ELECTRICAL TRANSMISSION LINE AS A MODEL FOR CATHETER-MANOMETER DYNAMIC RESPONSE

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Summary
For faithful representation of the blood pressure waveform, the most relevant property of the fluid-filled CM system isits dynamic response, which can be of a resonant nature. A model which is more closely related to physical reality, however, is the electrical transmission line (TL), the properties of which has been extensively studied in Electrical Engineering. In a simple analogue, the physical parameters of the TL can easily be translated into R, C, and L. The physical background of this response lies in the superposition of travelling waves *in the fluid, which reflect at both ends as well as at possible air bubbles, because the termination is far from characteristic. The computer program 'KatManW' to be presented allows to visualize and quantify the aspects of CM system dynamic behaviour as time-functions as well as in the frequency domain.*

Introduction

For faithful representation of the blood pressure waveform, the most relevant property of the fluid-filled CM system is its compliance (volume displacement per pressure change). Even small air bubbles within the system appear to give rise to serious distortion of the signal recorded. To ensure that the pressure is faithfully recorded, the system should have the same sensitivity at all frequencies within the relevant band and introduce a phase shift which is directly proportional to frequency. So, strict quality management is required in hospital practice. In literature, this behaviour is often described by an 'equivalent' second-order system. A model which is more closely related to physical reality, however, is the electrical transmission line (TL), the properties of which has been extensively studied in Electrical Engineering. In the simple analogue where pressure → voltage V and flow → electric current I, the physical parameters of the TL can easily be translated into an equivalent R, C, and L. According to the assumptions underlying their definitions in the hydraulic domain, also the frequency dependence of R and L has to be accounted for by the so-called Womersley-corrections from fluid dynamic theory. The transducer is modelled by a Ct terminating the line, allowing TL dynamic behaviour to be described in the *frequency domain* by the characteristic impedance Z, the propagation factor γ and the reflection coefficient ρ. The physical background of CM-response then appears to lie in the superposition of *travelling waves* in the fluid, which reflect at both ends as well as at the air bubbles, because the termination is far from characteristic, as described by TL theory. Thus, a more realistic and informative insight is given, compared to the second-order system approximation.

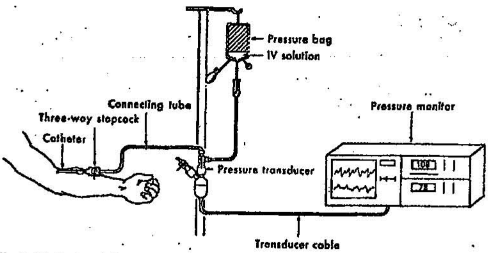

Fig 1 Catheter-Manometer System

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Electric Analogue

The catheter is considered as a *hydraulic transmission line*; fluid flow is necessary to detect pressure variations. Thereby, the inertance of the fluid column, the compliance of the catheter wall and the flow resistance are relevant. Some of these parameters are frequency-dependent. This hydraulic model can be adequately described with an *electric analogue*.

Electric	Hydraulic	Symbol	Energy type	Assumptions
Voltage	Pressure	P	Wave	Stationary
Current	Volume Flow	F	Wave	Incompressible
Charge	Fluid Volume	V		
Time	Time	t		
Resistance	Flow Resistance	$R = P/F$	Dissipation: $E_d = RF^2$	Laminar flow
Inductance	Inertance	$L = P(dF/dt)^{-1}$	Kinetic: $E_k = ½LF^2$	Plug flow
Capacitance	Compliance	$C = dV/dP$	Elastic: $E_p = ½CP^2$	Linearity

The *compliance* may augment strongly in the presence of *air bubbles*; the transducer membrane has a compliance C_T proportional to 1/E, with E Young's modulus of its material. The *flow resistance* is proportional to the *dynamic viscosity* η of the fluid (Poiseuille flow).

In the analogue, the electric transmission line is the *equivalent description* of the hydraulic case, with corresponding parameters, which now are frequency-dependent because of the underlying fluid dynamics (Womersley corrections).

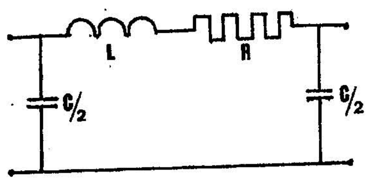


Fig 2 Section of thansmission line

Transmission Line

A transmission line is characterized by its

-propagation factor

$$\gamma(\omega) = \sqrt{[R(\omega) + j\omega\, L(\omega)]j\omega\, C}$$

and

-characteristic impedance

$$Z(\omega) = \sqrt{\frac{R(\omega) + j\omega\, L(\omega)}{j\omega\, C}}$$

constituting the *transfer function*:

$$\frac{P_o}{P_i} = \frac{1}{1 + j\omega\, C_T\, Z \sinh\gamma + \cosh\gamma}$$

Wave reflections

The frequency-dependent transfer of the line is a consequence of its *termination*. If the terminating impedances at its begin or end are non-characteristic, *reflected waves* will occur.
The reflected pressure-wave is given by:
with r the *reflection coefficient*.

$$P_r = P_o.r = P_o.\frac{Z_T - Z}{Z_T + Z}$$

The manometer impedance $Z_T = 1/j\omega C_T$ is imaginary. As both the numerator and the denominator of r comprise the factor $\sqrt{(Z_T^2 + Z^2)}$, $|r| = 1$ so that *total reflection* occurs here.
If the arriving pressure wave is denoted by P, then the measured pressure is:

$$P_o = P + P_r = \frac{2\,Z_T\,P}{Z_T + Z}$$

$$\frac{P_o}{2P} = \frac{Z_T}{Z_T + Z} = \frac{1/j\omega\,C_T}{Z + 1/j\omega\,C_T} = \frac{1}{1 + j\omega\,C_T Z} = \cos\phi$$

In practice $|Z_T| \approx 100|Z|$, so $P_0/P \approx 2$.
The phase shift caused by r, with $\phi = \arctan \omega C_T Z$, follows from:
The circulation system constitutes a voltage source, because of its large capacity, so total reflection with phase-reversal of the reflected wave will take place here (r = -1).
Thus, multiple reflections occur and the pressure-course is determined by the transmission factor γ and the reflection coefficient r alternately. With x the distance along the catheter:

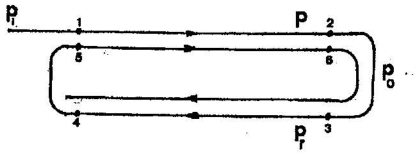


Fig 3 Wave Reflections

Path:	Factor:
1 - 2	$e^{-\gamma x}$
2 - 3	$e^{-2j\phi}$
3 - 4	$e^{-\gamma x}$
4 - 5	-1
5 - 6	$e^{-\gamma x}$, so:
2 - 6	$-e^{-2(\gamma x + j\phi)}$

etcetera.

If an *air bubble* is by mistake introduced during filling of the system, its effects on the wave transmission are considerable. It constitutes a discontinuity in compliance because of the compressibility of air. So, a third location of reflection occurs and no less than four traveling waves determine together the pressure measured, giving rise to severe signal distortion.

Second-order approximation
When the hyperbolic functions in the transfer function are expanded into the well-known power series and only the first-order terms in γ are kept, it follows:

$$\frac{P_o}{P_i} \approx \frac{1}{1 + j\omega\,C_T Z\gamma + \frac{1}{2}\gamma^2} = \frac{1}{1 + [R(\omega) + j\omega L(\omega)]\; j\omega\,(C_T + \frac{1}{2}C)}$$

This yields a 2nd order π-filter with the lumped-circuit diagram:

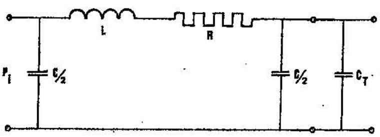


Fig. 4 Second-order π-filter

The manometer capacitance C_T appears in parallel with the ½C of the catheter.
Its transfer function is given by:

$$G(p) = \frac{1}{p^2 L C_p + pRC_p + 1} \qquad \text{with } C_p = C_T + ½C$$

A complex pair of poles occurs if $(RC_p)^2 - 4LC_p < 0$. From numerical values it follows that resonance is to be expected, especially in the frequency band where the pressure signal still has relevant components (in practice up to the tenth harmonic). The second-order system has a *natural frequency* f_n:

$$f_n = \frac{1}{2\pi\sqrt{LC}} = \frac{\frac{1}{T}}{\sqrt{1-\zeta^2}}$$

and a *damping factor* ζ:

$$\zeta = \frac{R}{4\pi}\sqrt{\frac{C}{L}} = \frac{1}{\sqrt{1+\left(\frac{\pi}{\ln\frac{A_2}{A_1}}\right)^2}}$$

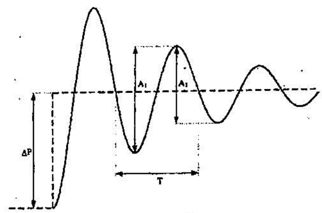


Fig 5 Step-response of a second-order system

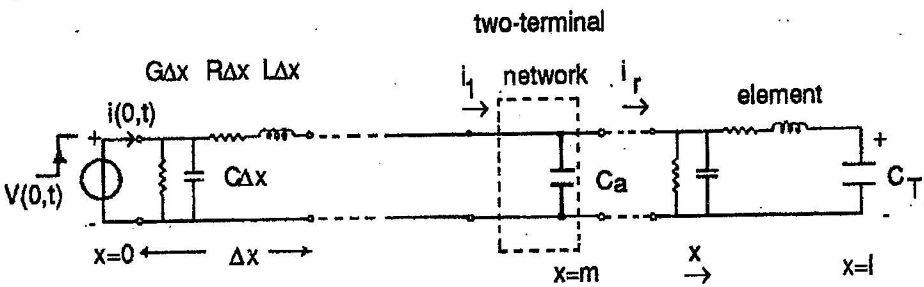
The values of A_1 and A_2 can be determined from a step-response measurement ("pop-test").

This approximation is widely used, but has limited relevance as it does not reflect the physical processes underlying the signal distortion.

Transmission Line Theory

Formulation of TL-behaviour in the complex frequency domain $j\omega$ with Z and γ yields:

$V(x,j\omega) = A \cdot e^{-\gamma x} + B \cdot e^{+\gamma x}$ and $I(x,j\omega) = \frac{1}{Z}(A \cdot e^{-\gamma x} + B \cdot e^{+\gamma x})$ where the term with e^{-x} describe wave fronts travelling in the forward direction and the one with e^{+x} refer to the backward direction. These equations are, however, not suitable for simulation of wave phenomena. Therefore, consider the *basic TL-model*:



Propagation of voltage waves V(x,t) along a TL is governed by the so-called "Telegraph Equation":

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial x^2} - \frac{R}{L}\frac{\partial V}{\partial t} \quad (G = 0)$$ of which the second term represents damping.

This can be decomposed into the set of coupled first-order partial differential equations:

$$C\frac{\partial V}{\partial t}+\frac{\partial I}{\partial t}=0 \text{ and } L\frac{\partial I}{\partial t}+\frac{\partial V}{\partial x}=-RI.$$

Solution requires *boundary conditions* at the transducer location x = l: $C_T\frac{\partial V(l,t)}{\partial t}=I(l,t)$

and for an air bubble at x = m: $C_a\frac{\partial V(m,t)}{\partial t}=I_{left}(m,t)-I_{right}(m,t)$.

These partial DV's can be solved by separation of variables, yielding forward and backward *travelling waves* along characteristic directions dx/dt for V and I with velocity $c=1/\sqrt{LC}$.

Simulation

The set of coupled DV's : $\frac{\partial}{\partial t}\begin{pmatrix}V\\I\end{pmatrix}=\begin{pmatrix}0 & -\frac{1}{C}\\ -\frac{1}{L} & 0\end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix}V\\I\end{pmatrix}+\begin{pmatrix}0 & 0\\ 0 & -\frac{R}{L}\end{pmatrix}\begin{pmatrix}V\\I\end{pmatrix}$ from above

can be transformed into: $\frac{\partial}{\partial t}\begin{pmatrix}\Phi\\\Psi\end{pmatrix}=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix}\Phi\\\Psi\end{pmatrix}+\begin{pmatrix}-\frac{R}{2L} & \frac{R}{2L}\\ \frac{R}{2L} & \frac{-R}{2L}\end{pmatrix}\begin{pmatrix}\Phi\\\Psi\end{pmatrix}$

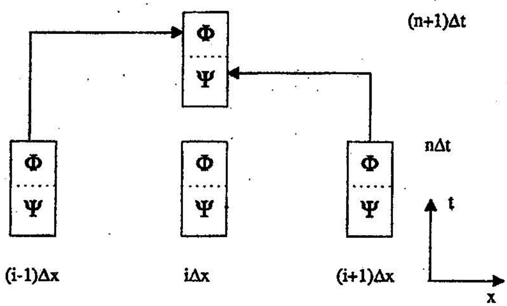
The λ's represent the *characteristic directions* $\lambda_{1,2}=\pm\frac{dx}{dt}=\pm\frac{1}{\sqrt{LC}}$.

The *Φ and Ψ* terms describe the *forward and backward travelling waves* along the characteristic directions. The equations have to be solved for all instances of time t and position x. Then, Φ and Ψ are coupled through: $\frac{d}{dt}\begin{pmatrix}\Phi\\\Psi\end{pmatrix}=\frac{R}{2L}\begin{pmatrix}-\Phi\\\Psi\end{pmatrix}+\frac{R}{2L}\begin{pmatrix}\Phi\\-\Psi\end{pmatrix}$

Pressure and flow waves are eventually found by:

$$V=\Phi+\Psi \text{ and } I=\sqrt{\frac{C}{L}}(\Phi-\Psi).$$

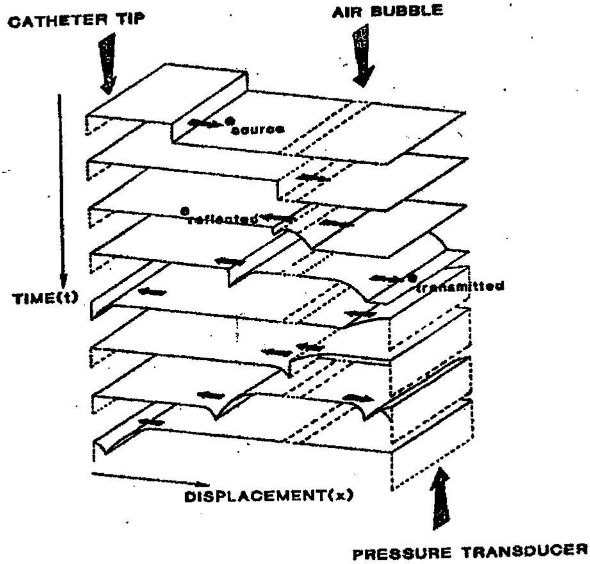
Thus, the *calculation scheme* becomes:



The time steps Δt are counted by n and depicted vertically; the space steps Δx are counted by i and shown horizontally. They are coupled by the wave velocity by: $\Delta t=\sqrt{LC}\Delta x$. At time nΔt the values of Φ and Ψ in each cell are known. To calculate cell (iΔx,(n+1)Δt), the values of Φ and Ψ in cells ((i-1)Δx, nΔt) and ((I+1)Δx, nΔt) are used. So, Φ at (n+1)Δt and iΔx is estimated from Φ and Ψ at the i-1[th] cell. Ψ is calculated backward in the same fashion from the i+1[th] cell. Two arrays have to be reserved in computer memory for the new values of Φ and Ψ, to be derived from their previous values.

After completing one computation, all recent Φ and Ψ values are kept in order to draw the waveform along the line. Since this calculation loop is repeated at fixed intervals of time, the reader is presented with a traveling wave on the screen.

Air-bubble effects



Discussion

Representation of CM-systems by the TL-model described above yields much more insight into the physical phenomena which cause signal distortion than the second-order simulation does. The application *"KatmanW"* allows the selection of input parameters for the TL, and transducer. Characteristic, open and short-circuited termination may be chosen. Step, impulse, sine and measured pressure curves are available as stimulation input. Air bubbles of various length and position can be introduced. Simulation results are offered as time responses, running waves and frequency characteristics. Screens and parameter lists may be printed for further reference.

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