A NEW VARIABLE DIGITAL SECOND-ORDER ELLIPTIC LOWPASS FILTER SECTION

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Abstract: A new digital multi-output IIR biquadratic filter section permitting independent tuning of the filter parameters (poles and zeros angles and distance to the unity circle) is proposed in this paper. For narrowband elliptic LP and wideband HP filters it is even possible to tune simultaneously both the poles and zeros angles by changing a single multiplier coefficient and to keep in the same time a constant unity-gain in the passband. The main describing relations are derived and the tuning characteristics and their limitations are investigated. No other section with such possibilities is known in the literature. All theoretical results are verified experimentally.

1. INTRODUCTION

Recently there was a constant interest in the design and investigation of variable digital filters [1]. There are, however, severe problems in the realization of variable IIR filters and the most popular method known – that of Mitra, Neuvo and Roivainen (MNR) [2], based on parallel allpass structures with real or complex coefficients and employing the allpass (frequency) transformations of Constantinides followed by truncated Taylor series expansions – is quite approximate. Two other promising approaches were advanced recently: a) cascaded realization of the filter followed by truncated Taylor series expansions of the coefficients of each second-order section [3]; b) realizations using equal first- or second-order sections without any truncations [4], [5]. In both approaches variable second-order sections with independent tuning of the characteristics are required. After an extensive search in the literature it was found that only one such circuit (that of Murakoshi, Watanabe and Nishihara – MWN [3]) is known, but it is creating serious practical problems when elliptic transfer functions are realized and is not able at all to realize non-elliptic transfer functions.

In this contribution, we develop and investigate a new biquadratic section with a canonic structure, which meets all requirements and realizes equally easy elliptic and non-elliptic transfer functions with independently tunable characteristics. And what is more important, it is possible to tune the poles' and zeros' angles simultaneously by changing the value of a single multiplier coefficient retaining thus the selectivity of the tuned filter. The proposed section possesses very low sensitivity for poles and zeros near z=1 (the most difficult case or pole-zero disposition) which means very high accuracy of tuning of narrowband filters or realizations with very short wordlength.

2. BASIC CONCEPT

Given a general second-order transfer function with zeroes on the unit circle

$$H(z) = g_0 \frac{1-2\cos\Theta_z z^{-1} + z^{-2}}{1-2r_p\cos\Theta_p z^{-1} + r_p^2 z^{-2}} = g_0 \frac{1+g_3 z^{-1} + z^{-2}}{1+g_1 z^{-1} + g_2 z^{-2}}, \qquad (1)$$

where:

$$g_o = \frac{1+g_1+g_2}{2+g_3}; \quad \text{(for LP elliptic transfer function)} \tag{2}$$

$$g_o = \frac{1-g_1+g_2}{2-g_3}; \quad \text{(for HP elliptic transfer function)} \tag{3}$$

$$g_o = 0.25(1+g_1+g_2); \quad g_3 = 2; \text{ (for LP transfer function)} \tag{4}$$

$$g_o = 0.25(1-g_1+g_2); \quad g_3 = -2; \text{ (for HP transfer function)} \tag{5}$$

and g_o is taken to ensure a unity gain for the most important frequency (DC for LP and $F_s/2$ for HP) in the passband.

It looks easy at first sight to tune independently Θ_z, Θ_p and r_p, by just trimming respectively g_3, g_1 and g_2 and to use for this the common direct form realization. But any of these tunings will change also the passband gain and in order to compensate it, a recalculation and trimming of g_o will be necessary. Thus, even for tuning of a single parameter two multiplier coefficients have to be recalculated and reprogrammed. Additional complication is the division operation in (2), (3) which is difficult for digital implementation. And, finally, it is impossible to tune simultaneously Θ_z, and Θ_p by changing a single multiplier coefficient.

The only known section permitting most of these independent and coupled tunings is the MWN-section [3], realizing the following transfer function:

$$H(z) = (1+\alpha)\frac{1+[\gamma(1-\beta)-2\beta]z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}, \tag{6}$$

where Θ_z and Θ_p might be tuned by changing β and γ, while $(1+\alpha)$ remains unchanged. Unfortunately, there is no easy way to realize non-elliptic transfer function: it is necessary to maintain γ as

$$\gamma = 2\frac{1+\beta}{1-\beta}, \tag{7}$$

in order to have $g_3 = 2$ (3) for LP filter for example and when β is changed to tune the LP filter cutoff frequency, γ must also be recalculated (incl. division operation) and reprogrammed. For narrowband LP filters $\beta \approx 1$ and it is producing (7) values for γ larger than 1000, that are hardly practical for digital implementation. Additionally, the DC gain in this non-elliptic case also has the large value of $H(0) = 4/(1-\beta)$ and must be compensated by a scaling multiplier with an inverse (and very small value) recalculated every time when β is changed. And as for narrowband LP filters $\alpha \approx 1$ and $\beta \approx 1$, this realization will have quite high sensitivity to the filter coefficients, which means low-tuning accuracy and bad behavior in limited wordlength environment.

The basic concept which we adopt for further investigations is to try to find or to develop a multi-output section, having a transfer function denominator like that in Eq. (6) and non-elliptic LP and HP outputs with unity gains for the most important frequencies in the passband. Then we can construct an elliptic transfer function as

$$H_E(z) = \gamma H_{LP}(z) + H_{HP}(z), \tag{8}$$

where γ will control not only the zero position, but also the type (LP or HP) of the elliptic filter.

3. NEW CIRCUIT DESCRIPTION

The only other than the MWN-section with a denominator similar to the one in Eq. (6), found in the literature, is the low-sensitivity biquadratic section investigated in [6]. There is another group of sections with denominator as in (6), proposed in [7], but they can realize only BP and BS transfer function and are not applicable in our case. After implementing the procedure (8) and introducing additional operations (mainly summing), we obtain the final structure, shown in Fig.1. Part of this structure was investigated and published in [5].

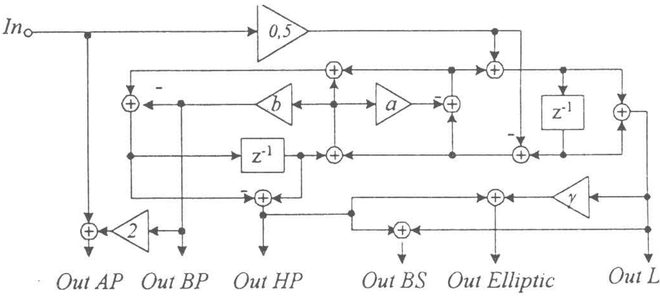


Fig. 1. New biquadratic multi-output variable filter section

The transfer function at the elliptic output is

$$H_E(z) = \frac{a(\gamma-1)+2-b}{2} \frac{1+2\dfrac{a(\gamma+1)-2+b}{a(\gamma-1)+2-b}z^{-1}+z^{-2}}{1+(-2+b+2a)z^{-1}+(1-b)z^{-2}}, \tag{9}$$

where the multiplier coefficients, obtained after sensitivity minimization, are calculated from those in (1) using the formulae:

$$a = 0.5(1+g_1+g_2); \quad b = 1-g_2; \quad \gamma = \frac{-1+g_1-g_2\ g_3+2}{1+g_1+g_2\ g_3-2}. \tag{10}$$

The numerators of the transfer functions at the other outputs are

$$N_{LP}(z) = 0.5a(1+z^{-1})^2; \quad N_{HP}(z) = 0.5(2-a-b)(1-z^{-1})^2 \tag{11}$$

$$N_{BP}(z) = -0.5b(1-z^{-2}); \quad N_{AP}(z) = (1-b)+(-2+2a+b)z^{-1}+z^{-2}; \tag{12}$$

$$N_{BS}(z) = 0.5(2-b)\left[1+\frac{-2+2a+b}{2-b}z^{-1}+z^{-2}\right]. \tag{13}$$

The most amazing quality of all these transfer functions (except $H_E(z)$) is that they have unity gain for the most important frequencies in their passbands. Concerning $H_E(z)$, it has DC gain equal to γ and thus the type (LP or HP) of the elliptic transfer function is controlled. It requires then an additional multiplier with a coefficient $1/\gamma$ at the input or at the output. It is a complication, if it has to be also tuned, but it will appear in all our further considerations that it will stay fixed.

It is clear from (9) – (13) that it is possible to tune independently Θ_p and r_p by changing a and b and also Θ_z (in the elliptic case) by trimming γ. Many other possibilities are available, as it will be shown next.

4. RANGE OF TUNABILITY INVESTIGATION

We have observed in [4] that an independent tuning of Θ_p and r_p by changing α and β in the MWN-circuit is possible within some limited frequency range out of which the passband ripples are increasing too much. The same was found also for our section in [5]. It means that the cutoff frequency of the non-elliptic LP and HP filters cannot be tuned freely over the entire frequency range. It was shown firmly, however, in [4], [5], that the range of tuning of the cutoff frequency, while the ripples are kept in the required limits, is much wider than that, achieved with MNR-circuit [2]. And there should not be limitations for the tuning of the central frequency and the bandwidth of the BP and BS filters.

Another very important feature can easily be sensed when inspecting Eq. (9). The core structure of the section in Fig. 1 was developed for realization of poles (and zeros) near $z=1$, i. e. narrow-band LP and wide-band HP filters. For such case (most interesting for variable realizations) $g_1 \approx -2$, $g_2 \approx 1$ and $g_3 \approx -2$. According to (10), it is producing $a < 0.01$, $b < 0.01$ and γ in the range from 0.1 to 10. From (9) and (1) it is easy to find that

$$\Theta_z = \arccos\frac{(2-b)-a(\gamma+1)}{(2-b)+a(\gamma-1)}; \quad \Theta_p = \arccos\frac{(1-0.5b)-a}{\sqrt{1-b}}. \tag{14}$$

It is clear that for the values of a,b and γ so mentioned, the second terms everywhere they exist in (14) are absolutely negligible compared to the first. Thus, the arguments of both arccos-functions are quite equal and near to unity. And, what is more important, they depend in about the same way on a. It means that Θ_z and Θ_p could be tuned simultaneously by changing only the multiplier coefficient a. It means that there will be no need to trim γ in order to tune Θ_z, there will be no need to have γ variable and no additional multiplier with a coefficient $1/\gamma$ at the input or at the output will be necessary. And if Θ_z and Θ_p are changed with the same speed, when trimmed by a, then the selectivity of the tuned filter will remain quite constant. In order to check all this, an elliptic LP filter section was designed and simulated and the results of the tuning are shown in Fig. 2a. It is seen that even for values of g_1, g_2 and g_3 (as given in the figure) not so close to the extreme values ($g_1 \approx -2$, $g_2 \approx 1$ and $g_3 \approx -2$.) the cutoff frequency and the zero-frequency are tuned very successfully in the same pattern (and thus retaining the magnitude steepness and the selectivity) by changing only a single coefficient a. This is illustrated even better in Fig. 2b, where the tuning characteristics of Θ_z and Θ_p for different values of the coefficient γ are given. It is seen that tuning curves of $\Theta_z(a)$ are staying almost parallel to $\Theta_p(a)$, even when γ, resp. the distance between Θ_z and Θ_p is changed considerably.

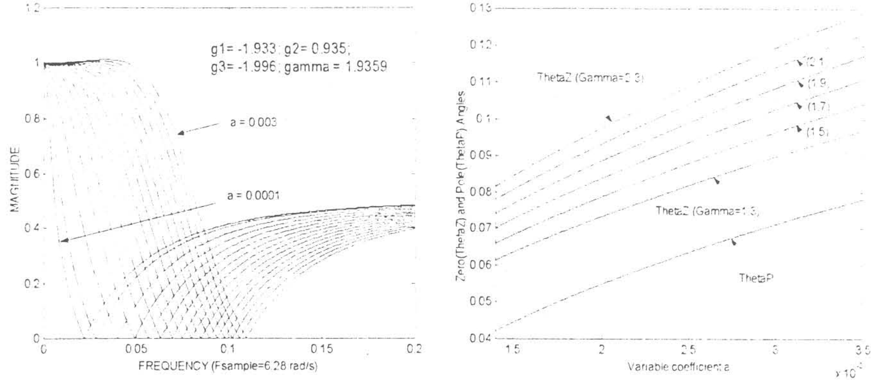


Fig.2. Simultanious tuning of the cutoff frequency and the zero-frequency of an elliptic LP by trimming only the coefficient a (a); tuning curves of Θ_z and Θ_p for different γ.

5. EXPERIMENTS

A number of experiments (simulations using MATLAB) have been performed in order to verify all the results and statements derived in this work.

In Fig. 3 the tuning of the cutoff frequencies of an LP and a HP non-elliptic sec-

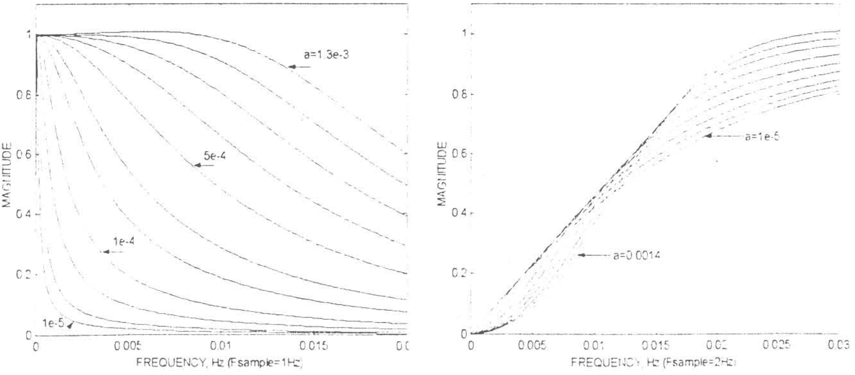
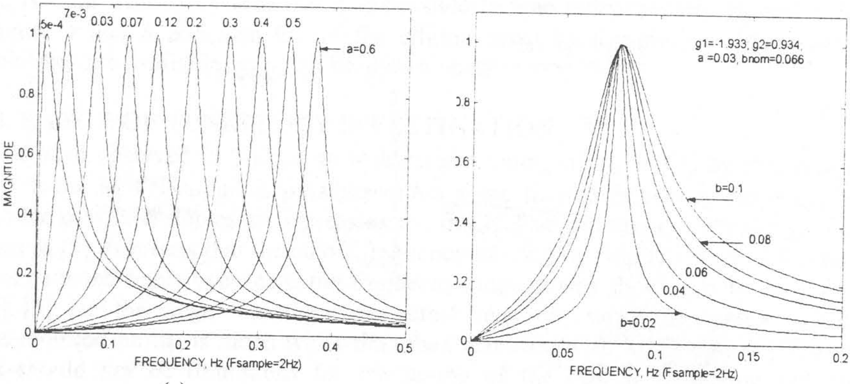


Fig. 3. Tuning of the cutoff frequency of non-elliptic LP (a) and HP (b) filters for g1=-1.933; g2=0.934; a_{nom}=5.0e-4

tions are demonstrated. The range of tuning in both cases is much wider, compared to the results with MNR-filters [2].

In Fig. 4 it is shown how successfully the central frequency and the bandwidth of a BP filters are tuned independently by changing the multiplier coefficients a and b.



(a) (b)

Fig. 4. Tuning of the central frequency (a_{nom} =5.0e-4)(a) and the bandwidth (a =0.03, b_{nom}=0.066)(b) of a BP section (g1=-1.933; g2=0.934) with by changing a and b.

6. CONCLUSIONS

The new section, proposed in this paper, provides an independent tuning of the filter parameters in a ways no other known circuits do. It is possible, moreover, to tune simultaneously both the poles and zeros angles by changing only a single multiplier coefficient and to keep in the same time a constant unity-gain in the passband. It has also very low sensitivity and permits a very high accuracy of tuning.

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