APPLICATION OF PASSIVITY TECHNIQUE IN LINEAR STATE-FEEDBACK CONTROL OF A BUCK-BOOST CONVERTER

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Abstract: - The paper presents both theoretical and experimental results of the passivity technique application on a circuit-oriented model of a buck-boost converter controlled in a state space feedback. Nonlinear analysis based on system passivity technique and implementation of the control law, comprised of linear combination of the state variables of a buck-boost dc-dc converter, provides converter's large-signal stability. The global stability condition is obtained by studying the absorbed and dissipated power in the two-port large-signal model of the regulator. Large-signal equivalent circuit for the converter is derived from its bilinear description, while the stability condition is obtained by exploring the resulting two-port for the condition of the network passivity. The influence of the control signal saturation on the circuit stability is taken into account as well. We used the linear model of the converter to approve its stability and the root locus method to achieve proper values of the loop gain in order the desired transient response to be obtained. Simulations based on analytically derived relations are compared with experimentally obtained results. It is confirmed that both simulation and experimental results presented are in good accordance with theoretically derived predictions.

Keywords - power converter design; passivity technique; state-feedback.

I. INTRODUCTION

A Buck-Boost dc-dc converter is nonlinear and variable structure system as the most of the switched-mode dc-dc converters are. The control signal, a PWM representation of the control law, is applied to the switch of a power converter thus making its topological structure to vary with time. Linear modeling validity is restricted around the small signal steady state and is not suitable for the case of large input signal perturbances or for the large-scale variations of the load.

Various nonlinear techniques have been applied to achieve a large signal stability of switched mode DC-DC converters. In [1] a stability of a converter is analyzed by tracking the state space trajectories of a nonlinear system, while for its transient response an averaged model is used. In [2] a global stability is approved by Ljapunov based control design for a closed loop system. Modified averaging model applied on a small signal control circuit for a pulsatory loaded converter to estimate the controlled converter for the case of large signals is implemented in [3].

One of the methods that have been implemented to achieve stabilization of non-linear systems is passivity-based control. The state-space interpretation of passivity is developed by the concept of dissipativity [4]. The close connection between passivity and stability is given in [5] and [6]. The use of passivity for stabilization of non-linear systems can be found in [7], while its application for stabilization of dc-dc power converters is presented in [8], [9], [10] and [11].

In [12] a method of large-signal stability analysis for the buck-boost converter topology is presented that is based on the similar analysis for the boost converter given in [13]. The global stability condition is obtained by studying the absorbed and dissipated power in the two-port large-signal model of the power converter. In this paper we applied a passivity technique to achieve large signal stability of a PWM controlled Buck-Boost converter (see [12] for details), and implemented a linear state feedback control of the converter by its bilinear model. Simulation as well as experimental results are given.

II. ON PASSIVITY OF BUCK-BOOST CONVERTER WITH CLOSED LOOP CONTROL

A dynamic system is considered as passive one if the rate of input energy is not smaller than the rate of the stored energy enlargement. The difference between the input and the accumulated energy must be dissipated by the system itself. This means that the passive system is unable to store more energy than externally received. For a nonlinear system decsribed by:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

to satisfy passivity criteria, according to Willems [7], there must exist a positive semidefinite continuous differentiable function *V(x(t))* so that:

$$u^t y \geq \frac{\partial V}{\partial x}\dot{x} + \varepsilon u^t u + \delta y^t y + \rho\varphi(x) \tag{2}$$

where ε, δ и ρ are non-negative constants, and $\varphi(x)$ is positive semidefinitive function so that: $\varphi(x) \equiv 0 \Rightarrow x=0$ for all solutions of (2) and for all control variables *u(t)* for which that solution exists.The item $\rho\psi(x)$ describes the state of dissipativity. It can be shown that for a nonlinear system with a feedback loop, consisting of passive subsystems, the overall system is also passive one.

III. APPLICATION OF PASSIVITY TECHNIQUE ON BUCK -BOOST BY LINEAR STATE SPACE CONTROL

A buck-boost converter topology is shown in fig.1. Since the circuit changes its structure depending on the value of the control variable u(t):{0, 1}, the state equations [14] describing its behavior during the switching period, when the switch is on (*u=1*) and when the switch is off (*u=0*), can be written. By introducing a duty cycle $d_t \in [0\ 1]$ as a new continuous variable, the state space equations can be written in a form:

$$\dot{x}_t = A_2 x_t + b_2 + (A_1 - A_2)x_t d_t + (b_1 - b_2)d_t, \tag{3}$$

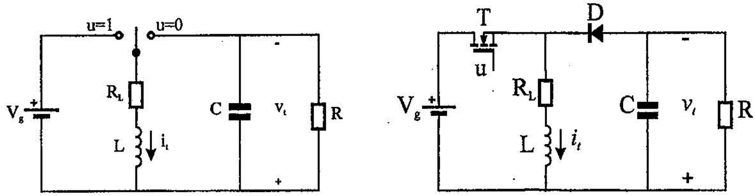


Fig. 1 Buck-Boost Converter: V_g is the input dc voltage, L and C are the elements of the converter, R is the load, and R_L stands for the ohmic losses in the inductor.

where, $x_t = \begin{bmatrix} i_t \\ v_t \end{bmatrix}$ is the state vector whose elements are the inductor current and the capacitor voltage, and

$$A_1 = \begin{bmatrix} -R_L/L & 0 \\ 0 & -1/RC \end{bmatrix} \quad b_1 = \begin{bmatrix} V_g/L \\ 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} -R_L/L & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

which represents the bilinear description of the buck-boost converter. By substituting the state variables and the duty cycle d_t as sums of the equilibrium point value and the coresponding perturbation values:

$$x_t = x_e + x \qquad d_t = d_e + d \tag{4}$$

and by separation of the equilibrium part of the equation from the dynamic one, allows us to rewrite the dynamics of the system in a form:

$$L\frac{\partial i}{\partial t} = -R_L i - d_e' v + vd + (V_g + v_e)d \tag{5}$$

$$C\frac{\partial v}{\partial t} = d_e' i - \frac{v}{R} - id - i_e d \tag{6}$$

These equations describe the incremental variable relationships. They can be represented by the equivalent two-port model circuit shown in fig. 2 [15]. In [12] is shown that the control law that guarantees large-signal stability can be written as:

$$d = -\alpha \cdot (V_g + v_e)i + \alpha \cdot i_e v = -\alpha \cdot \left((V_g + v_e)i - i_e v\right) \tag{7}$$

where α is a positive value parameter having a dimension of reciprocal energy, W^{-1}, while the dynamic part and equilibrium point are:

$$\dot{x} = Ax + Bxd + bd \tag{8}$$

$$x_e = \begin{bmatrix} i_e \\ v_e \end{bmatrix} = \begin{bmatrix} \dfrac{V_g d_e}{R_L + (1-d_e)^2 R} \\ \dfrac{V_g d_e (1-d_e) R}{R_L + (1-d_e)^2 R} \end{bmatrix} \tag{9}$$

$$A = A_2 + (A_1 - A_2)d_e = \begin{bmatrix} -R_L/L & -d_e'/L \\ d_e'/C & -1/RC \end{bmatrix}, b = (A_1 - A_2)x_e + B_1 = \begin{bmatrix} (V_g + v_e)/L \\ -i_e/C \end{bmatrix} \text{ and } d_e' = 1 - d_e$$

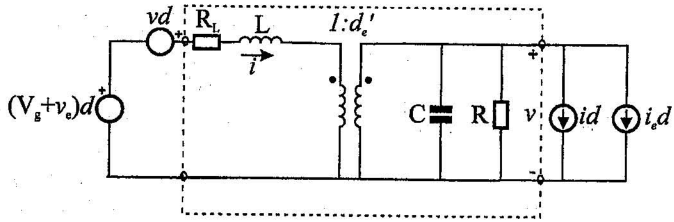


Fig. 2 Modified equivalent two-port model of the converter

For the system passivity to be achieved, the power absorbed by the two-port should be less than the power dissipated by the converter's resistive elements. The passivity of the system is used to guarantee that the equivalent circuit will tend to zero incremental energy which means that the converter state will tend to its equilibrium point. The control signal (duty cycle) can go to saturation, which means that it is bounded between d_{min} and d_{max}. The control signal saturation must be taken into consideration in order to account for system stability. To satisfy this requirement, the parametar α becomes a nonlinear function for d in saturation, [12, [13]:

$$\alpha(u)=\begin{cases}\alpha_{\max} & \dfrac{d_{\min}}{\alpha_{\min}}\le u\le\dfrac{d_{\max}}{\alpha_{\max}}\\[2mm] \dfrac{d_{\max}}{u} & u>\dfrac{d_{\max}}{\alpha_{\max}}\\[2mm] \dfrac{d_{\min}}{u} & u<\dfrac{d_{\min}}{\alpha_{\max}}\end{cases}\text{, while the control law is } u=-\alpha\left((V_g+v_e)i-i_e v\right) \quad (10)$$

IV. DESIGN OF THE CONTROL LOOP AND SIMULATION RESULTS

Once the large-signal stability is guaranteed, the design of the desired dynamics of the switching regulator can be done. For this aim we can apply a linear analysis, which does not take into account the bilinear terms in the differential equations. The linear incremental model of the buck-boost converter is:

$$\begin{bmatrix}\dot{i}\\ \dot{v}\end{bmatrix}=\begin{bmatrix}-R_L/L & -d_e'/L\\ d_e'/C & -1/RC\end{bmatrix}\begin{bmatrix}i\\ v\end{bmatrix}+\begin{bmatrix}(V_g+v_e)/L\\ -i_e/C\end{bmatrix}d \quad (11)$$

For the control law $d=-\alpha_{\max}\left((V_g+v_e)i-i_e v\right)$ the loop gain is given by:

$$T(s)=-\alpha_{\max}\left((V_g+v_e)\frac{I(s)}{D(s)}-i_e\frac{V(s)}{D(s)}\right)=$$

$$=-\alpha_{\max}\frac{\left(\dfrac{(V_g+v_e)^2}{L}+\dfrac{i_e^2}{C}\right)s+\dfrac{1}{LC}\left(\dfrac{(V_g+v_e)^2}{R}+R_L i_e^2\right)}{s^2+\left(\dfrac{R_L}{L}+\dfrac{1}{RC}\right)s+\dfrac{1}{LC}\left(\dfrac{R_L}{R}+d_e'^2\right)} \quad (12)$$

As an experimental case we used the following set of parameters for which the linear analysis and the simulations were performed:

	Simulation:	Equilibrium point:	Test:
$L=234\ \mu H$	$V_g=12$ V	$v_e=7.18$ V	$V_g=13$ V
$R_L=0{,}044\ \Omega$	$Ts=20\ \mu s$	$i_e=1.85$ A	
$C=224\ \mu F$	$d_e=0.5$		$d_e=0.5$
$R=10\ \Omega$	$\alpha_{max}=0.949\cdot10^{-3}$		

The control law described above is implemented in a circuit shown in fig. 3. To show the good correspondence between the theoretically observed results and the practical circuit behavior, inductor current and output voltage responses during the startup are given respectively on fig 4.

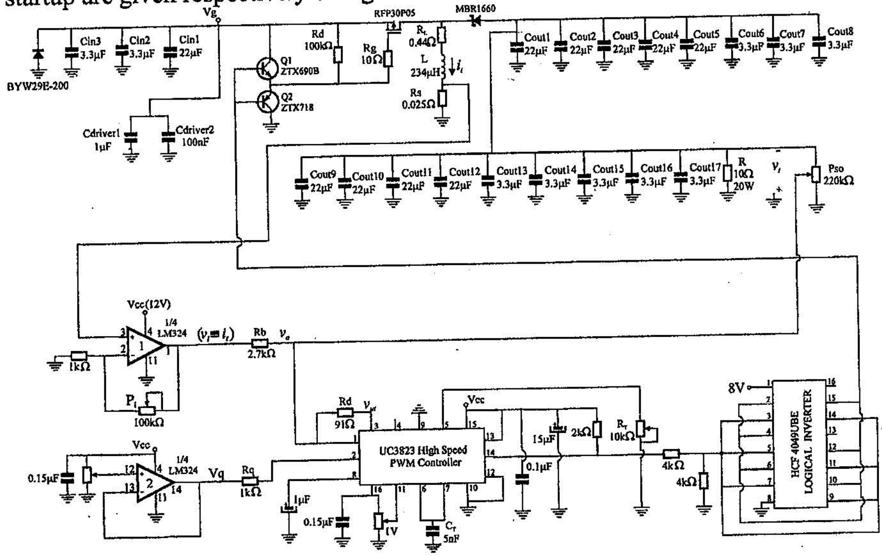


Fig. 3 Schematic of the linear state feedback controlled Buck-Boost converter

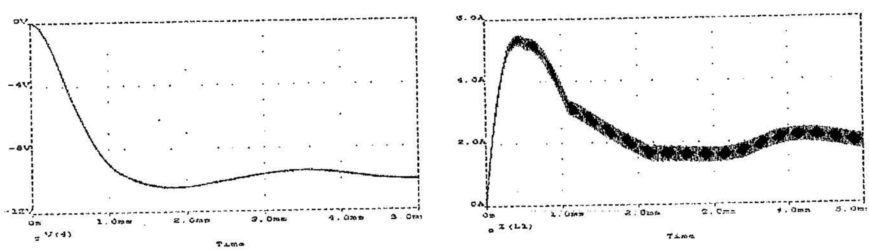
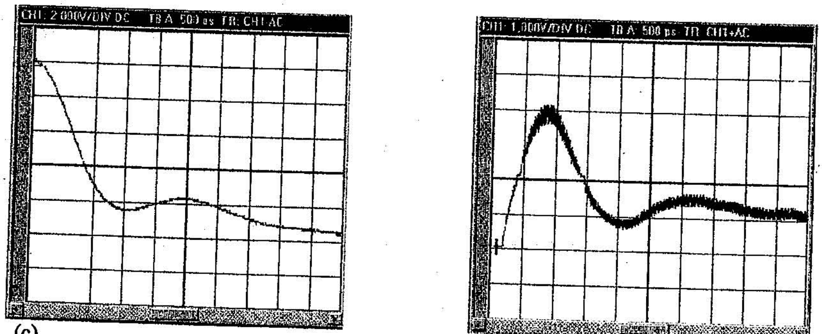


Fig. 4 Simulation: (a) Inductor voltage response b)Output current response during start-up



(c) (d)

Fig. 4 Responses for the inductor voltage (c) and output current (d) obtained experimentally

IV. CONCLUSION

Passivity based technique is implemented for a buck-boost converter design. The condition for large-signal stability of the regulator is obtained for a linear control law. Providing large-signal analysis the bilinear description is taken into consideration. Having the global stability approved, the conventional linear control theory design is implemented to achieve appropriate transient response. Results obtained by simulation as well as experimentally are presented showing a good accordance with theoretically derived predictions.

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