OPTIMUM NUMBER OF CHECKS IN DIAGNOSTICS ELEMENTS OF FIXED SYSTEM

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We consider the checking problem under the case that if system failure is detected by checking, then the problem ends. By the methods of calculus of variations, we find the optimum number of checks which minimizes the expected loss up to detection of the first failure subject to the condition that the expected cost of checking is restricted. Applying the obtained results to the Gamma distribution with a shape parameter 2, we show the curve of the optimum number of checks graphically.

INTRODUCTION

Many different checking problems of a one-unit system have been treated in the literature. Especially, Barlow and Proschan [1] and Tassev [3] proposed a typical inspection policy and discussed the optimum checking procedures. In such a checking problem as they formulated, it is supposed that:

1. system failure is detected only by checking with probably one,
2. checking does not degrade the system,
3. the system cannot fail while being checked and
4. each checked incurs a constant cost and the time elapsed between system failure and its discovery at the next checking results in a constant cost per unit of time.

However, the results derived by them become complicated when there are many checks. To treat this case Keller [2] discussed the optimum checking schedules supposing that checking is so frequent that it can be described by a continuous density $n(t)$ of checks per unit of time. By the methods of calculus of variations, he found the optimum $n(t)$ minimizing the expected cost of checking and the expected loss up to detection of the first failure.

In this paper we assume that each checking is made instantaneously and if system failure is detected by checking, then the problem ends: There no replacements. That is, we consider the model, which has the same assumptions as those of inspection policy in Zelen [3] and Tassev [5]. According to Keller's discussion we find the optimum number of checks which minimizes the expected loss up to detection of the first failure subject to the condition that the expected cost of checking is restricted. It follows that our model is more practical than of Keller.

FORMULATIONS AND ANALYSIS

Introduce a smooth density $n(t)$, which denotes the number of checks per unit of time. Let $F(t)$ be the failure time distribution of the system (or the unit). Then, $dF(t)/dt = f(t)$ is the probability of failure per unit of time. The time between two successive checkings in $1/n(t)$ and the total number of checks up to time t is $\int_0^t n(s)ds$.

If we suppose that a cost C1 is suffered for each checking and the loss due to elapsed time between failure and its detection incurs a constant cost C2 per unit of time, then the total expected cost of checking is

$$K(n(t)) = \int_0^x C_1 \int_0^t n(s)ds\ f(t)dt, \tag{1}$$

and the expected loss up to detection of the first failure is

$$J(n(t)) = \{2/n(t)\} f(t)dt\ . \tag{2}$$

In particular, we assume that there is a constraint on the allowable investment of the total expected cost of checking. That has the specific form

$$K(n(t)) = \int_0^x C_1 \int_0^t n(s)ds\ f(t)dt = A, \tag{3}$$

where A is a constant. From the above discussion, we seek the function $n(t)$, which minimizes (2) subject to (3):

$$J(n(t)) = \int_0^x C_2/n(t)[f(t)dt] = \min$$

subject to

$$K(n(t)) = \int_0^x C_1 \int_0^t n(s)ds\ f(t)dt = A.$$

Using the Lagrange multiplier γ, the Euler – Lagrange differential equation corresponding to this conditional problem of variation can be written easily by treating $\int_0^t n(s)ds$ as the unknown function and $n(t)$ as its derivative. Then, the Euler – Lagrange differential equation becomes

$$d/dt\{[C_2/n(t)^2]f(t)\}. \tag{4}$$

Integrating (4) with respect to t, we obtain

$$1/n(t)^2 = \{C_2/\gamma[a - F(t)]\}/C_2 f(t), \tag{5}$$

where a is integration constant. The result for $n(t)$, for which (5) can be solved in terms of the right-hand side, depends on the constant a. It follows from (5) that if $\gamma \le 0$ then $a \le 0$ since $C_1 > 0, C_2 > 0$, $f(t) > 0$, and $F(\infty) = 1$. (5) yields

$$n(t) = \{C_2 f(t)/\gamma[a - F(t)]\}^{1/2}. \tag{6}$$

Inserting the solution $n(t)$ of (6) into (2) in order to determine a, we obtained

$$J(n(t)) = \int_0^x (C_1 C_2 f(t))^{1/2} \{\gamma[a - f(t)]\}^{1/2} dt. \tag{7}$$

From (7), if $\gamma \ge 0$ then $J(n(t))$ is minimized when $a = 1$ and if $\gamma \le 0$ then $J(n(t))$ is minimized when $a = 0$. We can determine γ for specified a from (3) and (6). Since $n(t)$ obtained for $a = 0$ cannot satisfy the assumption $\int_0^\infty n(s)ds$ is sufficiently great value, $n(t)$ obtained for $a=1$ is optimal for the failure rate function $r(t)$ corresponding to $F(t)$ is IFR (Increasing Failure Rate), where

$$r(t)=f(t)/[1-F(t)]$$

(see, Barlow and Proschan [1,pp.22-35]). Thus, as the optimum $n(t)$ we obtain

$$n(t)=(A/C_1)[r(t)]^{1/2}\Big/\left\{\int_0^\infty\int_0^t\{r(s)\}^{1/2}ds\ f(t)dt\right\}. \qquad (8)$$

Actually, we verify that $n(t)$ satisfying (8) minimizes $J(n(t))$. Let $n(t)+n_0(t)$ be any admissible function in the extremum problem, which is not equal to $n(t)$. From (3), we have

$$\int_0^\infty\int_0^t n_0(s)ds\ f(t)dt=0. \qquad (9)$$

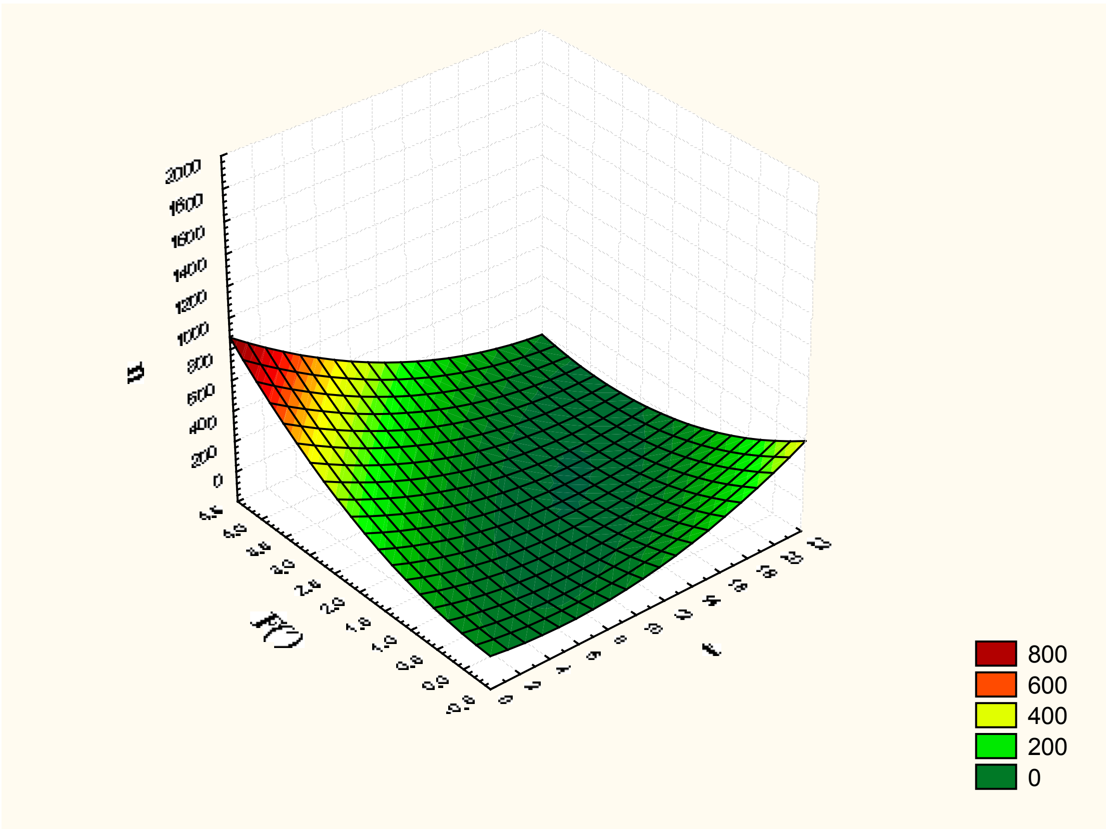

Fig.1 Variation of the smooth density ***n*** of the number of checks in the connection of time ***t*** and the function of distribution of time for system refusing ***F(t)***

Then

$$J(n(t)+n_0(t))-J(n(t))=C_2\int_0^\infty\{1/[1+n_0(t)/n(t)]-1\}[f(t)/n(t)]dt>$$

$$>(-C_2)\int_0^\infty n_0(t)/n(t)^2 f(t)dt=$$

$$=[(-C_2)/p^2]\int_0^\infty n_0(t)[1-F(t)]dt=[(-C_2)/p^2]\int_0^\infty\int_0^t n_0(s)ds\ f(t)dt=0$$

where

$$p = (A/C_1) \Big/ \int_0^\infty \int_0^t \{r(s)\}^{1/2} ds\ f(t) dt .$$

In the above inequality, we used the relation that

$$1 \Big/ (1+x) \begin{cases} = 1 & (for\ x = 0) \\ > 1 - x & (for\ x > 0) \end{cases} .$$

Hence n(t) satisfying (8) minimizes $J(n(t))$ and is the functional that we seek to find. Finally we obtain the following theorem on the optimum number of checks:

Theorem. If r(t) is IFR, then there exists the optimum number n(t) of checks which minimizes the expected loss $J(n(t))$ subject to the expected cost $K(n(t)) = A$, and which is given by

$$n(t) = \langle A/C_1 \rangle \{r(t)\}^{1/2} / \int_0^\infty \int_0^t \{r(t)\}^{1/2} ds\ f(t) dt .$$

Example

Let us apply the obtained results to the Gamma distribution with a shape parameter 2, $F(t) = 1 - (1 + \lambda t)\exp(-\lambda t), \lambda > 0$.

From (8), since

$r(t) = \lambda^2 t / (1 + \lambda t)$,

we obtain the density of checks

$$n(k) = k[t/(1+\lambda t)]^{1/2}, \qquad (10)$$

where k is a positive constant and

$$k = (A/C_1)(-1/2)\lambda^{-3/2} \Big/ \int_0^\infty \log\left((1+\lambda t)^{1/2} - (\lambda t)^{1/2}\right) t \exp(-\lambda t) dt . \qquad (11)$$

Since $n(0) = 0$ and $n(\infty) = k/\alpha^{1/2}$ and for any $t > 0$, $dn(t)/dt > 0$ and $d^2 n(t)/dt^2 < 0$.

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