# ARITHMETIC COMPLEXITY ESTIMATION OF A PN CODES PROCESSING WAVELET ALGORITHM

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*Known properties of cyclic signals and his circular convolution processing has been interest in one alternative processing, that take place for some application recently – Discrete Wavelet Transform(DWT). There exist powerful signal classes, assumed such processing kind. They are applicable in area of DS - CDMA, Software Defined Radio (SDR) etc.*

*In this paper the computational complexity of one known wavelet algorithm for n – shift invariant signals class processing is estimated. Some estimation for MatLab realization of algorithm model is presented. Some disadvantages of this wavelet processing are analyzed.*

## 1. INTRODUCTION

Cyclic convolution array calculations are widely used in many DSP fields. Due to their practical importance, some new interpretations will be present.

**Def. 1: Cyclic convolution** is defined, [1,3] as a vector

$$\mathbf{Z}_C = \mathbf{CY} \ , \tag{1}$$

where $\mathbf{C}_{N,N}$ – right-circulant matrix with the first row generation code sequence(GCS), $\{h(i)\}, i = \overline{0, N-1}$. A *right-circulant* matrix is a matrix whose $k$ th row is equal to row $0$ shifted to the right circularly by $k$ columns,

$\mathbf{Y}$ – vector of input mixture S + N, i. e.

$$z(j) = \sum_{k=0}^{N-1} y(k) h(j-k), \ j = \overline{0, N-1}, \quad N = n^s . \tag{2}$$

**Def. 2: n - convolution (hypocyclic convolution)** is defined [1, 2, 3], as a vector,

$$\mathbf{Z}_N = \mathbf{C}_N \mathbf{Y} \, , \tag{3}$$

where $\mathbf{C}_N$ - is n – shift invariant matrix , i.e.

$$z(j) = \sum_{k=0}^{N-1} y(k)h[(j-k) \bmod n], \qquad (4)$$

where in generally $\mathbf{Z}_\mathrm{N}$ calculates faster than $\mathbf{Z}_\mathrm{C}$.The volume about $\mathbf{O}(N^2)$ arithmetic operation needed for direct $\mathbf{Z}_\mathrm{C}$ calculation make difficult decoding schemes realization for large numbers $N$.

**This work aim is** to propose one geometric interpretation of n – adic convolution (hypocyclic convolution). It is represent estimation of arithmetic operations quantity needed to fast all-decoding of normal systems of cyclic and n-shift invariant signals, and one MatLab model.

## 2. GEOMETRIC INTERPRETATION OF $\mathbf{C}_{cir}$ AND $\mathbf{C}_\Theta$

It is consider one row of $\mathbf{C}_{cir}$, where his elements corresponds $N$ equidistant points $|O, N|$.

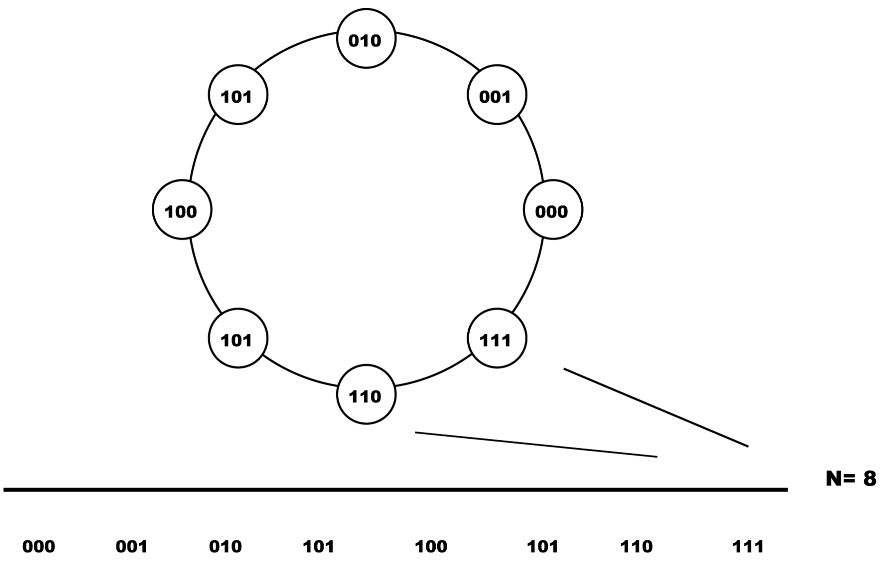


Fig. 1 Geometric interpretation of $\mathbf{C}_{cir}$.

The point indexes are $0,1,\ldots,N-1$. For interpretation of a cyclic shift let convolve $|O, N|$ in circle, as it shown in Fig. 1. The cyclic shift will correspond to indexes rotation(right or left rotate for given case of $\mathbf{C}_{cir}$), to k steps.

Generalizing the geometric interpretation for simple cyclic shifting may be achieved to geometric interpretation of $n$ - shift ($\mathbf{C}_\Theta$) case. Here the circles magnitude, which one are gained when the segments (arrays) convolve, will be varying for each stage (for each $\bmod\ n$).

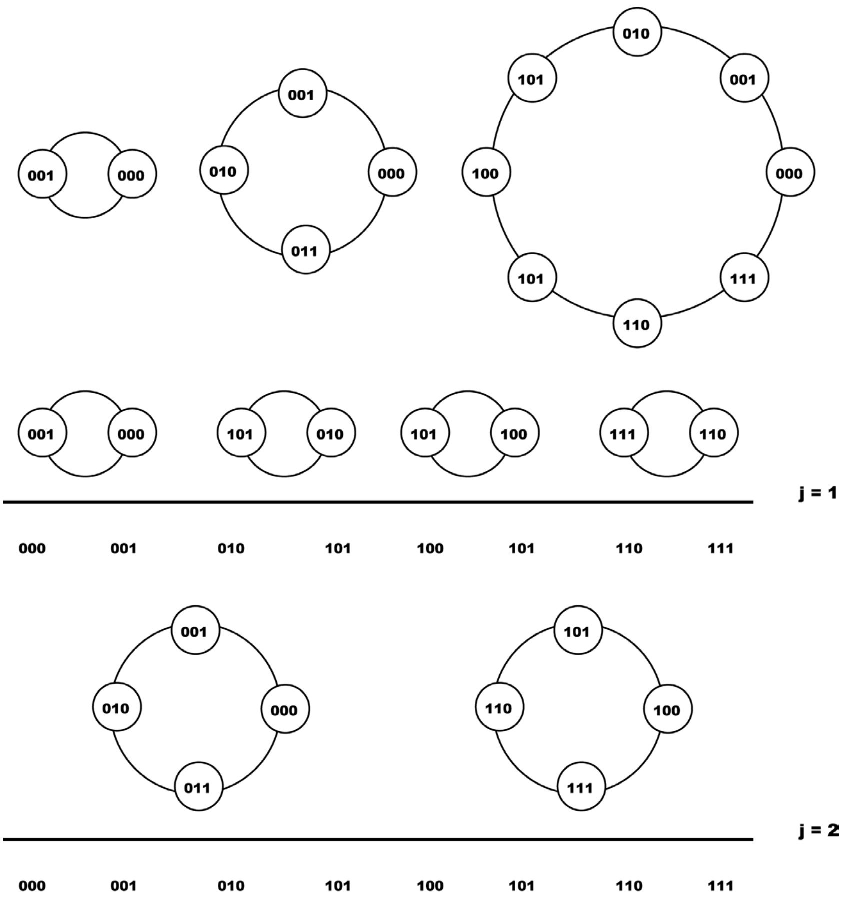


Fig. 2 Geometric interpretation of $\mathbf{C}_{\Theta}$.

In the case $N = 2^S$, $\mathbf{C}_{\Theta}$ can be seen, analogically. The point indexes are $0,1,\ldots,N-1$, too. For n –shift interpretation let $|O, N|$ is convolve as a set of circles with diameters growing in geometrical progression from $2/\pi$ to $N/\pi$ - fig. 2. (in accordance of concrete form of $\mathbf{C}_{\Theta}$).

## 3. CONDITIONS FOR EQUIVALENCE OF CYCLIC AND N-ADIC CONVOLUTIONS

So, consider an orthogonal cyclic signals class, for which one calculation of $\mathbf{Z}_{\mathrm{C}}$ and $\mathbf{Z}_{\mathrm{N}}$ are equivalent. It is known the following theorem **T**, [2]:

**T**: The circulant and **n** - ade matrix(hypocyclic) matrix order of **N**, are equivalent , i.e. **C** = $\mathbf{C}_{\mathbf{n}}$ if and only if the elements of GCS($\mathbf{C}_{0,N}$) satisfy the follow conditions:

$$h\left(in^{k-1}\right) = h\left(in^{k-1} + pn^{k}\right),\ k = \overline{1, s-1},\ i = \overline{1, n-1},\ p = \overline{1, n^{s-k} - 1}$$

$$h\left(in^{s-1}\right),\ i=\overline{0,n-1}\text{-independent elements of GCS.} \qquad (5)$$

*Geometric interpretation* of the theorem **T**:

Since cyclic shift interpreted already was shown, that $|O, N|$ must convolve to circle as it was displayed in Fig. 1. and Fig. 2. It can be shown that points with indexes defined from system (5) hits to the vertexes(intersection points) of *hypocycloids* built inside to the main circle. The ratio of the circumference radiuses is in geometric progression, $n=2$, i.e. $r=\{N/2, N/2^2, ..., 2\}$.

## 4. ARITHMETIC OPERATIONS QUANTITY

Can be noted that the $n$ – convolution algorithm, known from [4] is a discrete wavelet processing that is an attractive tool in signal processing area [3, 5,7].

Estimations for computing of $n$ – convolution vector $\theta_{coa}$ ( number of complex additions) and $\theta_{com}$ (number of complex multiplies) are presented in [4]:

$$\theta_{coa}=2(n-1)\sum_{k=1}^{s-1} n^k-n+N+N(n-1)=(n+2)n^s-3n_{coa} \quad , \qquad (6)$$

$$\theta_{com}=nN+(n-1)\sum_{k=1}^{s-1} n^k=(n+1)n^s-n_{com} \quad . \qquad (7)$$

Estimations for $\theta_{coa}$ and $\theta_{com}$ when calculated $Z_c$ with algorithm FFT, discrete exponential function (FFT-DEF) bases and $N=2^s$ and $N=3^s$, [4 ]:

$$\theta_{coa}\left[2^s\right]_{\text{FFT-DEF}}=s2^{s+1}{}_{coa};\theta_{coa}\left[3^s\right]_{\text{FFT-DEF}}=3^s\left(2s+\log_3 4\right)_{coa} \quad , \qquad (8)$$

$$\theta_{com}\left[2^s\right]_{\text{FFT-DEF}}=(s-2)2^s+4_{com};\theta_{com}\left[3^s\right]_{\text{FFT-DEF}}=(4s-1)3^s+2_{com} \ . \qquad (9)$$

Comparing correspondingly (6) and (8) on $\theta_{coa}$, and (7) and (9) on $\theta_{com}$ can be conclude about algorithm's efficiency for fast direct processing.

## 5. MATLAB SIMULATIONS

Algorithm described in [4] is constructive. With the standard Simulink blocks (MatLab) is build one digital convolver schema that produce hypocyclic convolution for concrete values of $n$ and $N$ ($n=2, N=2^3$).

For concrete convolver it was clear that, Fig. 3, $\theta_{coa}=26$, $\theta_{com}=22$.

As a disadvantage can be noted the decreasing of volume of class of signals, that will be suited for this convolver type. It is naturally follows from the theorem **T** restrictions (5).

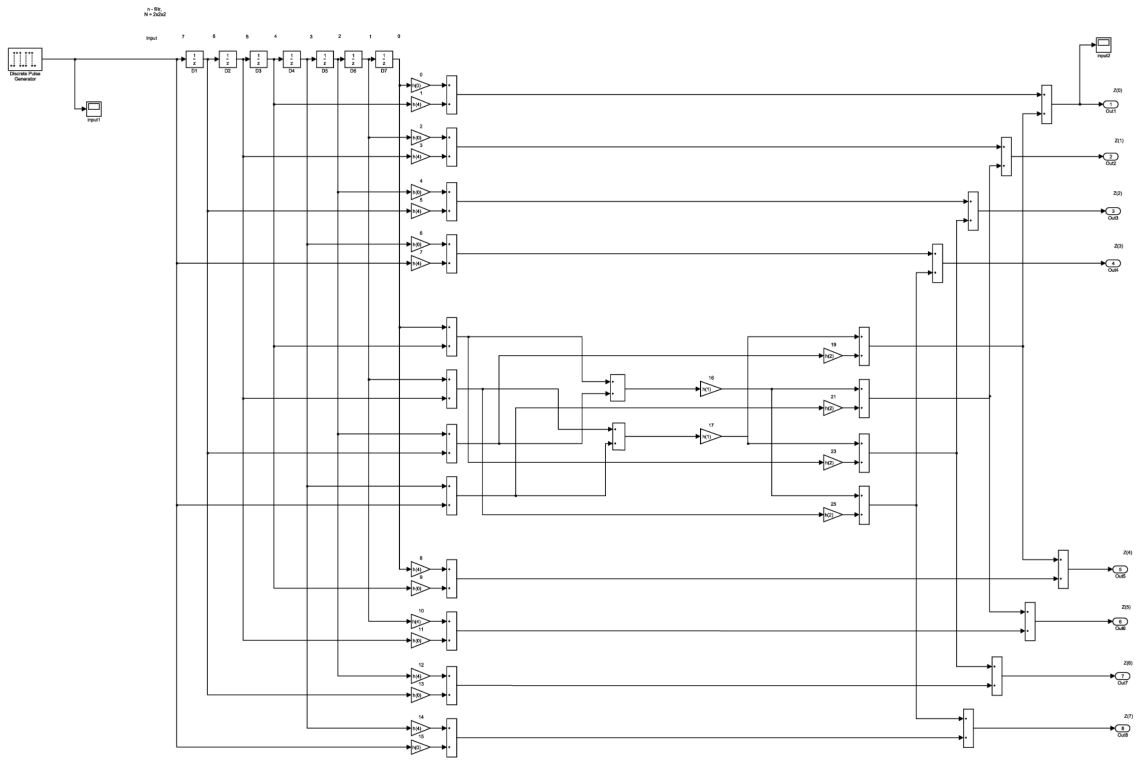

Fig. 3. Digital convolver model performed hypocyclic convolution, $n = 2, N = 2^3$.

## 6. CONCLUDING REMARKS

In the result in this paper was presented one geometric interpretation of n – convolution (hypocyclic convolution). The estimations of number of complex addition and multiplies, for fast all-decoding of normal systems of cyclic and n-shift invariant signals are presented. Practically realizable model of convolver in the Simulink, MatLab was demonstrated.

## 7. REFERENCES

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