TUNING ACCURACY INVESTIGATION OF VARIABLE IIR DIGITAL FILTERS REALIZED AS A CASCADE OF IDENTICAL SUB-FILTERS

Georgi Stoyanov¹, Ivan Uzunov² and Masayuki Kawamata³

¹Dept. of Telecommunications, Technical University of Sofia, Kl. Ohridski Str 8, 1000 Sofia, phone:+359 2 965 3255, e-mail: stoyanov@ieee.org, fax:68 60 89 ²Institute of Communication Engineering, Tampere University of Technology, P.O. Box 553, FIN-33101, Tampere, Finland, e-mail: uzunoviv@cs.tut.fi

³Graduate School of Engineering, Tohoku University, Aramaki Aza Aoba 05, Sendai 980-8579, Japan, e-mail: kawamata@mk.ecei.tohoku.ac.jp

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Abstract – The accuracy and range of tuning, the sensitivities and the overall performance of variable IIR filters, designed as a cascade of several identical sub-filters, have been studied in this work and compared with these of the corresponding classically designed variable filters. It was firmly shown that, at the price of a slight increase of the filter order, the cascaded sub-filter realizations are achieving much better performance. All results are verified experimentally.

1. INTRODUCTION

Variable IIR filters are often used in telecommunication and electronic equipment and all the known methods of their design are critically studied in Ref. [1], [2]. The most popular design procedure is based on the spectral (allpass) transformations of Constantinides (TC) [1], [2], but when the prototypes are IIR filters, delay-free loops appear after the TC. Due to the attempts to eliminate these delay-free loops, no precise, without limitations, real-time tuning of IIR filters is known until now - all methods are approximate and valid only in a narrow range of values of the tuned parameter and over some limited frequency range. Most methods are based on truncated Taylor series expansions, applied on parallel-allpass-structure [3] (called MNR-method after the names of the authors Mitra, Neuvo and Roivainen). This method is considered as the best known, but we have shown in Ref. [4] that the magnitude characteristics are degrading even when the LP/HP (lowpass/ highpass) filter cutoff frequency or the bandpass and bandstop (BP/BS) bandwidth (BW) are tuned over a very limited frequency range. We have proposed a new approach [5], [6], based on a cascaded connection of several identical sub-filters. It permits an easy tuning of the cutoff frequency of the LP filter without having to use TC and truncated Taylor series expansions when using sub-filters of first or second order. We have developed [7], [8] several new tunable sub-filter structures (of first and second order) very suitable for narrow-band realizations. In Ref. [2], [9] we have advanced the idea to realize the sub-filters of higher than second order as parallel allpass structures but to take special measures to reduce their stop-band (SB) sensitivity.

The main aim of the present work is to investigate the tuning accuracy of the variable filters obtained according to our method (as cascades of identical sub-filters) and to compare their performance with that of the MNR realizations.

2. DESIGN OF VARIABLE FILTERS AS A CASCADE OF IDENTICAL SUB-FILTERS

The main concept in our approach to design variable digital filters (VDF) is to use several cascaded identical filter blocks, each of them providing a very simple tuning of a given frequency parameter by varying a single multiplier coefficient. As the subfilters are identical, only one multiplier coefficient value per given parameter for the entire filter has to be recalculated in the process of tuning.

The magnitude specifications of the desired LP filter are: pass-band (PB) from 0 to Ω_p , stop-band (SB) from Ω_{SB} to $\Omega_s/2$, maximal PB attenuation A_p , dB and minimal SB attenuation A_s , dB. And we have to find a total transfer function (TF) H(z) represented as a product of N equal individual TFs $H_i(z)$, each of them of order n:

$$H(z) = H_i^N(z), \qquad H_i(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}.$$
 (1)

These TFs might be of Butterworth, Chebyshev or elliptic type and in the process of design we have to determine the minimal number N of the individual TFs $H_i(z)$ necessary to meet the specifications with given (selected) type (maximally flat, equiripple or other) and order n. A step-by-step design procedure for this is given in [5], [6]. An approximation using N equal terms is far from optimal and there are many limitations. It might be even impossible to meet some difficult filter specifications no matter how high the number N is taken. These limitations are investigated in [5], [6], [9].

We have developed [7], [8] several excellent variable first- (called LS1) (Fig. 1a) and second- order (called BQ3) (Fig. 1b) sections for cascade realization with identical first- (IFOS) and second-order (ISOS) sections with independent tuning without using any Taylor series. The elliptic and the LP TFs of BQ3 are:

$$H_E(z) = g_{0_E} \frac{1 + 2\frac{c(\gamma+1) - 1}{c(\gamma-1) + 1}z^{-1} + z^{-2}}{1 - 2(1 - d)(1 - 2c)z^{-1} + (1 - 2d)z^{-2}}, \quad g_{0_E} = (1 - d)[1 + c(\gamma - 1)]; \quad (2)$$

It appeared, however, to be impossible to synthesize such structures with higher than second order. No such structures have also been found in the literature. In order to solve the problem, we had to accept the MNR approach (Fig. 2) [3] but only for the realization of the sub-filters. In Fig. 2b it is shown how a coefficient a_1 is turned variable $(a_{1var} \approx a_1 + \alpha K_i)$ by adding a parallel branch, containing a variable coefficient α and an additional coefficient K_i properly calculated by using Taylor series expansions [3]. It provides an easy tuning of the cutoff frequency of the LP individual subfilters by varying a single multiplier coefficient. But as our structure is a cascade of several low-order order sections (even though obtained as parallel-allpass-structures), it has much lower SB sensitivity, compared to that of the totally parallel allpass structure, which is behaving really badly, as shown in [4]. And instead of using the most popular in the literature first- (called MH) (Fig. 3a) and second-order (called MH2B) (Fig. 4a) allpass sections, we have developed some very-low-sensitivity (for poles



Fig. 1. Very-low-sensitivity (for poles near *z*=1) first- (LS1)(a) and second-order (BQ3) (b) filter sections suitable for cascaded IFOS and ISOS variable filters



Fig. 2. Parallel-allpass-structure-based realization (a) and variable coefficient realization (b)



Fig. 3. First-order all-pass sections: (a) MH section, (b) ST section



near z=1) first- (called ST) (Fig. 3b) and second-order (called LS)(Fig. 4b) allpass sections, realizing the following TFs:

$$H(z)_{ST} = \frac{z^{-1} - (1 - \alpha_{01})}{1 - (1 - \alpha_{01})z^{-1}}, \quad H(z)_{LS} = \frac{1 - c_2 - (2 - 2c_1 - c_2)z^{-1} + z^{-2}}{1 - (2 - 2c_1 - c_2)z^{-1} + (1 - c_2)z^{-2}}.$$
 (3)

3. SENSITIVITY INVESTIGATIONS

If Θ_A and Θ_B are the phase responses of the two branches in Fig. 2a, we can derive expressions for the worst-case sensitivity of the magnitudes with respect to the changes in all multiplier coefficients m_i and m_j in the upper and lower branches:

$$WS_{allm}^{|H(j\omega)|} = \frac{1}{2} |\tan\frac{\Delta\Theta}{2}| [|\Theta_A|\sum_i S_{m_i}^{\Theta}A| - |\Theta_B|\sum_j S_{m_j}^{\Theta}B|]$$
(4)

$$WS_{allm}^{|G(j\omega)|} = \frac{1}{2} |\cot\frac{\Delta\Theta}{2}| [|\Theta_B|\sum_j |S_{m_j}^{\Theta}B| - |\Theta_A|\sum_j |S_{m_i}^{\Theta}A|], \qquad (5)$$

where $\Delta \Theta = \Theta_B - \Theta_A$. These sensitivities will be very high in the SB, where high SB attenuation is obtained with $\Delta \Theta \approx \pi$ or $k\pi$ for $H(\omega)$ and $\Delta \Theta \approx 0$ or $2k\pi$ for $G(\omega)$. The only way to decrease (4), (5) and thus to improve the accuracy of tuning and of the realization is to decrease the phase sensitivities S_m^{Θ} of the allpass circuits. It is even better clear for the specific points of the magnitude, like the cutoff frequency and the frequencies of the minimal attenuation (for elliptic approximations) in the SB:

$$WS_{m}^{|H(\omega_{c})|} = \frac{1}{2} \left[\sum_{k} \sum_{i} \left| \Theta_{A_{k}}(\omega_{c}) S_{m_{i}}^{\Theta_{A_{k}}(\omega_{c})} \right| + \sum_{l} \sum_{j} \left| \Theta_{B_{l}}(\omega_{c}) S_{m_{j}}^{\Theta_{B_{l}}(\omega_{c})} \right| \right]$$
(6)

$$WS_{m}^{|H(\omega_{\min})|} = M \left[\sum_{k} \sum_{i} \left| \Theta_{A_{k}}(\omega_{\min}) S_{m_{i}}^{\Theta_{A_{k}}(\omega_{\min})} \right| + \sum_{l} \sum_{j} \left| \Theta_{B_{l}}(\omega_{\min}) S_{m_{j}}^{\Theta_{B_{l}}(\omega_{\min})} \right| \right], \quad (7)$$

where Θ_{A_k} , Θ_{B_l} are the phase responses of the allpass sections in the two branches of Fig. 2a and *M* depends on A_s in the specifications (*M*= 16 for A_s =30dB, 50 for 40 dB and 158 for 50 dB). The proper selection of these sections is critically important.

We have investigated (Fig. 5a) the worst-case sensitivities of two MNR-realizations of same specifications ($F_p=0.01$, $F_{SB}=0.03$, $A_p=1$ dB, $A_{SB}=35$ dB) – one with MH, MH2B and one with our sections (ST and LS – Figs 3b, 4b). It is seen that the usage of proper sections is reducing the SB sensitivity more than 60 times.

Then we have realized a VDF with the same specifications, but using our new approach (cascaded ISOS in this case), employing our BQ3 section (Fig. 1b) and investigated the sensitivity. The results shown in Fig. 5b demonstrate a startling reduction of more than 300 times, which makes our approach the best possible.

It is clear that really low SB sensitivities, i.e. very high tuning accuracy, are possible only if IFOS or ISOS realizations are employed. When high filter selectivity (impossible to meet with IFOS or ISOS difficult specifications) is required, we have to use third- or fifth-order sub-filters (it is impossible to realize even-order LP subfilters with real coefficients and sub-filters of order higher that 5 are impractical, because we lose then all the merits of the cascade realization), designed according to MNR-approach, but with our very low sensitivity allpass sections of Fig. 3b, 4b.

It is very difficult (often impossible) to derive general formulae or to obtain numerical results about the sensitivities of higher order filters. This is why we have employed an indirect approach for a comparative study of the sensitivities of such filters. For quite difficult specifications ($F_p=0.01$, $F_{SB}=0.03$, $A_p=2dB$, $A_{SB}=55 dB$ and Butterworth approximation) we have obtained 7th order TF and realized it as MNR VDF using MH and MH2B (Figs. 3a, 4a) sections. Then a cascaded realization with third-order sub-filters was designed for the same specifications, which produced N=3or total order 9. The sub-filters were designed using our low-sensitivity allpass sections (Figs. 3b, 4b). Both filters have been tuned with factor $\alpha=0.033$ (Fig.2b) and



Fig. 5. WS sensitivities of third-order elliptic filter ($F_p=0.01$, $F_{SB}=0.03$, $A_p=1$ dB, $A_{SB}=35$ dB), realized with MH and with Low-sensitivity allpass sections (a); with two BQ3 sections (b)

then simulated with coefficients quantized to different word-length *B* (supposing "canonic sign-digit code"). The results are shown in Fig. 6. The MNR-filter characteristics (Fig. 6a) are destroyed even with B=7bit – the attenuation is changed from Butterworth type to something like elliptic and is getting some SB minimum of about 15 dB which is far below the limit of 55 dB. Our filter is behaving perfectly even with B=3 (Fig. 6b) and is changing slightly for B=2bit.



Fig. 6. Attenuation of a variable Butterworth filter (starting specifications $F_p=0.01$, $F_{SB}=0.03$, $A_p=2$ dB, $A_{SB}=55$ dB), realized as 7th order MNR structure (a) and as a cascade of 3 third-order identical sub-filters (b) for different coefficient word-length and $\alpha=0.033$

4. EXPERIMENTS

The range of tuning of the filters described in Fig. 6 has been studied through simulations and the results are shown in Fig. 7. The MNR filter is changing its type (from Butterworth to non-polynomial) for every $\alpha > |0.1|$, and for $\alpha > 0.11$ the SB specifications are already violated, while our filter is smoothly tuned from $\alpha = -0.3$ to $\alpha = 0.3$, covering thus very wide frequency range without any magnitude degradation.

5. CONCLUSIONS

The accuracy and range of tuning, the sensitivities and the overall performance of



Fig. 7. Attenuation of a variable Butterworth filter with starting specifications $F_p=0.01$, $F_{SB}=0.03$, $A_p=2$ dB, $A_{SB}=55$ dB, realized as 7th order MNR structure (a) and as a cascade of 3 third-order identical sub-filters (b)

variable IIR filters, designed as a cascade of several identical sub-filters, have been studied in this work and compared with these of the corresponding MNR-filters. It was firmly shown that at the price of a slight increase of the filter order our filters are achieving much better performance. These results are verified experimentally.

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