

# INVESTIGATION OF LEAST-SQUARES ALGORITHM BASED ADAPTIVE DIGITAL FILTER SECTION FOR DETECTION OR SUPPRESSION OF A SINGLE SINUSOIDAL SIGNAL

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*A new adaptive line enhancer using a recently advanced digital biquadratic filter section and a modified least-squares based algorithm is proposed in this paper. The new structure is having an independent tuning of the central frequency and the bandwidth of the bandpass/bandstop realizations, permitting a considerable reduction of the computations (by eliminating the matrix inversion operation), while ensuring very small residual error in low frequency applications, due to the very low sensitivity for poles near  $z=1$ . All theoretical results are verified experimentally.*

## 1. INTRODUCTION

Detection and enhancement (or suppression) of a narrow-band signal, existing together with some wide-band signal/noise, is a very common task in telecommunications and electronic measurements. Adaptive systems detecting, tracking and evaluating the frequency of one or several sinusoidal signals are called adaptive line enhancers (ALE). There is a trend in the recent years to realize ALEs around adaptive second-order IIR bandpass (BP) and bandstop (BP) filter sections [1]-[6] – at least one section per sinusoid when cascaded, but generally much more, connected in a complicated structures [1], [2], [3]. Adaptive algorithms used in the first realizations were Gauss-Newton's or its simplifications (in order to reduce the computational complexity due to the matrix inversion operation) [1], [3], including a modification of the gradient-search algorithm [2]. It was shown in [4] that most of these algorithms are having either complicated computations or slow convergence, depending even on the amplitude of the input signal (please refer to [4] for details). In [4] a very interesting modification of the least-squares algorithm, avoiding its computational complexity (due to the matrix inversion operation), was proposed. It is converging very fast and it is not using any step-size parameters.

The other important point in designing of second-order BS-section-based ALEs is the proper selection of the filter section structure. This structure must permit an independent tuning of the central frequency and the bandwidth (BW) of the BP/BS transfer function and should employ a canonic number of multipliers (two in this case). The sections, advanced in [1], [2], [3] (mainly state-variable based) are employing 3 multipliers and some additional conditions must be met. The other popular structure, used in [4], [5], [6] is lattice based, but it is not convenient for low

frequency applications because of its higher sensitivity for poles near  $z=1$ , as shown in [7]. The same, in addition to the higher number of multipliers, is true for the direct-form realization, investigated in [6]. Another biquadratic section (called BQ3) has been advanced and investigated recently in [7], [8] and it was shown in [7] that it is by far better than the lattice based section (called BQ1 in [7]) for applications in the low frequency band.

The main aim of this contribution is to try to apply the modified least squares algorithm from [4] to the BQ3-section and to investigate the behavior of the ALE so obtained.

## 2. SECTION DESCRIPTION

BQ3 section, shown in Fig.1 is realizing at different outputs, besides all possible second-order transfer functions, independent BP and BS transfer functions [3], [4].

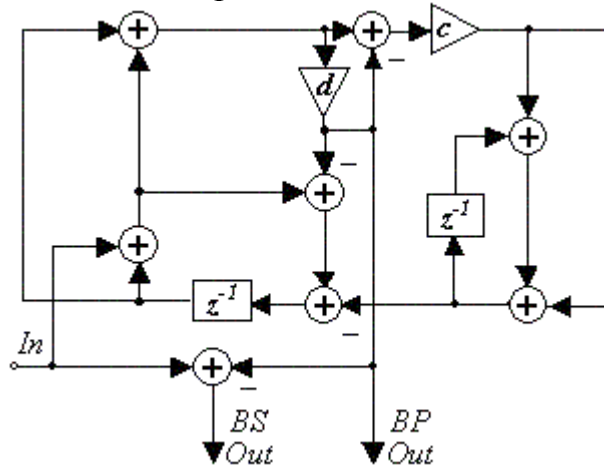


Fig.1 Biquadratic section BQ3 with its BP and BS outputs

The transfer functions at the BP and BS output of the structure are:

$$H_{BP}(z) = \frac{d(1 - z^{-2})}{1 + (-2 + 4c + 2d - 4cd)z^{-1} + (1 - 2d)z^{-2}}, \quad (1)$$

$$H_{BS}(z) = \frac{(1 - d)[1 - 2(1 - 2c)z^{-1} + z^{-2}]}{1 + (-2 + 4c + 2d - 4cd)z^{-1} + (1 - 2d)z^{-2}}. \quad (2)$$

It is possible, as seen from (1) and (2), to tune by  $d$  the bandwidth BW of those transfer functions independently from the central frequency[8]:

$$BW = \cos^{-1} \frac{1 - 2d}{1 - 2d + 2d^2}. \quad (3)$$

It was shown in [7] [8] that in the case of narrow band realizations it is possible to tune simultaneously the pole and zero frequencies  $\theta_p$  and  $\theta_z$  by changing only the multiplier  $c$ . It is described very simply in the BS case:

$$\theta_z = \cos^{-1}(1 - 2c). \quad (4)$$

An important advantage of BQ3 is the availability of BP and BS outputs within the same structure making it very attractive for ALE realizations.

### 3. GRADIENT GENERATION AND COEFFICIENT UPDATE

Let input signal consists of an unknown sinusoid and noise. The noise is white Gaussian with zero-mean and variance  $\sigma_n^2$ . The frequency of the input sinusoid is estimated by searching an optimum coefficient  $c$  which minimizes the adaptation error. If the function has minimum, the solution to get the optimum  $c$  is derived when the equation is equal to 0. Having a single coefficient for adaptation of the central frequency is providing a considerable reduction of the computational load – there is no need for matrix inversion in the process of adaptation. If  $e(i)$  is the error signal at the BS output of Fig.1 and  $x(i)$  is the input signal we can easily obtain from Eq. (2) the following difference equation:

$$e(i) = (1-d)x(i) + (-2+4c(i)+2d-4dc(i))x(i-1) + (1-d)x(i-2) - (-2+4c(i)+2d-4dc(i))e(i-1) - (1-2d)e(i-2). \quad (5)$$

In order to adapt the structure, first we have to generate the first-order derivative or gradient of the output error with respect to the filter coefficient  $c(i)$ , which is:

$$grad(i) = 4(1-d)[e(i-1) - x(i-1)] = \frac{\partial e(i)}{\partial c(i)}. \quad (6)$$

The total adaptation error of  $c(i)$  is denoted by  $J(c(i))$  and is given by the equation[4]:

$$J(c(i)) = \sum_{m=0}^i \lambda^m e^2(m), \quad (7)$$

where  $\lambda$  is the forgetting factor and must be set less than 1, but near to unity.

We can calculate the optimal value of  $c(i)$  from the equation obtained after equating the derivative of  $J(c(i))$  to zero:

$$\frac{\partial J(c(i))}{\partial c(i)} = 2 \sum_{m=0}^i \lambda^m e(m) \frac{\partial e(m)}{\partial c(m)} = 0. \quad (8)$$

By multiplication of equation (5) and (6) we estimate the frequency of the input sinusoid. The equation to be solved for getting the optimized  $c(i)$  is:

$$c(i+1) = \frac{\sum_{m=0}^i \lambda^m grad(m) [(1-d)x(m) - 2(1-d)x(m-1) + (1-d)x(m-2) + 2(1-d)e(m-1) - (1-2d)e(m-2)]}{4 \sum_{k=0}^i \lambda^m grad(m)(1-d)[e(m-1) - x(m-1)]} \quad (9)$$

### 4. SIMULATIONS

Simulation results were made when the signal-to-noise ratio was 20dB or 10dB. For SNR=20dB the amplitude of the input sinusoid is unity and the noise variance  $\sigma_n^2=0.1$ . The forgetting factor is set to  $\lambda=0.9999$  and coefficient  $d$  is changed in order to investigate the adaptation process for different values. Such a forgetting factor has

been chosen in order to reach good convergence and stability of the adaptation process. Experiments were done with  $f_s=8$  kHz. All the theoretical results were verified experimentally and a very good adaptation was observed for different SNR values.

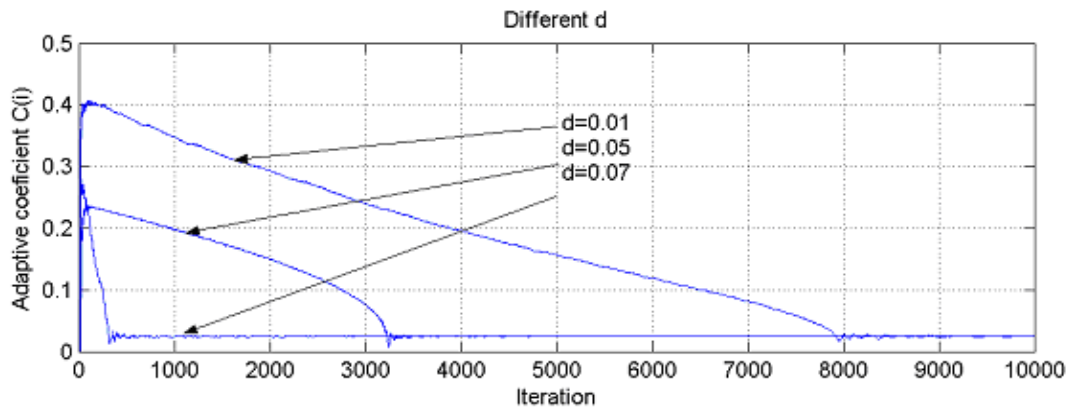


Fig. 2. Adaptation with different bandwidths to sinusoid with frequency 200Hz and SNR=20dB

On Figure 2 are curves of coefficient adaptation obtained for different bandwidths. As the BW increases, the speed of the adaptation process grows. When  $d=0.07$ , which is  $BW=0.15$ (formula(3)) the adaptation is accomplished almost immediately, whereas for  $d=0.01$ ( $BW=0.024$ ), adaptation takes two thirds of the analyzed period.

Figure 3 is governed by SNR=10dB and different frequencies:

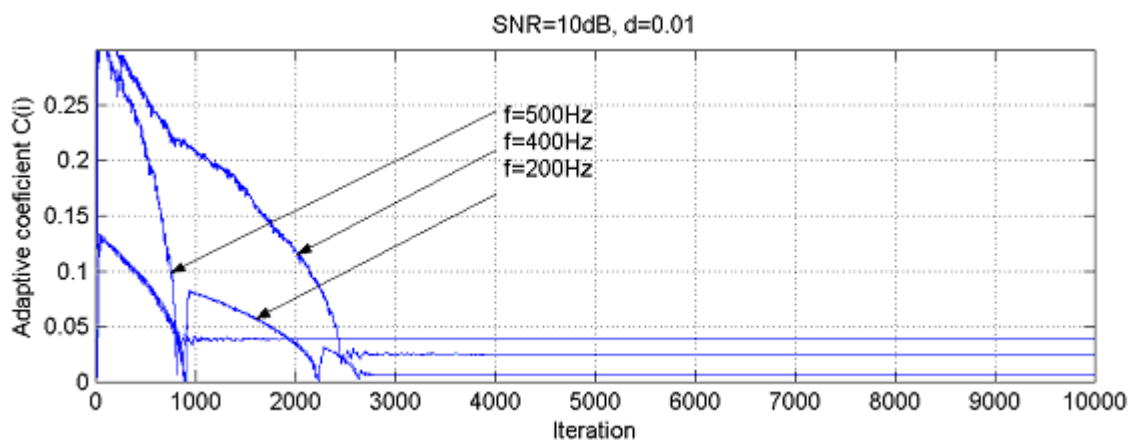


Fig.3. Investigation of tracking of different frequencies for SNR=10dB ( $f_s=8$ kHz)

According to this figure we can make the conclusion that in even such small bandwidth  $d=0.01$  ( $BW=0.024$ ) coefficient  $c(i)$  adapts considerably fast.

On plot 4 are curves for SNR=20dB and bandwidth set to  $d=0.01$ .

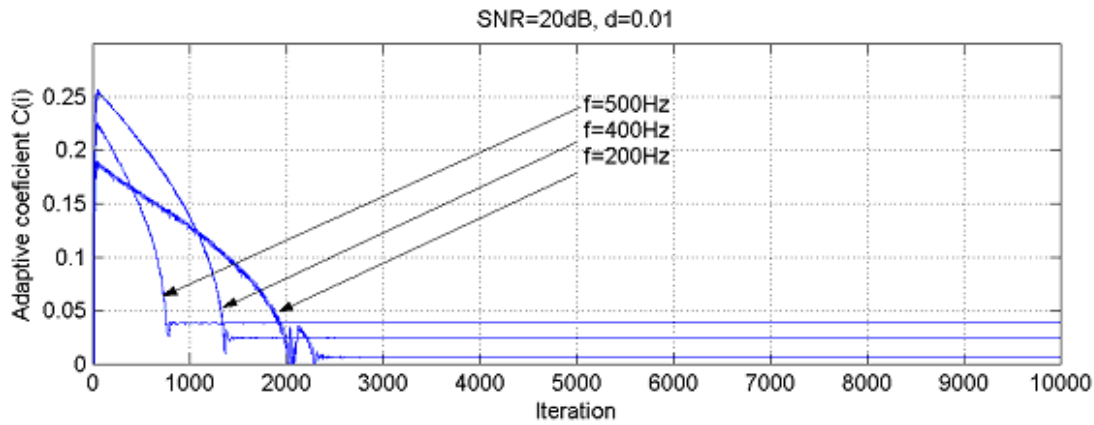
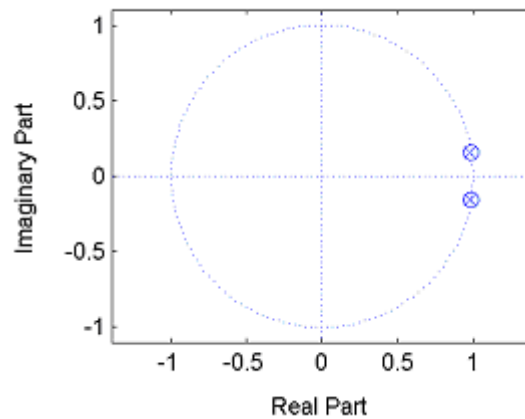


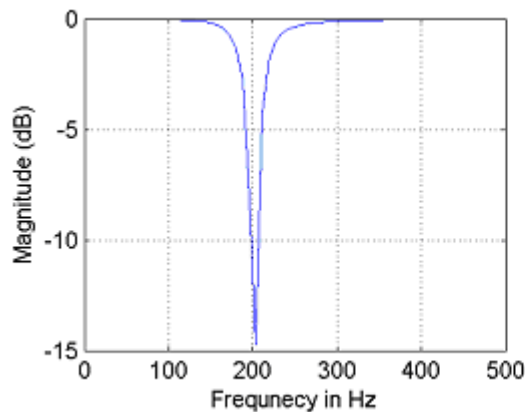
Fig.4. Adaptation to different frequencies for SNR=20dB and  $d=0.01$ , ( $f_s=8\text{kHz}$ )

It is observed from figure 4 that for those frequencies the plot is similar to previous one and the speed of adaptation process increases. Here the process is finished by the end of 2500th iteration for the investigated frequency range.

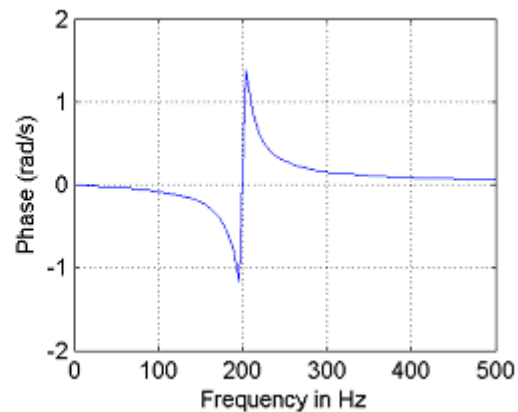
In order to verify the obtained value of  $c=0.0062$  after the adaptation was finished for frequency of 200Hz, bandwidth  $d=0.01$  and SNR=20dB was plotted pole-zero diagram figure 5(a), magnitude and phase response figure 5(b), 5(c).



(a)



(b)



(c)

Fig.5. Pole-zero diagram (a), magnitude (b) and phase response (c) of the BS transfer function after adaptation to 200Hz.

As we said in introduction this section is suitable for realization of adaptive filters with poles near  $z=1$ . On the Fig.5 this is observed clearly. The input frequency of 200Hz is recovered using the obtained coefficient  $c=0.0062$ .

## 5. CONCLUSIONS

In this paper modification of Least-Squares algorithm, applied to adapt BQ3 section is used. The same approach was employed for theoretical investigations of biquadratic filter section for detection or suppression of a single sinusoidal signal and graphical results showing that this structure is suitable for adaptive filtering realization have been obtained. As the BQ3 section was chosen to be turned to adaptive, because of its low sensitivity in the low frequency band all investigations have been made in this band.

The structure has shown good convergence properties and performance for different signal-to-noise ratios. By the end of 2500th iteration (in the worst case – Figure 3) for investigated frequencies and SNRs, the adaptation process was completed. The curves of investigated coefficient  $c(i)$  have shown low fluctuation, while having enough higher adaptation speed as seen in all figures.

Besides the Least-Squares, investigations based on other algorithms have been performed, but there are additional problems remaining to be solved.

## 6. REFERENCES

- [1]. Kwan, T. and K. Martin: *Adaptive detection and enhancement of multiple sinusoids using a cascade of IIR filters*, IEEE Trans. Circuits Syst., Vol. CAS-36, No. 7, pp. 937-947, July 1989.
- [2]. Soderstrand, M. et al., *Suppression of multiple narrow-band interference using real-time adaptive notch filters*, IEEE Trans. Circuits Syst.-II, Vol. 44, No. 3, pp. 217-225, March 1997.
- [3]. Martin, K and M. T. Sun, *Adaptive filters suitable for real-time spectral analysis*, IEEE Trans. Circuits Syst., Vol. CAS-33, No. 2, pp. 218-229, Feb. 1986.
- [4]. Matsuura, K., E. Watanabe and A. Nishihara, *Adaptive line enhancers on the basis of least-squares algorithm for a single sinusoid detection*, The Institute of Electronics, Information Communication Engineers (IEICE) Trans. Fundamentals, Vol. E82-A, No. 8, pp. 1536-1543, Aug. 1999.
- [5]. Mvuma, A., S. Nishimura and T. Hinamoto, *Adaptive narrow-band interference suppression in DSSS communication systems using IIR notch filter*, IEICE Trans. Fundamentals, Vol. E84-A, No. 2, pp. 449-455, Feb. 2001.
- [6]. Nishimura, S. and A. Mvuma, *Steady-state performance of IIR adaptive notch filters – comparison between direct form and lattice structure implementations*, Proc. Asia Pacific Conference on Circuits And Systems'2000, Beijing, China, pp. TT24-2.1-TT24-2.4, May 2000.
- [7]. Stoyanov G., *A comparative study of variable biquadratic digital filter sections sensitivities and tuning accuracy*, Proc. Int. Conf. "Telecom'2002", Varna, Sv. Konstantin, pp. 674-681, Oct. 10-12, 2002.
- [8]. G. Stoyanov, G. and M. Kawamata, *Variable biquadratic digital filter section with simultaneous tuning of the pole and zero frequencies by a single parameter*, Proc. Interantional Symposium on Circuits And Systems'2003, Bangkok, Thailand, vol. 3, pp. III-566–III-569, May 2003.