Frequency Domain Design of Mixed IIR/FIR Quadrature Mirror Filter Banks

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Abstract– A new mixed IIR/FIR near-perfect-reconstruction quadrature mirror filter bank is proposed. The design is based on the method described in [1]. The modification consists in replacing Butterworth filters in the analysis part of the bank with more general class of flat delay IIR filters due to Selesnick. The banks obtained in such a way have very low reconstruction errors. Experimental results are presented to support this claim.

I. Introduction

Two-channel quadrature mirror-image filter (QMF) banks have found significant applications in one-dimensional and two-dimensional signal processing. Many techniques for design of perfect reconstruction (PR) or nearly perfect reconstruction (NPR) QMF banks have been developed. Most of these techniques focus on the design of finite-duration impulse response (FIR) filter bank. These banks do not suffer from phase distortion and instability. However, the analysis and synthesis filters in the bank require a large number of coefficients in order to satisfy the magnitude specifications. It is known that the infinite-duration impulse response (IIR) filter banks are more efficient from the aspect of decreasing the number of filters coefficients. On the other hand, the phase distortion in these banks is unavoidable. Designing an IIR filter that has to meet both magnitude and phase specifications simultaneously is generally very difficult problem: the objective function is rather complex and the stability of the filter has to be considered.

Recently, a different approach for the design of NPR QMF banks has been presented by M. Zhu, M. Ahmed and M. Swamy [1]. This approach employs all-pass based IIR filters and FIR filters to realize the analysis and synthesis parts of the bank, respectively. It is well known that all-pass based architecture has a low-complexity structure with low roundoff noise behavior. The phase distortion caused by the nonlinear phase IIR analysis filters has been reduced by an efficient compensation provided by the FIR synthesis filters. Butterworth filters have been used for the analysis part. In comparison with the IIR filter banks, the proposed mixed bank enjoys a low design and realization complexity, while giving a good reconstruction performance. Inspired by the very new class of maximally flat all-pass based low-pass recursive digital filters proposed by I. Selesnick [2], we have replaced the filters of Butterworth in the analysis section of the bank with these more general IIR filters. As a result, the NPR QMF banks obtained in such a way have significantly smaller reconstruction error.

The paper is organized as follows. Section II contains the formulation of the design problem along with an outline of the method of Zhu, Ahmed and Swamy. The results obtained by implementing Selesnick's filters in the analysis part of the bank are given in Section III.

II. FORMULATION OF THE DESIGN PROBLEM

We assume the following notation for a two-channel QMF bank: H_0 and H_1 are the lowpass and highpass filters, respectively, of the analysis section, and G_0, G_1 are the corresponding filters of synthesis section. The perfect reconstruction condition can be expressed as

$$G_0(z)H_0(z)+G_1(z)H_1(z)=2z^{-d} \tag{1}$$

where d is the overall delay of the bank.

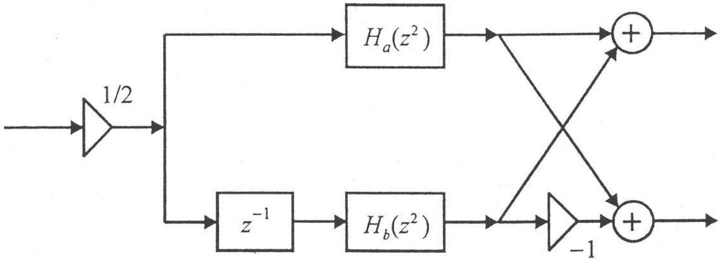

Fig. 1. All-pass realization of complementary quadrature mirror filters.

A pair of complementary quadrature mirror filters H_0 and H_1 of order N can be realized efficiently [3, 4] by a parallel connection of two all-pass filters H_a and H_b as

$$H_0(z)=\frac{H_a(z^2)+z^{-1}H_b(z^2)}{2} \tag{2}$$

$$H_1(z)=\frac{H_a(z^2)-z^{-1}H_b(z^2)}{2} \tag{3}$$

when N is odd. The upper branch in Fig. 1 gives $H_0(z)$. The lower branch gives $H_1(z)$ without additional filtering. The two all passes are given by

$$H_a(z^2)=\prod_{k=1}^{N_1}\frac{a_{2k-1}+z^{-2}}{1+a_{2k-1}z^{-2}} \quad (4)$$

$$H_b(z^2)=\prod_{k=1}^{N_2}\frac{a_{2k}+z^{-2}}{1+a_{2k}z^{-2}} \quad (5)$$

The coefficients a_i and the order N can be determined from the specifications of the analysis filters. The values of N_1 and N_2 depend on the value of N.

The design problem is formulated as follows. Let

$$H_0(e^{j\omega})=\left|H_0(e^{j\omega})\right|e^{j\phi_0(\omega)} \quad (6)$$

be a frequency response of the low-pass analysis filter H_0. The low-pass synthesis filter G_0 is to be designed such that its phase response can compensate the nonlinear phase response of H_0, i.e.,

$$\hat{G}_0(e^{j\omega})=\left|\hat{G}_0(e^{j\omega})\right|e^{j\psi_0(\omega)}=\left|H_0(e^{j\omega})\right|e^{-j[\omega d+\phi_0(\omega)]}. \quad (7)$$

Of course, the frequency response $\hat{G}_0(e^{j\omega})$ is the desired frequency response for the low-pass synthesis filter. The coefficients of G_0 are obtained by minimizing the weighting squared error defined as

$$E=\sum_{k=0}^{M-1}W(\omega_k)\left|G_0(e^{j\omega_k})-\hat{G}_0(e^{j\omega_k})\right|^2 \quad (8)$$

The objective function E is evaluated on a dense grid of M frequencies ω_k ($k = 0, 1, \ldots, M-1$) linearly distributed in the range from $\omega=-\pi$ to $\omega=\pi-2\pi k/M$. A rough approximation $\widetilde{G}_0(e^{j\omega})$ of the desired frequency response $\hat{G}_0(e^{j\omega})$ is computed as L-point DFT ($L \le M$) of M-point IDFT of $\hat{G}_0(e^{j\omega})$ that has been truncated to L taps. The weighting function is defined as

$$W(\omega)=\left|\widetilde{G}_0(e^{j\omega})-\hat{G}_0(e^{j\omega})\right|^2. \quad (9)$$

III. DESIGN

We propose a design of mixed IIR/FIR QMF bank that follows the main line of the method given in [1]. Our design differs in the analysis part of bank. We have replaced the filters of Butterworth with more general filters proposed by Selesnick [2]. These filters allow adjustment of the delay. Moreover, the tradeoff between the delay and the phase linearity can be chosen. It is worth to note that, with the classical Butterworth filter of degree N which is retrieved as a special case, it is not possible to adjust the delay (or phase linearity). For the details of the design procedure we refers to the original paper [2].

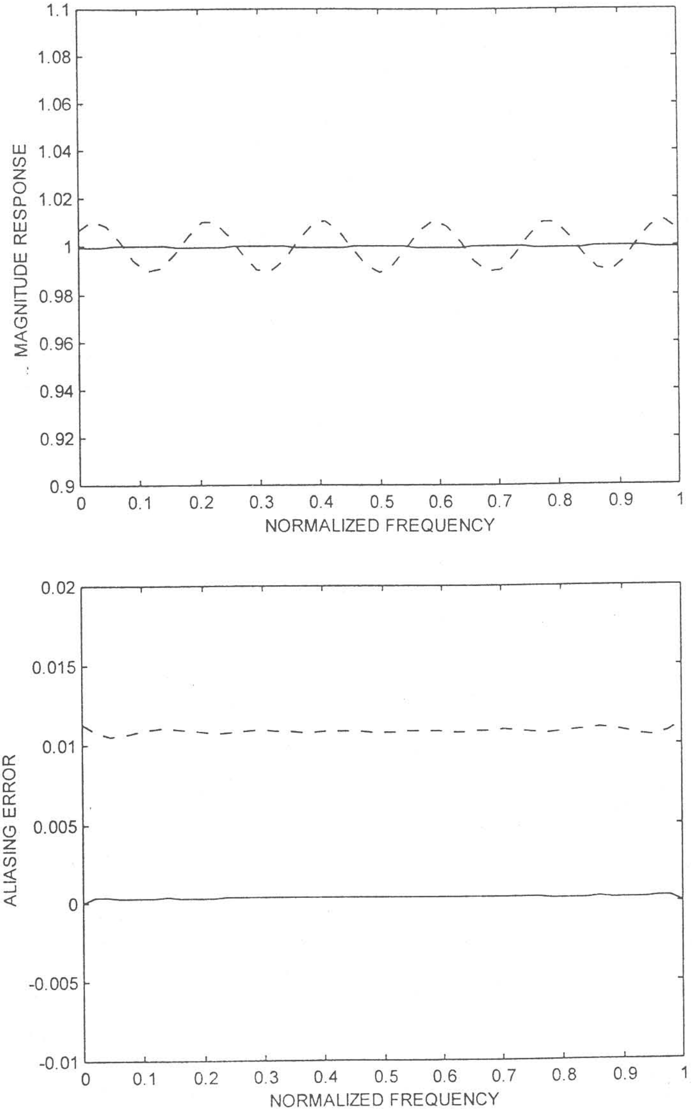

Fig. 2. Magnitude responses and aliasing errors for 15-taps FIR synthesis filters. The dotted lines correspond to the five-order Butterworth analysis filters.

Using the method of Zhu and others to design the banks having analysis filters proposed by Selesnick leads to very efficient filter banks. We present two examples of banks realized with 15-tap and 18-tap FIR synthesis filters, respectively. The flatness parameters of the analysis filters are specified to 3 at $\omega = 0$ and 3 at $\omega = \pi$. A fifth-order Butterworth filter is used for comparison. The value for M is chosen to be 100. The magnitude responses and the aliasing errors are shown in Fig. 2 and Fig. 3.

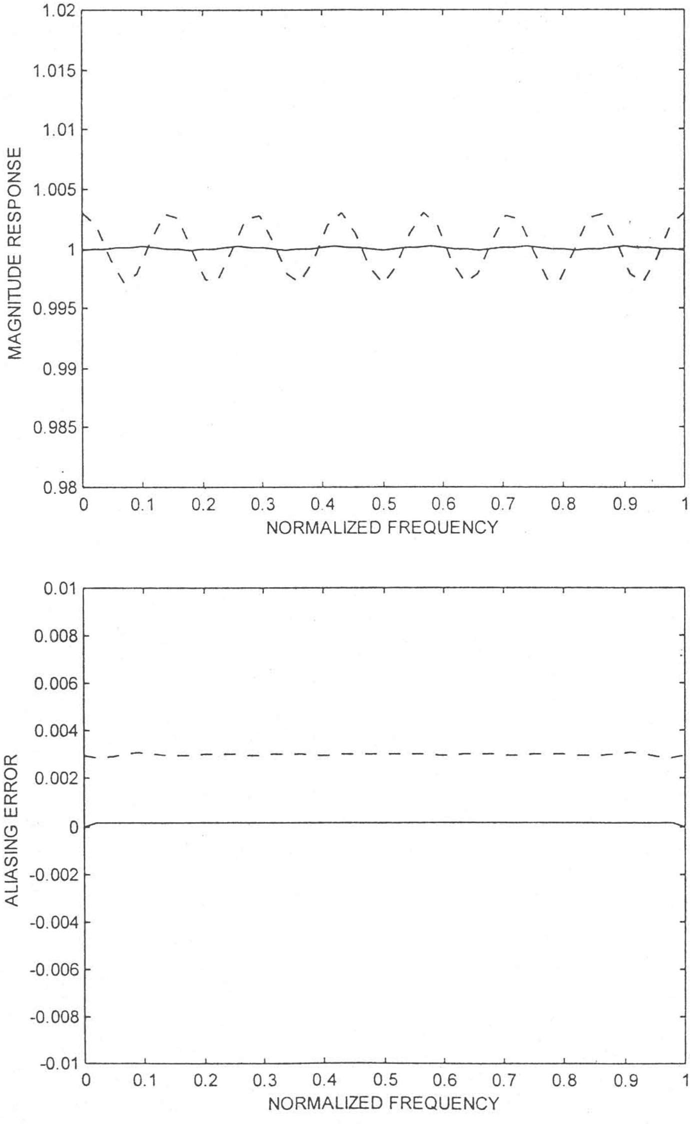

Fig. 3. Magnitude responses and aliasing errors for 18-taps FIR synthesis filters. The dotted lines correspond to the five-order Butterworth analysis filters.

It is seen that the aliasing error exhibits a constant behavior. Table 1 lists the maximum magnitude and aliasing errors.

TABLE 1: COMPARISON OF THE PROPOSED IMPROVEMENT

	Maximum magnitude error		Maximum aliasing error	
	$L = 15$	$L = 18$	$L = 15$	$L = 18$
Butterworth	0.01080	0.00300	0.01130	0.00310
Selesnick	0.00038	0.00013	0.00039	0.00015

IV. CONCLUSION

An improved design of mixed IIR/FIR quadrature mirror filter banks based on the method of Zhu, Ahmed and Swamy [1] has been developed. The improvement consists in replacing IIR filters in the analysis part of the bank. As a result, we have obtained near-perfect-reconstruction QMF banks having significantly smaller aliasing and magnitude errors.

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