process has severe impact on the measurement accuracy and repeatability. The effect of jitter can be associated with a random timing variable denoted as *tj*, and added to the period *T* of the sampled signal. The result of the amplitude measurement of frequency bin corresponding to the frequency of interest will bring information not only for the spectral bin of interest but also for the neighboring ones in the signal spectrum. This is easily seen in fig. 1, where the overlapped results in frequency domain of the several times repeated measurement of a sampled signal with timing jitter is presented.

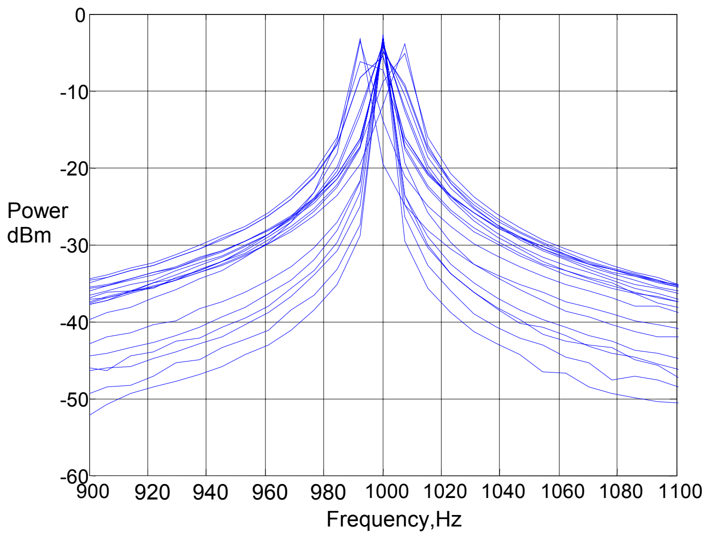

Fig. 1

The frequency change Δf of the measured signal, caused by the timing jitter component – tj, is be denoted as:

$$\Delta f = f - fj\,, \qquad (1)$$

Where f is the fundamental tone, and fj is the changed frequency according to:

$$fj = \frac{1}{(T \pm tj)}, \qquad (2)$$

Where $T = \frac{1}{f}$.

For the signal with test results shown in fig. 1, the ratio tj/T=1%. The distribution of the test results in this case would be widened as shown in fig. 2. This result presents significant parasitic increase of the standard deviation σ = 5.4123 dBm.

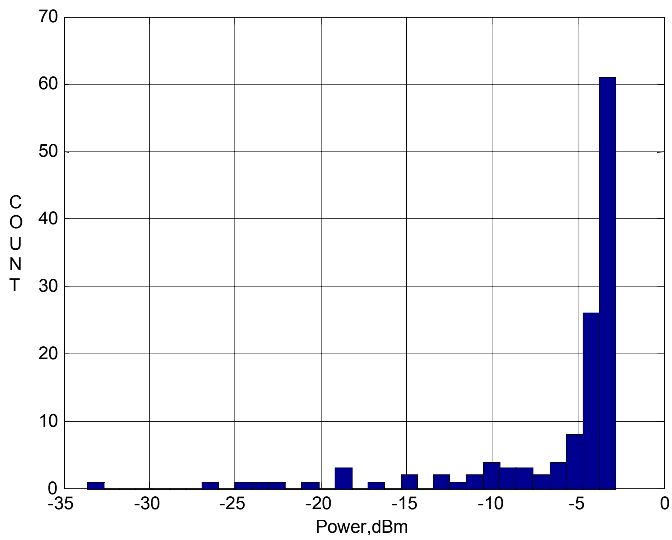

Fig. 2

3. MATHEMATICAL MODEL OF THE SIGNAL SAMPLING PROCESS IN PRESENCE OF TIMING JITTER

The simulated model of the DSP measurement in presence of timing jitter is a key point for the analytical study of the process. The parameters of the mathematical simulation are as follows. The frequency of the sampled sinus signal is f =1kHz. The signal-to-noise-ratio is SNR=40dB. The sampling frequency is fs=4kHz. Every measurement takes 128 signal periods. The jitter timing component tj is added up to the signal period T. The tj/T ratio defines the maximum value of the frequency deviation. tj/T takes random Gaussian values between 0% and the maximum specified value in %. Fig. 3 shows the distribution of the added up values of tj corresponding to the group of measurements with maximum value of tj/T=0.5%. The negative part of the distribution corresponds to the negative change of the signal period.

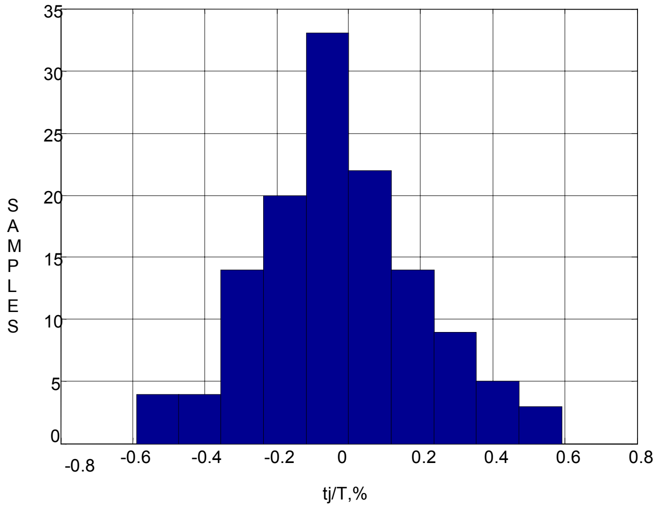

Fig. 3

Because of the random character of the added to the signal timing variable and noise, for deriving the equation σ = f (tj/T) is used the method of "Monte Carlo". 100 measurements are performed for every maximal value of tj/T, %.

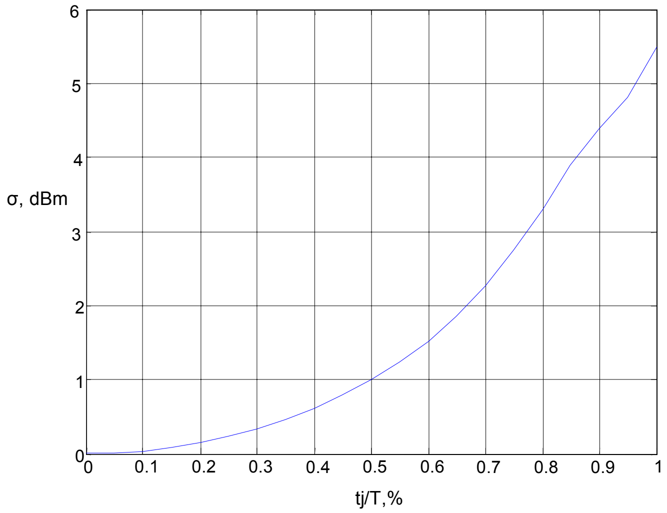

Fig. 4

The simulation output result shown in fig. 4 represents the averaged value of the standard deviation for every group of 100 measurements as a function of the ratio *tj/T*, %.

4. ANALYTICAL EXPRESSION OF THE RELATION BETWEEN THE STANDARD DEVIATION AND THE JITTER QUANTITY

Interpolating the simulation output data gives the mathematical equation, describing the relation between σ and the jitter quantity. By the means of the specialized software tool XlXtrFun™ for functions interpolation based on experimental data, the following graphical and analytical results are obtained. The curve of the experimental data is interpolated with the least squares fit criteria. After the synthesis of the interpolating polynomial equation, it is solved for 160 points – values of the variable *x*. The analytical expression of least order (in this case 5th) providing least square error for the overall range of values is:

$$\sigma(x) = -30.2157 \cdot x^5 + 64.206 \cdot x^4 - 42.4253 \cdot x^3 + 14.8393 \cdot x^2 - 1.0179 \cdot x + 0.0139 , \quad (3)$$

Where *x* denotes *tj/T*, %. Fig. 5 represents the coincidence results of the experimental graph – discrete values in red, and the graph of the interpolating equation - in black.

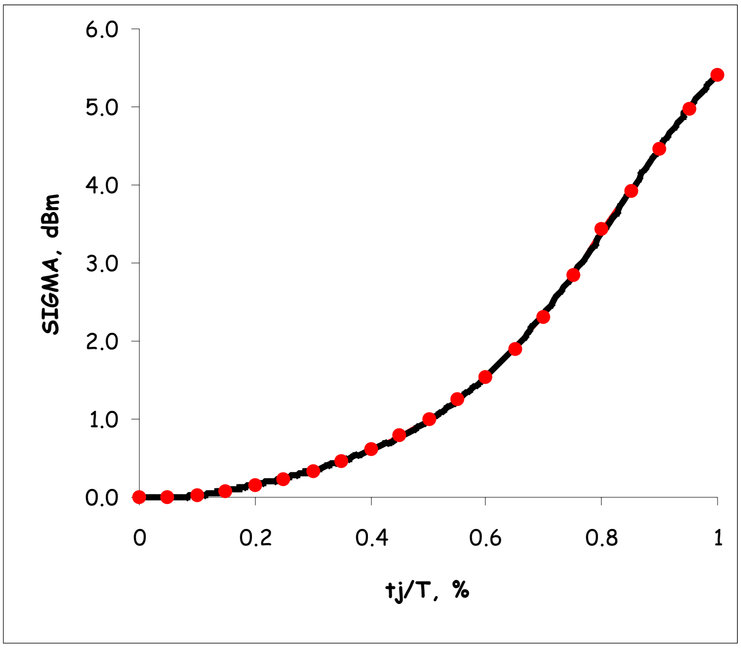

Fig. 5

5. CONCLUSIONS

The analyses of the statistical methods used for judgment of the process quality in the IC manufacture show that the presence of timing jitter in the sampled signal leads to severely worsened test results distributions, statistical indicators, stability and repeatability of the measurements.

The created mathematical model simulating DSP measurement in presence of timing jitter, successfully represents the real-life measurement conditions and results, thus playing a key role in the analytical analyses of the problem.

The relation between the standard deviation and the quantity of timing jitter is investigated and analyzed based on the results of the model simulation. The interpolating polynomial equation is solved for 160 points and the coincidence with the experimental data curve is demonstrated.

In this paper is presented a research of the statistical indicators of the quality tests problems caused by the presence of timing jitter in the sampled signal. It is created mathematical model of the sampling process in the presence of jitter, helping the study and analyzes of the problem. Using the powerful method of "Monte Carlo" it is obtained and analyzed the relation between the standard deviation and the quantity of timing jitter in the sampled signal. By the means of a specialized software tool for functions interpolation based on experimental data, it is derived the analytical equation, describing this relation.

The following numerical and graphical data are demonstrated:

- The results of the mathematical simulation of the sampling process in the presence of jitter
- The result of the coincidence between the interpolating polynomial curve and the curve of the experimental data

The author would like to extend his gratitude to eng. Rossen Miletiev, who has helped through the process of creating and debugging the jitter model software and provided the necessary simulation results data.

6. REFERENCES

[1] Alcatel Microelectronics, C150 ASSP ATE test specification version 0.4, 2002

[2] Alcatel Microelectronics, C150 ASSP ATE test report, October, 2002

[3] Alan V. Oppenheim, Ronald W. Schafer, Discrete-Time Signal Processing, Prentice Hall, Englewood Cliffs, NJ, March 1989, ISBN: 013216292X

[4] Alan V. Oppenheim et aI., Signals and Systems, Prentice Hall, Englewood Cli*ffs*, NJ, August 1997, ISBN: 0138147574

[5] Forrest W. Breyfogle, Implementing Six Sigma: Smarter Solutions Using Statistical Methods, 2nd edition, June 7,1999, John Wiley & Sons, New York, NY, ISBN: 0471296597

[6] L. Shiano, M. Momenzadeh, F.-M. Zhang, Y.-J. Lee,Y. Kim, F. J. Meyer, F. Lombardi, S. Max, ``Frequency Domain Measurement of Timing Jitter", 2004 IEEE IMTC

[7] Mark and Gordon W. Roberts, An introduction to Mixed-Signal IC Test and Measurement, Oxford university press, 2001

[8] Mark J. Kiemele, Stephen R. Schmidt, Ronald J. Berdine, Basic Statistics Tools for Continuous Improvement, Fourth Edition, Air Academy Press, 1155 Kelly Johnson Blvd., Suite 105, Colorado Springs, CO, 80920,1997, ISBN: 1880156067

[9] Statistical control tools for quality DVD manufacturing, Dr Stephan Kueper, www.oto-online.com/jan01/stats.html.