A RELIABILITY ANALYSIS OF THE DISTRIBUTED PROCESSING SYSTEMS

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Some important practical systems exhibit memory. And other distributions than Markov were found to have a better match with experimental data. This paper proposes, from this point of view, a state of the art in the reliability analysis of the computing distributed systems: concept of memory profile, periodic tests for transient failures, minimising the cost and maximising the probability of detecting the faults, the dynamic computer network reliability by deriving time dependent expressions for reliability in the context of the distributed processing systems.

1. Digital systems faults

Digital systems have two types of faults from the point of view of operational failures: permanent faults, due to hardware failures or software errors, and intermittent faults due to transient failures [1]. Intermittent faults are automatically detected by the error correcting code and corrected by the error control or the restart. Some tests are applied to detect and isolate faults, but it would waste time and money to do frequent tests. It can be considered continuous and repetitive tests for a continuous parameter Markov model with intermittent faults [2].

Redundant systems with s-independent modules were studied in [3]. Malaiya extended them for non-Markov models [4] and redundant systems with s-independent modules. Nakagawara and Yasui consider the same model as [5], where tests are scheduled at periodic times KT $(k=1,2,\ldots)$ to detect intermittent faults.

2. Distributed systems reliability modelling

In a study of multiple-processor systems, the Weibull distribution is found to have a better match with experimental data. This implies that in such systems, the transition probability rate from a state depends on the time the system spent in that state. There are established some methods that allow the analysis of this important class of non-Markov systems.

2.1. Probabilistic graph modelling

For reliability analysis, a distributed processing system is represented by a probabilistic graph $G(V,E)$ where V and E are respectively, the set of nodes representing the computers, and the set of directed and undirected arcs representing the communication links. Some of the events whose probabilities of success are of interest in a network are: terminal-pair connectivity, broadcast connectivity and multi-terminal connectivity. Occasionally, these events are also required to satisfy some performance constraints specified by the user. The most common constraints include delay (time delay or hop length), flow (capacity or throughput), and survivability of distributed programs and data.

Several methods are reported in the literature for terminal reliability analysis and computing using Boolean algebra [6-8]. These methods start by considering all the simple paths between a given pair of nodes, and then performing some Boolean operations to arrive at the Boolean expression for the probabilistic event of interest. Using the corresponding network element reliabilities, it results the expression for obtaining the terminal reliability expression.

Another useful reliability measure for computer networks is tree connectivity. The probabilistic event of interest is that there exists at least one path from a particular node to a set of nodes. This is useful in studying the reliability of broadcasting of information from a given node to a set of nodes.

The network reliability measures should always qualified with:

1) the probabilistic event for which the measures are evaluated,
2) performance constraints, and
3) the time interval.

For example, the reliability and availability are denoted as:

R(event, constraint, time)

A(event, constraint, time).

We may be interested in finding the time for which the availability of terminal-pair connectivity is greater than a value. To include performance constraints, we may want to find the terminal-pair reliability $R(t)$ such that the message delay between source and destination is less than some value.

For complex networks the Markov modelling approach becomes very difficult and time consuming because of the state space explosion. For each probabilistic event considered, the number of states in the Markov model is directly proportional to the branching factor, existence of cross links and the depth of the network. When availability is needed, we have to expand the state diagram to account for different elements.

2.2. Transient failures modelling

In a study of multiple-processor systems [9], the Weibull distribution is found to have a better match with experimental data. This implies that in such systems, the transition probability rate from a state depends on the time the system spent in that state. Malaiya and Su established some methods that allow analysis of this important class of non-Markov systems.

The motivation in developing the methods was to analyse transient failures in digital systems more accurately. The methods are very general and are not restricted to Weibull (which is more general than exponential) distribution. It can be used to compute the reliability and the availability of digital systems with (or without) redundancy. It can also be used either to design proper test experiment for digital systems with transient faults and for other systems which exhibit similar non-Markov behaviour.

For a discrete-time 2-state system

$$p_x(j) = \Pr\{\text{system in the state } x \text{ at instant } t_j\}$$

$$= \sum_{i=1}^{\infty} \pi_x(i,j) \quad (1)$$

The function $\pi_x(i,j)$, $i = 1$ to ∞ is called *memory profile*, and describes the probabilistic history of the state $x = 0$ or 1, at instant t_j. The first index i refers to the steps (with the dimension of time) and the second index j refers to the instant in time.

Since $p_0(j) + p_1(j) = 1$ and (2)

$\lambda(n)$ ➢ Pr{system is in state 1 at instant t_j | (it was in state 0 at instant t_{j-1}) ∩ (system was in state 0 for a period $n\tau_s$)} (3)

$\mu(n)$ ➢ Pr{system is in state 0 at instant t_j | (it was in state 1 at instant t_{j-1}) ∩ (system was in state 1 for a period $n\tau_s$)} (4)

it was determined that

$$\pi_0(1,j)[\Lambda + M] = 1 \quad (5)$$

$$\Lambda \equiv 1 + \sum_{i=2}^{\infty} \prod_{k=1}^{i-1} (1 - \lambda(k)) \quad (6)$$

$$M \equiv 1 + \sum_{i=2}^{\infty} \prod_{k=1}^{i-1} (1 - \mu(k)) \quad (7)$$

$$p_0(j) = \Lambda / (\Lambda + M), \quad p_1(j) = M / (\Lambda + M) \quad (8)$$

For continuous-time systems cases, the memory profile $\pi_x(\tau,t)$, $\tau = 0$ to ∞, is continuous, the $\lambda(\tau)$, $\mu(\tau)$, $\pi_x(\tau,t)$ all have dimension of $(\text{time})^{-1}$.

$\pi_x(\tau,t)dt$ ➢ Pr{(system is in state x at instant t) ∩ (it entered state x between τ to $\tau + d\tau$ time ago)} (9)

$\lambda(n)$ ➢ Pr{system is in state 1 at instant t | (it was in state 0 at time $t - dt$) ∩ (system was in state 0 for a period τ)} (10)

$\mu(n)$ ➢ Pr{system is in state 0 at instant t | (it was in state 1 at time $t - dt$) ∩ (system was in state 1 for a period τ)} (11)

$p_x(t)$ ➢ Pr{system is in state 0 at time t}; $x = 0$ or 1 (12)

$$p_x(t) = \int_0^{\infty} \pi_x(\tau,t)d\tau, \ x = 0,\ 1 \quad (13)$$

$$p_0(t) = \Lambda / (\Lambda + M), \quad p_1(t) = M / (\Lambda + M) \quad \text{where} \quad (14)$$

$$\Lambda = \int_0^{\infty} \exp[-\int_0^{\tau} \lambda(\tau)d\tau]d\tau \quad (15)$$

$$M = \int_0^{\infty} \exp[-\int_0^{\tau} \mu(\tau)d\tau]d\tau \quad (16)$$

3. Conclusions

In digital systems, the probability of a transient failure occurrence is related to the period of time that the system has been operating correctly. Because analytic methods do not exist for accurate modelling of this kind of systems, it was introduced the concept of memory profile.

It was also done the reliability analysis of a distributed system, particularly a computer network. In view of a drawback of Markov modelling, Boolean algebra provides an attractive means to achieve both efficiency and functionality.

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