EFFECTIVE DESIGN ALGORITHMS FOR FINITE IMPULSE RESPONSE DIGITAL FILTERS

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Abstract. *Linear-phase FIR filters are known to have some very desirable features like guaranteed stability, free from limit cycles and phase distortion, and low coefficient sensitivity. This paper gives briefly a mathematical basis of the problem – symmetry/antisymmetry constraints imposed on impulse response, different types of FIR characteristics and their main features. Some important aspects and design problems of one- and two-dimensional FIR filters are considered, too (input specifications, accuracy, and practical recommendations). Two new algorithms based on the Least Squares weighted criterion are presented. The first method is aimed to reject the Gibbs' phenomenon of one-dimensional linear-phase FIR filters using a set of equally-spaced fixed levels (margins) in the magnitude response. The second one could be used for design of two-dimensional (2-D) FIR filters (special application is given for fan 2-D filters).*

I. Introduction

Digital filters with a finite-duration impulse response (FIR, nonrecursive) have characteristics that make them useful in many applications. They can achieve exactly linear phase and can not be unstable. The problem of optimum frequency domain design of such kind of filters can be easily formulated as a real approximation problem and efficient algorithms for its solution exist. General- or special-purpose hardware could be used for the realization. Other attractive features of FIR filters are low coefficient sensitivity and free from the limit cycles.

The nonrecursive filter is naturally suited for certain specific applications, e.g. to perform numerical differentiation or integration; it is also suited for applications where the prescribed specifications can not be met by conventional Butterworth or elliptic approximations. Other application areas are speech and image processing, stochastic filtering, phase equalization for communication systems, etc.

II. Design methods for linear-phase FIR filters

Design algorithms for linear-phase FIR filters could be divided into two main groups: (i) methods for design of one-dimensional filters, and (ii) methods for design of multidimensional filters (special case are two-dimensional filters). Frequently, the second type of methods are extension of the first one using suitable transformation procedures.

2.1. One-dimensional (1-D) FIR filters

There is a big variety of methods and approximation criteria for design of one-dimensional FIR filters. The oldest ones are those based on the fourier series, window functions or numerical-analysis formulas [1-3]. Window functions give a good alternative of other techniques for the reduction of Gibbs' oscillations, which show as a 'ripple' near to the passband edge of the amplitude response. The most frequently used window functions are Rectangular, Hann, Hamming, Blackman, and Kaiser windows.

Other widely used approximation method is frequency-sampling approach [3,4]. Direct design with this method is possible by applying the inverse DFT to equally spaced samples of the frequency response. If frequency-sampling design with an ideal desired frequency response having a discontinuity causes too much oscillation or overshoot between the samples, a transition region can be added to the ideal response. The shape of the transition function can have an important influence on the overall design. Other different techniques for reducing Gibbs' oscillations are based on spline function [5], straight line [1], trigonometric functions, etc.

The weighted Chebyshev method can be used to design optimal linear-phase FIR filters. A classical technique uses iterative Remez exchange method (Remez, 1934) which can be applied to determine the location of the required critical frequencies (local externals) of an equiripple filter. Later Parks and McClellan [6,7] have developed a particularly useful software interpretation of the Remez method. The user can specify the desired magnitude response in a piecewise-constant fashion over a maximum of 10 contiguous frequency bands. Relative weights could be added to each of these bands.

Least squares (LS) method represents an alternative of 'Parks-McClellan' algorithm. The first definition [1] of this approach is the sum of the squares of the error measured at a finite set of frequency sample points. The second one is the integral of the square of the error over a finite or infinite range of frequencies. There are different modifications of LS idea in the literature [8-12]. Vaidyanathan *et al.* [8] defined a new term 'eigenfilter' - filter, completely constructed according to the LS method, which coefficients are the components of eigenvector of a real, symmetric and positive-definite matrix. Weighted LS approach for design of filters with equiripple passband and stopband is given in [9]. Other different variant of the method is discussed in [10] with minimax passband and LS stopband of the filter.

2.2. Two-dimensional (2-D) FIR filters

Over the years an extensive array of techniques for designing 2-D FIR filters has been accumulated [13-22]. These techniques can be classified into the two categories of *general* and *specialized* design. First category of techniques are intended for approximation of *arbitrary* desired frequency responses, usually with no structural constraints on the filter. They include approaches such as windowing [15] of the ideal impulse response or the use of suitable optimality criteria possibly implemented with iterative algorithms. Methods of the second category are applicable to *restricted classes* of filters. The stopbands and passbands of filters encountered in practice are often defined by straight-line, circular or elliptical boundaries. Specialized design methodologies have been developed for handling these cases.

According to the filter length and symmetric characteristics, there are *four* major types of magnitude response for linear-phase 1-D FIR filters, and they are denoted as Case I, Case II, Case III and Case IV [2]. A similar case exists for quadrantally symmetric linear-phase 2-D FIR filters in which there are *sixteen* possible types of filters. As a whole, the theory for designing 1-D FIR filters can be extended to two or more dimensions. This is true for eigenfilter approach [16,17], minimax design [18,19], frequency-sampling method, and LS approach [20-22].

III. New LS methods for design of FIR filters

3.1 Mathematical basis of the methods

The properties of the two new LS methods are compared in Table 1. Detailed description of these methods is given in [23,24].

	LS Method I	**LS Method II**
FIR filters	1-D	2-D
Frequency response	$H(e^{j\omega})=\sum_{l=0}^{N-1} h(l).e^{-jl\omega}$	$H(e^{j\omega_1},e^{j\omega_2})=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} h(n_1,n_2).e^{-jn_1\omega_1}.e^{-jn_2\omega_2}$
Type of impulse response, length	Symmetrical $h(l)$ N odd (Case I)	Quadrantally symmetrical $h(n_1,n_2)$ N_1, N_2 odd
Impulse response	$h\left(\frac{N-1}{2}+l\right)=h\left(\frac{N-1}{2}-l\right)$ for $1\le l\le (N-1)/2$	$h\left(\frac{N_1-1}{2}-k_1,\frac{N_2-1}{2}-k_2\right)=h\left(\frac{N_1-1}{2}-k_1,\frac{N_2-1}{2}+k_2\right)$ $=h\left(\frac{N_1-1}{2}+k_1,\frac{N_2-1}{2}-k_2\right)=h\left(\frac{N_1-1}{2}+k_1,\frac{N_2-1}{2}+k_2\right)$ for $1\le k_1\le (N_1-1)/2$, $1\le k_2\le (N_2-1)/2$
Amplitude response	$M(\omega)=\sum_{l=0}^{(N-1)/2} c(l).\cos l\omega$ $c(l)=2h((N-1)/2+l)=$ $=2h((N-1)/2-l)$, for $1\le l\le (N-1)/2$ $c(0)=h((N-1)/2)$	$M(\omega_1,\omega_2)=\sum_{n_1=0}^{\frac{N_1-1}{2}}\sum_{n_2=0}^{\frac{N_2-1}{2}} a(n_1,n_2).\cos n_1\omega_1.\cos n_2\omega_2$ $a(n_1,n_2)$ – see [24]

Table 1

3.2 Formulation of LS design problem and solution

Summary of design steps for the two new techniques is given in Table 2. In the first method we have introduced f equally spaced fixed levels in the transition band in order to reduce Gibbs' oscillations. In other words, we redefine the 'standard' LS method with stepwise form of $D(\omega)$ [23]. By analogy, $Q(\omega)$ is extended with f new values corresponding to the levels in transition band.

The second LS method is applied to the special type of 2-D filters, so called 'fan' filters (Fig.1). Desired amplitude response $D(\omega_1,\omega_2)$ is given in Table 2. The contour plot of designed fan filter with $N_1=N_2=17$ is shown in Fig.2.

	LS Method I	LS Method II
Least-mean square error	$E = \int_0^{0.5} Q(\omega)[D(\omega) - M(\omega)]^2 d\omega$ $\omega \in [0,0.5]$ – normalized frequency region	$E = \alpha.\iint_p [D(\omega_1,\omega_2) - M(\omega_1,\omega_2)]^2 d\omega_1 d\omega_2 +$ $+ \beta.\iint_s M^2(\omega_1,\omega_2)\, d\omega_1 d\omega_2 = \alpha E_p + \beta E_S$, p - passband, s - stopband
Desired amplitude response $D(\omega)$	$D(\omega)$– different depending on the type of the filter (lowpass, highpass, bandpass, or bandstop) NEW: Redefinition in transition band	$D(\omega_1,\omega_2) = \begin{cases} 1 & p: 0 \le \omega_1 \le \pi \;,\; \omega_1 \le \omega_2 \le \pi \\ 0 & s: \omega_a \le \omega_1 \le \pi \;,\; 0 \le \omega_2 \le \pi - \omega_a \end{cases}$ Fan 2-D filter (Fig.1)
Weighted function	$Q(\omega)$ NEW: Redefinition in transition band	α, β
System of linear equations	$\sum_{l=0}^{k} d_{n,l}.c(l) = d_{n,k+1}$ $d_{n,l} = \int_0^{0.5} Q(\omega).\cos(2\pi.n\omega).\cos(2\pi.l\omega)d\omega$ $d_{n,k+1} = \int_0^{0.5} Q(\omega).D(\omega).\cos(2\pi.n\omega)d\omega$ $n=0,\dots,k \quad k=(N-1)/2$	$(\alpha.\mathbf{Q} + \beta.\mathbf{R}).\mathbf{a} = \alpha.\mathbf{d}$ $\mathrm{Q} = \iint_p \mathrm{c}(\omega_1,\omega_2).\mathrm{c}^T(\omega_1,\omega_2)d\omega_1 d\omega_2$ $\mathrm{R} = \iint_s \mathrm{c}(\omega_1,\omega_2).\mathrm{c}^T(\omega_1,\omega_2)d\omega_1 d\omega_2$ $\mathrm{d} = \iint_p D(\omega_1,\omega_2).\mathrm{c}(\omega_1,\omega_2)d\omega_1 d\omega_2$ $\mathrm{c}(\omega_1,\omega_2)$ – see [24]

Table 2

The coefficients of the filters in the above discussed methods are obtained by solving a system of linear equations. The absence of iteration procedure is the main advantage of these methods. Also, closed form expressions are derived for the elements of matrices which appear in LS approach [23,24].

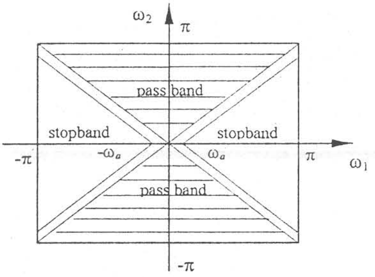

Fig.1 Specification chart of a fan filter

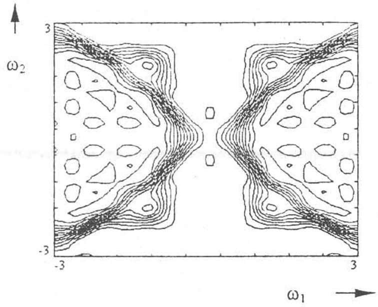

Fig.2 A contour plot for 17 x 17 fan filter with $\omega_a = 0.3\pi$

IV. Conclusion

This article presents a comparative review of different methods for design of 1-D and 2-D FIR linear-phase filters. Least squares technique gives a good alternative of other existing approaches with a lower complexity and an absence of iteration procedure. A mathematical background of two new LS algorithms is compared in Table 1 and Table 2. The second method which is applicable for 2-D filters leads to more complex formulas due to the (ω_1,ω_2) – plane. Nevertheless, these analytical methods enable fast calculation and simplicity compared with other iterative algorithms.

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