DESIGN OF MULTIPLIERLESS LINEAR PHASE FIR FILTERS WITH MINIMUM NUMBER OF POWER-OF-TWO-TERMS-COEFFICIENTS

Valentina Anzova, Kamelia Ivanova and Georgi Stoyanov

Department of Telecommunications, Technical University of Sofia, 8 Kl. Ohridski 1756, Sofia, Bulgaria, phone: +359 2 965 2134, e-mail: via@tu-sofia.bg, ksi@tu-sofia.bg, stoyanov@ieee.org

Keywords: linear phase FIR filters, multiplierless realizations, mixed integer linear programming

An optimization problem for designing of a non-uniformly spaced, linear phase FIR filter with coefficients consisting of minimum number of signed-power-of-two terms is formulated. The aim of this paper is first to reduce the region that contains the optimal solution in order to decrease the computation time and, then FIR filters are designed using mixed integer linear programming (MILP). In this paper we minimize the number of signed-power-of-two-terms per coefficients for defined range of stopband ripple and fixed passband ripple, subject to filter specification, filter order and number of coefficient bits. We optimize both coefficients representation and stopband ripple. In our method all possible coefficient values for given stopband ripple are calculated and compared and thus the one containing a minimum number of terms is selected. It is shown that our optimization procedure compares very favorably with other known methods.

1. INTRODUCTION

Recently, numerous algorithms have been proposed for designing multiplierless finite impulse response (FIR) filters. In multiplierless digital filters multiplications are replaced with a sequence of shifts and adds (or subtracts). Therefore only adders (or subtracts) are required for the coefficient implementation. This leads to significant reduction in the computational complexity and power consumption.

In order to find an optimum digital filter implementation satisfying all the design criteria in some cases optimization methods can be used. FIR filters with discrete coefficient values are designed using the methods of integer programming. When the coefficients are represented by signed-power-of-two (SPT) terms the most used method is mixed integer linear programming (MILP). Different objective functions are being considered in existing literature. MILP has been used earlier in [1]-[3] where the objective function was to minimize the ripple subject to filter specification. In [4] and [5] the aim was to minimize the number of SPT terms. This leads to reduction of hardware cost. An optimization problem for designing linear phase FIR filter with minimal complexity was formulated and solved by MILP in [6]. Another approach for design of linear phase FIR filters with minimum number of adders required to meet the specifications was proposed in [7].

In this paper we minimize the number of SPT terms for defined range of stopband (SB) ripple and fixed passband (PB) ripple, subject to filter specification, filter order and number of coefficient bits. First we reduce the region that contains the optimal solution in order to decrease the execution time, as proposed in [8]. In our method we optimize coefficient representation for fixed PB ripple and search an optimal solution, using different values for the SB ripple. We have written a simple program, realizing the proposed algorithm. The main advantages of our method compared with other existing algorithms are its simplicity and reduction of the execution time.

2. BASIC THEORY

2.1 Linear phase FIR filters

The filter specifications must fit in given limits. The amplitude response of the desired filter should range in $[1+\delta_p \; 1-\delta_p]$ for the PB and in $[\delta_s \; -\delta_s]$ for the SB, where δ_p and δ_s are ripples in the PB and SB respectively.

Specifications for a lowpass (LP) filter are thus formulated as:

$$1-\delta_p \le |H(\omega)| \le 1+\delta_p, \quad \omega \in [0 \; \omega_p]; \quad -\delta_s \le |H(\omega)| \le \delta_s, \quad \omega \in [\omega_s \; \pi], \tag{1}$$

where ω_p and ω_s are the PB and SB frequencies respectively.

The transfer function of FIR filter of length N+1 is given by

$$H(e^{j\omega}) = \sum_{n=0}^{N} h(n)e^{-j\omega n} . \tag{2}$$

This filter has linear phase if its impulse response h(n) is either symmetric, i.e.,

$$h(n) = h(N-n), \quad 0 \le n \le N, \tag{3}$$

or is antisymmetric, i.e.,

$$h(n) = -h(N-n), \quad 0 \le n \le N \tag{4}$$

and thus four types of symmetry for the impulse response are possible. The magnitude response for the first type, for example, is

$$|H(\omega)| = h(\frac{N}{2}) + 2\sum_{n=1}^{N/2} h(\frac{N}{2}-n)\cos(\omega n) . \tag{5}$$

2.2 Multiplierless realizations

The basic operations in digital signal processing are addition, multiplication and delay. In VLSI implementations, a multiplier element is very costly. Therefore, it is very attractive to replace the multiplication of a data sample by each filter coefficient value with a series of adder and/or subtractors and shifters. The shifts are often hard-wired hence only adders are required for the coefficient implementation. This kind of multiplier representation is very effective in terms of area, power and delay compared with general multipliers. In this paper we focus on designing filters such that all the coefficients are representable using several power-of-two terms. In this case canonic signed digit (CSD) representation is the most proper.

2.3 CSD principles

CSD number representation is the unique representation of a given number as a sum of powers of two. Multiplication to a CSD number is very cheap since it is a sum of shifted versions of the multiplicand.

For a given M-bit representation of a number $h_m \in [-2^{M-1} \; 2^{M-1}]$, CSD representation is given by:

$$h_m = \sum_{k=1}^{M} s_k 2^{-k}, \tag{6}$$

where $s_k \in \{-1,0,1\}$. CSD representation is the signed digit representation in which

$$s_k s_{k-1} = 0 \quad \text{for } k = 1 \ldots M-1. \tag{7}$$

Among all signed digit representations, the CSD representation has the minimum number of nonzero digits.

In our algorithm we use linear programming and the objective function has to be linear. In this case power-of-two terms representation in (6) must transform to:

$$h_m = \sum_{k=1}^{M} (a_{m,k}^{+} - a_{m,k}^{-}) 2^{-k}, \tag{8}$$

where $a_{m,k}^{+} \in \{0,1\}$ and $a_{m,k}^{-} \in \{0,1\}$. In this way we have linear constraints on number of nonzero digits.

3. NEW METHOD OUTLINE

The desired linear phase FIR filter can be conveniently determined using a few steps procedure. First step involves determining initial closed space including the feasible space, where the filter meets the given criteria. To find this space we use the following constraints:

1. Discrete space constraints.
2. CSD code constraints.
3. Filter specification constraints.

The second step involves finding the filter coefficients with the minimum SPT terms using MILP. The objective function in the proposed algorithm compared with other existing algorithms minimizes the number of SPT terms for fixed PB ripple and various SB ripple values. In this way the optimization of both, the number of SPT terms and SB ripple value, leads to an optimal filter which exactly meet the initial conditions. The received filter is compared with Remez one. In addition, our algorithm is much faster than the conventional one used in [9].

4. LIMITATIONS INVESTIGATION

The aim of the first part of the proposed algorithm is to find initial closed space subject to some constraints. In discrete space filter coefficients can take only discrete values within a specified range [-1 1]. The quantization step depends on available wordlength representing numbers being used. For M-bit CSDC number representation the maximal possible value is:

$$\sum_{k=0}^{\lfloor M/2 \rfloor - 1} 2^{-2k-1}. \tag{9}$$

If we would use 7 bit number representation, the limitation (9) will produce discrete space filter coefficients within the range [-0.65625 0.65625]. In order to narrow further the searched space we introduce uniformly distributed grids of frequencies in PB and SB. The bigger the number of selected frequencies is the smaller the searched space. On the other hand, extremely big amount of frequency points leads to increased time of computations for optimal solution. In our approach we have chosen the frequency step one time smaller than cutoff frequencies. This step is a compromise between computational time and size of the searched space. The selected grid leads to strong reduction of the space that contains the optimal solution as shown in Fig. 1. The magnitude response of the desired linear phase FIR filter for given frequency points has to meet the filter requirements (1). In order to illustrate the graphical coefficients representation we consider the following example:

Example: 5 *bits wordlength, linear phase FIR filter of order N*=3, $\delta_p = \delta_s = 0.3$ *and cutoff frequencies* $\omega_p = 2\pi 0.2\,\mathrm{rad/s}$, $\omega_s = 2\pi 0.337\,\mathrm{rad/s}$.

This simplification allows representation of the searched space in two dimensions.

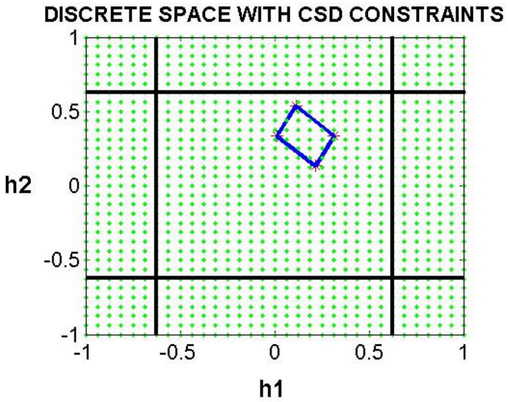

Fig. 1. Localization of the initial space containing optimal solution.

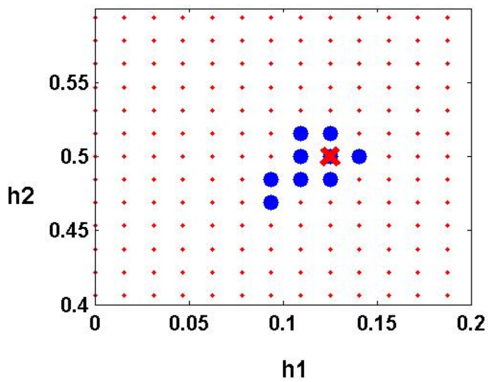

Fig. 2. All possible solutions obtained with proposed algorithm '•' and optimal solution '**x**'

In second part of our algorithm we find the optimal solution using MILP. We concentrate on optimization of linear phase FIR filter with odd length symmetric impulse response. Starting from (5) and utilizing (1), (6) and (8) we can formulate the following optimization problem:

$$\text{minimize} \qquad \sum_{m=1}^{N/2+1}\sum_{k=1}^{M}(a_{m,k}^{+} + a_{m,k}^{-})$$

$$\text{subject to} \qquad \begin{cases} (1+2\sum_{n=1}^{N/2}\cos(\omega n))\sum_{k=1}^{M}(a_{m,k}^{+} - a_{m,k}^{-})2^{-k} \le 1+\delta_p & \omega\in[0\ \omega_p] \\ -(1+2\sum_{n=1}^{N/2}\cos(\omega n))\sum_{k=1}^{M}(a_{m,k}^{+} - a_{m,k}^{-})2^{-k} \le \delta_p - 1 & \omega\in[0\ \omega_p] \\ (1+2\sum_{n=1}^{N/2}\cos(\omega n))\sum_{k=1}^{M}(a_{m,k}^{+} - a_{m,k}^{-})2^{-k} \le \delta_s & \omega\in[\omega_s\ \pi] \\ -(1+2\sum_{n=1}^{N/2}\cos(\omega n))\sum_{k=1}^{M}(a_{m,k}^{+} - a_{m,k}^{-})2^{-k} \le \delta_s & \omega\in[\omega_s\ \pi]. \end{cases} \qquad (10)$$

The solution of this optimization problem yields coefficients set that is minimal in terms of SPT terms. As a result of the optimization some solutions with equal minimal number of SPT terms can be obtained (Fig. 2). In this case the optimal result is chosen in terms of SB ripple.

5. EXPERIMENTS

A linear phase LP FIR filter with $\omega_p = 2\pi 0.2\,\mathrm{rad/s}$ and $\omega_s = 2\pi 0.337\,\mathrm{rad/s}$ is considered. The filter order is N=6 with symmetric impulse response, PB ripple is $\delta_p = 0.1$ and coefficient wordlength is M=7 bits. The filter was optimized in terms of the number of SPT terms and SB ripple. We consider two cases:

1. Maximum ripple is $\delta_p = \delta_s = 0.1$ in both the SB and PB. The obtained multiplierless linear phase FIR filter with 7 SPT terms for the 4 coefficients is compared with Remez one as shown in Fig. 3.

2. SB and PB ripple are different. In this case all possible coefficient values for given stopband ripple range [0.08 0.2] are calculated and compared and the one containing a minimum number of SPT terms is selected. As a result of the optimization three possible solutions with 6 SPT terms for the 4 coefficients (Fig. 4) and $\delta_p = 0.1$ $\delta_s = 0.16$ are obtained. Dashed line (Fig. 4) shows the magnitude response for optimal multiplierless linear phase FIR filter that meet exactly the filter specification. The received filters are compared with Remez one (solid line).

The magnitude responses of the filters designed using various algorithms are shown in Fig. 5. The filter obtained with [9] has the same number of SPT terms for filter coefficients as ones obtained with our algorithm (case 2). This example shows that in our algorithm the execution time is reduced significantly (Table 1.).

Table 1.

Method	δ_p, δ_s	Number of SPT terms for filter coefficients	Execution time
Proposed method	$\delta_p = 0.1$ $\delta_s = 0.16$	6	27.937 s
Matlog Toolbox [9]	$\delta_p = 0.1$ $\delta_s = 0.16$	6	69.047 s

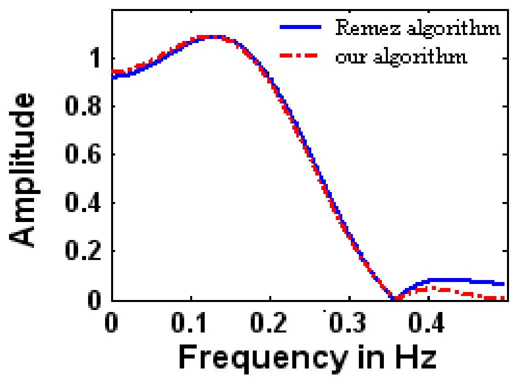

Fig. 3. Magnitude responses for the optimized multiplierless filters using proposed algorithm and Remez approach.

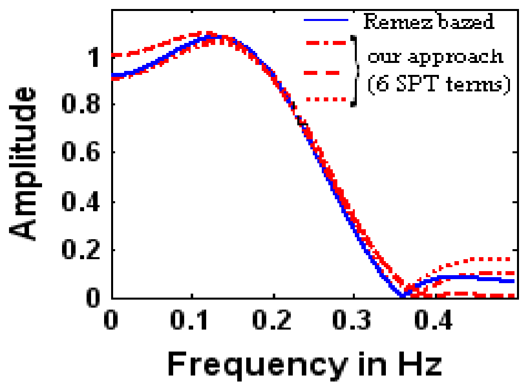

Fig. 4. Magnitude responses for our design (3 different curves) and Remez based filter.

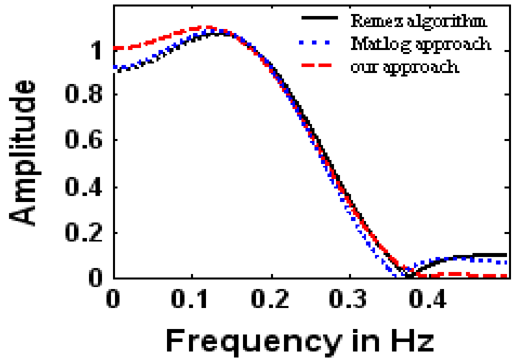

Fig. 5. Magnitude response for the optimized multiplierless filters using proposed algorithm, Matlog Toolbox algorithm and Remez algorithm.

6. CONCLUSION

In this paper an efficient MILP algorithm for design of linear phase multiplierless FIR filters was developed. It is minimizing simultaneously both the number of SPT terms for filter coefficients and the SB ripple, while in many other publications the aim of optimization is minimization only of the number of SPT terms. The filter designed using our method meets more precisely the filter specifications while taking shorter execution time. The proposed algorithm compares very favorably with the other existing methods concerning its simplicity and it is easy to be used. The efficiency of the method and its advantages over the other methods are verified experimentally.

7. REFERENCES

[1] Y.C. Lim, S. R. Parker, and A. G. Constantinides, *Finite word length FIR filter design using integer programming over a discrete coefficient space*, IEEE Trans. Acoust., Speech, Signal processing, vol. 30, pp. 661-664, Aug. 1982.

[2] Y.C. Lim and S. R. Parker, *FIR filter design over a discrete powers-of-two coefficient space,* IEEE Trans. Acoust., Speech, Signal processing, vol. 31, pp. 583-591, June. 1983.

[3] Y.C. Lim, *Design of discrete-coefficient-value linear phase FIR filters with optimum normalized peak ripple magnitude,* IEEE Trans. Circuits Syst., vol. 37, pp. 1480-1480, Dec. 1990.

[4] O. Gustafsson, H. Johansson, and Lars Wanhammar, *An MILP approach for the design of linear-phase FIR filters with minimum number of signed-power-of-two terms*, Proc. European Conf. Circuit Theory Design, Espoo, Finland, Aug. 28-31, 2001.

[5] H.Samueli, *An improved search algorithm for the design of multiplierless FIR filters with powers-of-two coefficients*, IEEE Trans.Circuits Syst., vol.36,pp 1044-1047,July 1989.

[6] J. T. Kim, W. J. Oh and Y. H. Lee, *Design of nonuniformly spaced linear-phase FIR filters using mixed integer linear programming,* Signal Processing, IEEE Transactions, vol. 44 , issue 1, pp. 123 – 126, Jan. 1996.

[7] J. Yli-Kaakinen and T. Saramaki, *A systematic algorithm for the design of multiplierless FIR filters*, Circuits and Systems, 2001. ISCAS 2001, The 2001 IEEE International Symposium, vol. 2 , pp. 185 – 188, 6-9 May 2001.

[8] B. Boulerial, M. F. Belbachir and A. N. Belbachir, *Reduced Search Space for the Optimal Design of Linear Phase FIR Filter,* 12th European Signal Processing Conference, Vienna, Austria, September, 2004 submitted.

[9] The Matlab Toolbox Matlog, http://www.ie.ncsu.edu/kay/matlog