Resolution Enhancement of Images Using Multiresolutional Basis Fitting Reconstruction

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Abstract– The wavelet basis method for reconstruction of nonuniformly sampled signals is applied to resolution enhancement of images. The size of an image is doubled by interpolating a new sample between every two samples of the image in horizontal and vertical direction. Larger degrees of magnification are obtained by iterating this procedure. The effectiveness of the proposed method using different wavelet bases is investigated.

I. Introduction

One problem of image interpolation is to magnify a small image without loss in image clarity. This involves interpolating samples using existing samples of the image. If the interpolation method is chosen appropriately, the magnified image has good resolution. There are various traditional methods of interpolation. The wavelet basis reconstruction method proposed by C. Ford and D. M. Etter [1] can be applied as an alternative to other interpolating methods. This multiresolutional basis fitting reconstruction (MBFR) method is very effective. It is suitable for performing reconstruction of signals even from nonuniformly spaced data samples.

We use the method of Ford and Etter to enhance the image resolution. We interpolate a new sample between every two samples of the original image in horizontal and vertical direction, thereby doubling its size. Larger degrees of magnification can be obtained by applying this procedure several times. The aim of our paper is to examine the effectiveness of this wavelet based method in resolution enhancement of images in the case when original small image is treated as downsampled version of some larger image (2x or 4x).

This paper is organized as follows. Section II briefly describes the method for interpolating one-dimensional signals by fitting data in multiresolutional framework [1]. In Section III we have shown that the above method can be extended to two-dimensional signals. Section IV contains experimental results that we obtained by applying the wavelet basis reconstruction method to resolution enhancement of images. Different wavelet bases are used for 2x and 4x magnification of 64x64 picture of Barbara. We conclude this paper in Section V.

II. Overview of Signal Reconstruction by Multiresolutional Fitting

A function $f(t)$, which resides in the space V_0, spanned by the orthogonal set of basic function $\{\phi(t-n)\}$, can be decomposed to an arbitrary resolution level J by the representation [2]

$$f(t)=\sum_{n} c_{J,n}\phi\left(\frac{t}{2^J}-n\right)+\sum_{j=1}^{J}\sum_{n} d_{j,n}\psi\left(\frac{t}{2^J}-n\right). \tag{1}$$

The multiresolutional decomposition of f (1) requires a series of nested subspaces of V_0, given by $V_{j+1}\subset V_j$, such that $V_{j+1}=V_j\oplus W_j$. The subspace V_j is spanned by the scaling basic functions $\{\phi(t/2^j-n)\}$, while W_j is spanned by the wavelet basic functions $\{\psi(t/2^j-n)\}$.

The problem of signal reconstruction by multiresolutional fitting, considered in [1], can be formulated as follows. Find M uniformly distributed samples of a discrete signal

$$\mathbf{f}=[f(0),f(1),f(2),\ldots,f(M-1)]^{\mathrm{T}}$$

from only $P<M$ samples of $\mathbf{f}$ on a nonuniformly sampled subset, indexed by $\{t_k\in\{0, 1,\ldots,M-1\}, k=0,1,\ldots,P-1\}$. It is assumed that the available signal

$$\mathbf{f}_s=[f(t_0),f(t_1),f(t_2),\ldots,f(t_{P-1})]^{\mathrm{T}}$$

is undersampled with respect to the Nyquist frequency of the complete, uniformly sampled signal $\mathbf{f}$.

By viewing the evenly spaced samples on $[0,\ldots,M-1]$ as the subspace V_0, (1) leads to the following system of equations at any resolution level $J\geq 1$:

$$f(t_0)=\sum_{n} c_{J,n}\phi\left(\frac{t_0}{2^J}-n\right)+\sum_{j=1}^{J}\sum_{n} d_{j,n}\psi\left(\frac{t_0}{2^J}-n\right)$$

$$f(t_1)=\sum_{n} c_{J,n}\phi\left(\frac{t_1}{2^J}-n\right)+\sum_{j=1}^{J}\sum_{n} d_{j,n}\psi\left(\frac{t_1}{2^J}-n\right)$$

$$\vdots$$

$$f(t_{P-1})=\sum_{n} c_{J,n}\phi\left(\frac{t_{P-1}}{2^J}-n\right)+\sum_{j=1}^{J}\sum_{n} d_{j,n}\psi\left(\frac{t_{P-1}}{2^J}-n\right).$$

A matrix form of the system is

$$\mathbf{f}_s=\mathbf{G}_J^s\mathbf{c}_J+\sum_{j=1}^{J}\mathbf{H}_j^s\mathbf{d}_j \tag{2}$$

where $\mathbf{G}_J^s$ is a matrix of shifts of the scaling function samples at level J associated with the time index t_k of each sample, and each $\mathbf{H}_j^s$ is the matrix of the shifts of the wavelet at each level j. If the signal $f(t)$ is approximated by its low-frequency components represented by the scaling function in (1) (temporarily ignoring the high-frequency terms), the system (2) can be viewed as

$$\mathbf{f}_s \approx \mathbf{G}_J^s \mathbf{c}_J. \tag{3}$$

The above system (3) is solved in a least-squares sense for estimates of the low frequency scaling function coefficients, denoted by $\hat{\mathbf{c}}_J$. The level J is chosen to be the minimum (or finest) resolution level. The low frequency estimate of **f**, denoted by $\hat{\mathbf{f}}_0$, is computed by solving the system

$$\mathbf{f}_0 = \mathbf{G}_J \hat{\mathbf{c}}_J \tag{4}$$

where $\mathbf{G}_J$ is the matrix of shifts of the scaling function at level J at all integer shifts $[0, \ldots, M-1]$. The error signal $\mathbf{e}_0$ at each available sample can be viewed as

$$\mathbf{e}_0 = \mathbf{f}_s - \hat{\mathbf{f}}_0 \,|_{t=t_k,\, k=0,1,\cdots,P-1} = \mathbf{f}_s - \mathbf{G}_J^s \hat{\mathbf{c}}_J \approx \mathbf{H}_J^s \mathbf{d}_J. \tag{5}$$

The error signal $\mathbf{e}_0$ can be approximated by its next-finer frequency components, represented by the lowest frequency band of the wavelet portion of (1). The system (5) provides an estimate of the coefficients $\mathbf{d}_J$, denoted by $\hat{\mathbf{d}}_J$. The first refinement of the original function estimate $\hat{\mathbf{f}}_1$ can be computed at every point on the even grid from

$$\hat{\mathbf{f}}_1 = \mathbf{G}_J \hat{\mathbf{c}}_J + \mathbf{H}_J \hat{\mathbf{d}}_J$$

where $\mathbf{H}_J$ is the matrix of shifts of the wavelet at level J at all integer shifts $[0, 1, \ldots, M-1]$.

The same procedure can be repeated to find another refinement of the function. The error signal $\mathbf{e}_1$ can be viewed as

$$\mathbf{e}_1 = \mathbf{f}_s - \hat{\mathbf{f}}_1 \,|_{t=t_k,\, k=0,1,\cdots,P-1} = \mathbf{f}_s - \mathbf{G}_J^s \hat{\mathbf{c}}_J - \mathbf{H}_J^s \hat{\mathbf{d}}_J = \mathbf{H}_{J-1}^s \hat{\mathbf{d}}_{J-1} \approx \mathbf{H}_{J-1}^s \mathbf{d}_{J-1}. \tag{6}$$

This system can be solved in a least-square sense to find $\hat{\mathbf{d}}_{J-1}$. Then $\hat{\mathbf{f}}_2$ is

$$\hat{\mathbf{f}}_2 = \mathbf{G}_J \hat{\mathbf{c}}_J + \mathbf{H}_J \hat{\mathbf{d}}_J + \mathbf{H}_{J-1} \hat{\mathbf{d}}_{J-1}.$$

The process continues until the finest possible resolution level has been reached. That is when aggregate number of coefficients $\hat{\mathbf{c}}_J, \hat{\mathbf{d}}_J, \hat{\mathbf{d}}_{J-1}, \cdots$ is less than or equal to the number of available samples of the signal.

III. Implementation

To magnify a small image we actually interpolate samples between some known samples and in that way the image becomes larger. We have used the method described in previous section to perform this interpolation. The known samples of image are actually obtained by uniformly subsampling some larger image with 2x size. Thus we obtain the samples $f(1), f(3), f(5)$, ... and setting $t_0 = 1$, $t_1 = 3$, $t_2 = 5$, ... we should find $f(2), f(4), f(6)$, ... This method is applied on every row first. So from 64x64 image we have obtained 64x128. Then method is applied on every column so from 64x128 image we have obtained 128x128 image. Then process could be repeated again on the resulting image so we can obtain 4x magnification with 256x256 image. In this process, different wavelet bases could be used to obtain different image quality. All results are obtained with MATLAB. Barbara is taken as the testing image. Results are shown in next section.

IV. Experimental Results

This section contains experimental results obtained with different wavelet basis. For every wavelet base two images are given: middle (128x128) and large (256x256)

Figure 1. Small 64x64 picture

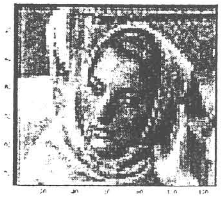

Figure 2. Middle 128x128 picture (haar wavelet basis)

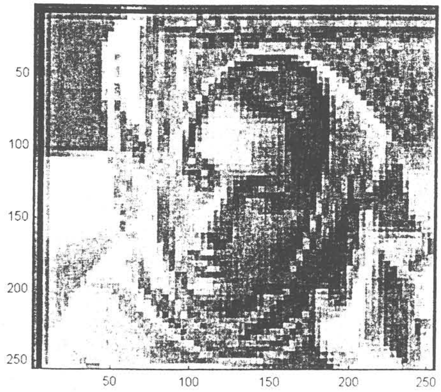

Figure 3. Large 256x256 picture (haar wavelet basis)

Figure 4. Middle 128x128 picture
(wavelet basis db2)

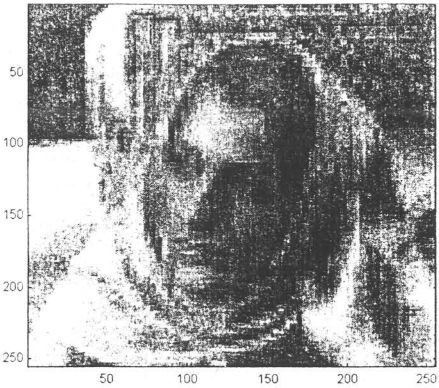

Figure 5. Large 256x256 picture
(wavelet basis db2)

Figure 6. Middle 128x128 picture
(wavelet basis db3)

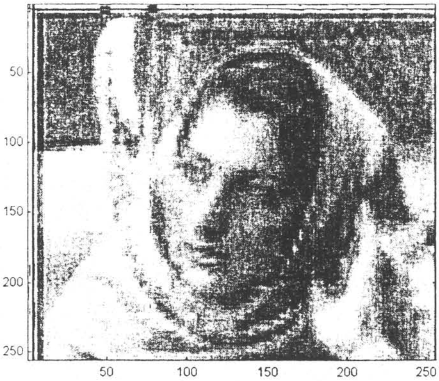

Figure 7. Large 256x256 picture
(wavelet basis db3)

V. CONCLUSION

Experimental results show that MBFR method can be implemented for magnifying images but processing time is long. As expected, processing time is longer for longer wavelet bases. From previous section one can see that blocking effects are noticeable when haar base is used (figure 2 and figure 3). When

Daubechies bases are used these effects are less noticeable but there are some blurring effects in the resulting images. The best result is obtained with implementation of db3 wavelet base. In this case the optimal tradeoff between duration of wavelet base (time resolution) and its approximation order (number of vanishing moments) is reached. It should be mentioned that we didn't implement any other technique for image sharping. Otherwise resulting images could look better. Some other wavelet bases could be used. Also further research could be done for finding more efficient algorithm that would decrease processing time.

References

[1] C. Ford and D. M. Etter, "Wavelet Basis Reconstruction of Nonuniformly Sampled Data", *IEEE Trans. Circuits Syst. II*, vol. 45, pp. 1165–1168, Aug. 1998.

[2] G. Strang and T. Nguen, *Wavelets and Filter Banks*, Wellesley MA,Wellesley-Cambridge Press, 1996.

[3] S. G. Chang, Z. Cvetkovic, and M. Vetterli, "Resolution Enhancement of Images Using Wavelet Transform Extrema Extrapolation," *Proc. IEEE ICASSP*, Detroit, MI, vol. 4, pp. 2379–2382, May 1995.