Synchronization with the Grid During the Measurement of Power

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Abstract

The error which results from mismatching of measuring interval and Power Net Period when measuring the power is analyzed in this work. An analytical expression of this error is also derived and proved here. The reaction of this error to the factors on which it depends is also researched. When the measuring interval is synchronized to the zero crossing point of the voltage curve, the error is said to approach a value, defined by the relative mismatching of the measuring interval and the Power Net period. A new start point scheme for measuring the power is proposed in which the above error tends to be zero.

Introduction

Availability of advanced, highly accurate, high-speed analogue to digital converter integrated circuits and high speed microcontrollers in the market enable the designers of measurement equipment to develop grid power measuring equipment with higher accuracy.

There are two basic problems connected with the accuracy of digital measurement of grid power.

The first problem arises from the sampling along the level and discretization by the time of the signal..

Sampling error δ_S, according [3] is given by

(1) $$\delta_S = \left(\alpha / \sqrt{m}\right)(1/N) ,$$

N - a weighting code;

α - a coefficient derived from the low of sampling error distribution;

m - number of samples.

Inaccuracies from discretizations arise due to application of approximate numeric computation methods. This is widely investigated by a great number of authors. A comprehensive study of the discretization theory is given in [1] and [2]. An evaluation of discretization error ΔY_D, according [3], can be given by

(2) $$\left|\Delta Y_D\right| \le \frac{T}{12.m^2}\left[\sum_j \left|\Delta f_j'\right| + T\left|f''(t)\right|_{\max}\right] ,$$

where

T is the period of the discretized signal;

$f(t)$ - is a function of the processed signal.

The second problem comes from the practical aspects of digital measurement of power. It mainly arises from crossing over of the period of the grid and the measuring interval, for which the calculations are being made for one period according to differential definition of active power, given by (3). It is also possible these two intervals to have different phase, during which the phase shifting angle is not a constant value. This leads to fluctuations of the readings during real measurements.

There are two methods to reduce these fluctuations. The first one is to increase the measurement time and averaging of the measurements. It has got an inherent drawback of being time consuming.

The second approach introduces synchronization of the measuring device with the measuring variable.

In this paper, the errors, which arise during the measurement of power by the first method is analyzed. Analyzed are the factors with which this error depends. Methods for synchronization with the grid aiming to reduce the above mentioned errors are also proposed.

Study of the electrical power measurement error due to the mismatching between the Power Net period and the time interval for which power is calculated.

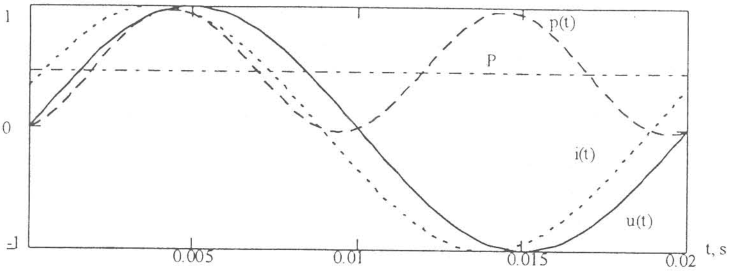
A few reasons set up power measurement for a period as an appropriate one. Firstly it follows the active power definition (3); secondly this is the shortest possible time interval this could be done, thus reducing overall time for instant power reading and computational burden.

$$(3)\ P = \frac{1}{T}\int_0^T u(t).i(t)dt \quad ,$$

where

T is the period of the Power Net current and voltage;

$u(t)$, $i(t)$ are the instantaneous voltage and current signals.



Фиг. 1 Моментни стойности на ток, напрежение, мощност и средна мощност за период

Fig.1 Instantaneous voltage and current signals and average power

Instantaneous current, voltage and active power are shown in fig.1. It is presumed that the voltage and current are sinusoidal function with a period T and angular frequency $\varpi=2.\pi/T$

$$u(t) = u_m.\sin(wt + \Theta)$$

$$i(t) = i_m.\sin(wt + \Theta - \varphi) \ .$$

where

u_m, i_m are the amplitude values of the voltage and current;

w is the angular Power Net frequency;

φ is the current to voltage phase difference;

Θ is the starting of the measuring interval according the zero crossing point of the voltage curve, $u(t)=0$.

The mismatching between Power Net period and measuring interval is introduced by the fact that the first is not integral cycles of sampling clock. There also exist ±2% instability of power frequency, which further increase the above mentioned error. The following equation gives the relation between the true Power Net period T and the interval Tz, for which calculations are being performed.

(4) $$Tz = T + \Delta T$$

where ΔT is the absolute error between the power period and the time interval for which the average active power is measured.

Let us denote the relation between the two time intervals with k

(5) $$k = \Delta T / T$$

If the time interval for which measurement is being performed is not integral power periods the definition given in (3) might be expressed as

(6) $$P_1 = \frac{1}{T + \Delta T}\int_0^{T+\Delta T} u(t).i(t)dt$$

Subtracting both integral definition (3) and (6) for average active power we can define the absolute error when the sampling frequency is not synchronized with the grid.

(7) $$\Delta P = \left| \frac{1}{T+\Delta T}\int_0^{T+\Delta T} u(t).i(t)dt - \frac{1}{T}\int_0^{T} u(t).i(t)dt \right|$$

Following some obvious transformation we finally come to

(8) $$\Delta P = \left| \frac{1}{2.w(T+\Delta T)}\left[\sin(2.w.\Delta T + 2\Theta - \varphi) - \sin(2\Theta - \varphi)\right] \right| U.I\,,$$

where U and I are rms values of the voltage and current respectively

Considering (5), the relation $\varpi=2.\pi/T$ and a very known trigonometric formula for summing two sinusoids we may easily come to

(9) $$\Delta P = \left| \frac{\sin(2\pi k).\cos(2\pi k + 2\Theta - \varphi)}{2\pi.(k+1)} \right| U.I$$

Then the definition of the relative error $\delta P=\Delta P/P$ becomes obvious as

(10) $$\delta P = \frac{\Delta P}{P} = \frac{\sin(2\pi k).\cos(2\pi k + 2\Theta - \varphi)}{2\pi(k+1).\cos\varphi} = \delta P(k,\Theta,\varphi)$$

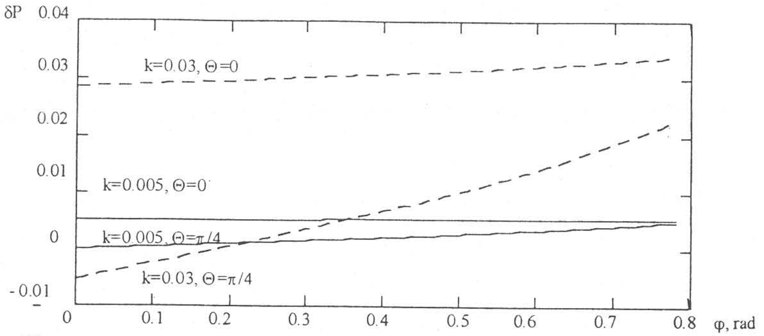
A closer look at (10) leads to the following conclusions regarding the factors which defines the relative error :

k - that is defined as a relevant mismatching between the time interval for which the power is being measured and the real power period. In real digital watt-hour meters the value of k usually is less than 0.03 and it depends primarily on sampling rate;

Θ - that is defined as an angle set by the time difference between the starting point of the measuring interval and the zero crossing point of the voltage curve, u(t)=0. This angle can be of any value and it is a designer's responsibility to choose it up;

φ - that is defined as a phase angle between the voltage and current signals. Usually its value is less than $\pi/4$ and $\cos(\varphi)<0,7$.

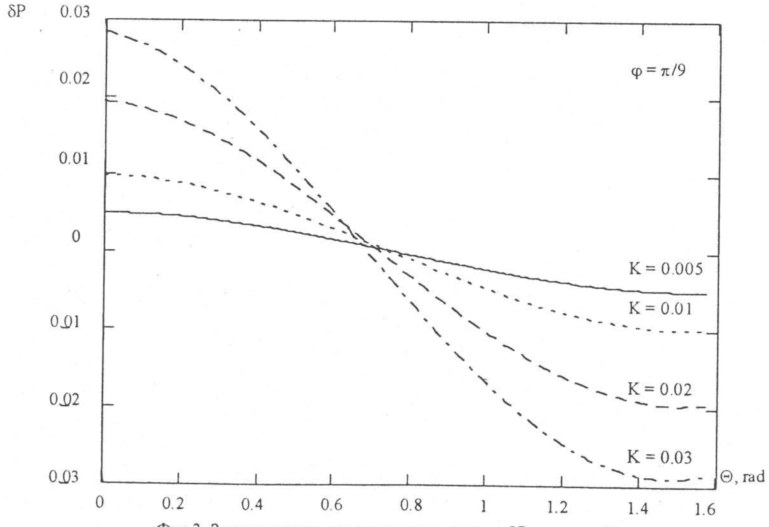
Fig. 2 shows the relation between the relative error δP and the current to voltage phase angle φ. It is shown a field of error deviation for two values of the relevant mismatching coefficient. For k=0.03 (the field closed by the dashed lines) δP gets a minimal value when it is on the line defined by k=0.03, $\Theta=\pi/4$. The same case is observed when k=0.005 (the field closed by the black lines)



Фиг.2 Зависимост на относителната грешка δP от дефазирането φ между ток и напрежение

Fig.2 The relative error δP fields for different values of k @ φ∈[0-π/4]

The relative error increases with the phase angle in the observed interval φ∈[0-π/4] when Θ is of constant value. When Θ has a small value the relative error δP approaches *k* in the whole observed interval of the argument φ. Furthermore there exists certain values of φ and Θ for which δP approaches zero.



Фиг.3 Зависимост на относителната грешка δP от ъгъла Θ

Fig.3 The relative error δP at Θ ∈[0-π/2]

This has lead to the conclusion that a study of the function $\delta P(k,\Theta,\varphi)$ due to variation of its parameter Θ is necessary at constant power factor. We have defined earlier Θ as the starting point of the measuring interval according to the zero crossing point of the voltage curve, u(t)=0. $\delta P(k,\Theta,\varphi)$ is being studied @ $\Theta \in [0, \pi/2]$

A family of traces for four different values of the mismatching coefficient k [0.005, 0.01, 0.02, 0.03] is shown in fig.3. All plots are primarily defined by the cosine function at Θ in $\delta P(k,\Theta,\varphi)$.

The error function gets a value of zero for Θ defined by

$$\Theta = \frac{\pi}{4} - k.\pi + \frac{\varphi}{2} \approx \frac{\pi}{4} + \frac{\varphi}{2} \qquad (11)$$

Conclusions

It is shown in the present paper that there exists an error δP when measuring active power with digital aquisition systems and this error comes from the mismatching of the real power period and the period for which the power is being acquired and calculated. Furthermore, it is given an analytical expression (10) that defines the above mentioned error. And finally based on the study described in this paper the following conclusions regarding the relative error δP can be derived:

- It primarily comes from crossing over of the period of the grid and the measuring interval, for which the calculations are being made for one period according to differential definition of active power
- Its value slightly depends on the phase difference between the voltage and the phase current;
- Its value primarily depends on the angle Θ, that defines the beginning of the measuring interval according to the zero crossing point of the voltage curve, u(t)=0;
- Its value can be zeroed with a careful determination of the angle Θ according equation (11);

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