OPTIMUM PROCESSING OF SATELLITE NAVIGATION SIGNALS WITH TIME DELAYS SEPARATION

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Summary

A multi-positioning radio-navigation system (MRNS) is synthesized for determining the mobile object coordinates and motion parameters by means of the quasi-range method. The developed here algorithm for radio-navigation signals processing is based on the optimal filtration. The information concerning the time delays of the range-finder code and the signal high frequency filling is used in the synthesized system with multiplex synchronization. The quasi-range MRNS takes into account a disagreement of the on board standard generator with respect to the global system time.

1. Problem formulation

The satellite radio navigation systems (SRNS) as NAVSTAR and GLONASS are of great interest lately. Unfortunately, at present these systems do not provide the required high accuracy in such important navigation tasks such as airplanes landing, river and coastal ship navigation. So, the problem of developing optimal algorithms for a navigation information processing providing higher positioning accuracy is very topical. This can be done owing to the more complete extracting of the information contained in the received from navigation emission sources (ES) signals. Such is the algorithm that makes use of the time delays separation method known as the method of additional variable (MAV). [1, 4]

The signals emitted from K in number ES are observed on board a mobile object (MO). The coordinates of ES toward a preliminarily chosen frame of reference are known.

The state vector $\lambda^T = \left(x, V_x, y, V_y, z, V_z, \Delta, V_\Delta\right)$ includes: the MO coordinates x, y, z; the scale disagreement of the on board generator of the MO with respect to the system time Δ and V_x, V_y, V_z, V_Δ are the respective velocities of x, y, z and Δ.

At the on board receiver's input an additive mixture of a signal and noise $\xi(t)$ is observed

(1) $$\xi(t) = \sum_{k=1}^{K} f_k\left[t - T_k(\lambda)\right] cos\left[\omega_0\left(t - T_{dk}\right)\right] + n(t),$$

where $f_k\left(t - T_k(\lambda)\right)$ is a envelope of the signal emitted from the k-th ES; $\omega_0 = 2\pi f_0$ is a circular frequency of the high-frequency filling (HFF);

$T_k(\lambda)=\tau_k(\lambda)+\Delta$ is a time delay of this envelope; $\tau_k(\lambda)$ is the true time of the mentioned signal; T_{dk} is a time delay of the HFF of this signal; $n(t)$ is a White Gaussian Noise (WGN) with zero mean and the correlation function $M\{n(t)n(t+\tau)\}=\frac{N}{2}\delta(\tau)$; N is a one-sided spectral density of $n(t)$; $\delta(\tau)$ is the delta function.

The amplitudes of the received signals emitted from different ES in the zone of radio-visibility are assumed to be equal.

The state vector λ can be described through Gaussian diffusion Markov process satisfying the system of stochastic differential equations [1]:

(2)
$$\begin{cases} \dot{x}=V_x \\ \dot{V}_x=-\alpha_x V_x+n_x(t) \\ \dot{y}=V_y \\ \dot{V}_y=-\alpha_y V_y+n_y(t) \\ \dot{z}=V_z \\ \dot{V}_z=-\alpha_z V_z+n_z(t) \\ \dot{\Delta}=V_\Delta \\ \dot{V}_\Delta=-\alpha_\Delta V_\Delta+n_\Delta(t) \end{cases}$$

where $n_x(t), n_y(t), n_z(t)$ and $n_\Delta(t)$ are independent WGN with one-sided spectral densities N_x, N_y, N_z, N_Δ and with zero means.

2. Optimal filtration algorithm with time delays separation

A vector of additional variables $T_d^T=(T_{d1},T_{d2},\ldots,T_{dK})$ is introduced here. Its components are the time delays of the HFF of the signals received from each ES of interest. Then the state vector λ is expanded to the new state vector $\lambda_d^T=\{\lambda^T,T_d^T\}$.[1]

By using the MAV and applying of discreet-continuos filtration the equation, expressing the system dynamic behavior takes the form [1]:

(3)
$$\lambda_{d(\nu+1)}=\Phi_d\lambda_{d\nu}+n_{\lambda_{d\nu}}$$

where $\lambda_{d\nu}=\lambda_d(t_\nu)$; $t_\nu=\nu T$; $\nu=1,2,3,\ldots$; Φ_d is the state transition matrix for sampling interval (T) for the expanded state vector λ_d; $n_{\lambda_{d\nu}}$ is a sequence of independent Gaussian random vectors with zero mean $M\{n_{\lambda_{d\nu}}\}=0$ and correlation matrix $\Psi'_d = M\left\{n_{\lambda_{dv}}, n^{l}_{\lambda_{dv}}\right\}$. The elements of the matrices Φ_d and Ψ'_d for $K=4$ are described in [1]. The extended Kalman filter algorithm has the following form [2]

(4)
$$\begin{cases} \hat{\lambda}_{v+1} = \tilde{\lambda}_{v+1} + \sum_{k=1}^{K}\left(\Gamma_{\lambda_k} B_k + \Gamma_{\lambda T_{dk}} B_{dk}\right) \\ \hat{T}_{d(v+1)} = \tilde{T}_{d(v+1)} + \sum_{k=1}^{K}\left(\Gamma_{T_d\lambda_k} B_k + \Gamma_{T_{dk}} B_{dk}\right) \end{cases}$$

where $\tilde{\lambda}_{d(v+1)} = \left\{\tilde{\lambda}^T_{v+1}, \tilde{T}^T_{d\,v+1}\right\}^T = \Phi_d \hat{\lambda}_{dv}$ is the predicted state vector;

$\hat{\lambda}^T_{dv} = \left\{\hat{\lambda}^T_v, \hat{T}^T_{dv}\right\}$ is the estimated state vector;

$$\Gamma_{\lambda_k} = R_{\lambda(v+1)} \frac{\partial T_k\left(\tilde{\lambda}_{v+1}\right)}{\partial \lambda}; \qquad \Gamma_{\lambda T_{dk}} = R_{\lambda T_d(v+1)} \frac{\partial \tilde{T}_{dk(v+1)}}{\partial T_d};$$

$$\Gamma_{T_d\lambda_k} = R_{T_d\lambda(v+1)} \frac{\partial T_k\left(\tilde{\lambda}_{v+1}\right)}{\partial \lambda}; \qquad \Gamma_{T_{dk}} = R_{T_d(v+1)} \frac{\partial \tilde{T}_{dk(v+1)}}{\partial T_d};$$

$$B_k = -\frac{2}{N} \int_{t_v}^{t_{v+1}} \xi(t) \frac{\partial f_k\left(t - T_k\left(\tilde{\lambda}_{v+1}\right)\right)}{\partial t} cos\left[\omega_0\left(t - \tilde{T}_{dk(v+1)}\right)\right] dt;$$

$$B_{dk} = \omega_0 \frac{2}{N} \int_{t_v}^{t_{v+1}} \xi(t) f_k\left(t - T_k\left(\tilde{\lambda}_{v+1}\right)\right) sin\left[\omega_0\left(t - \tilde{T}_{dk(v+1)}\right)\right] dt.$$

On the base of the estimated on each step $(v+1)$ state sub-vectors $\hat{\lambda}_{v+1}, \hat{T}_{d(v+1)}$ the vector $T\left(\hat{x}_{v+1}, \hat{y}_{v+1}, \hat{z}_{v+1}, \hat{\Delta}_{v+1}\right)$ of the time delays of the range-finder code of the signals emitted from all ES of interest can be computed [2]. By using the obtained estimates $T_k\left(\hat{x}_{v+1}, \hat{y}_{v+1}, \hat{z}_{v+1}, \hat{\Delta}_{v+1}\right), \left(k = \overline{1,K}\right)$ a minimization of the expression $\left\{\hat{T}_{dk(v+1)} + r_{k(v+1)} T_0 - T_k\left(\hat{x}_{v+1}, \hat{y}_{v+1}, \hat{z}_{v+1}, \hat{\Delta}_{v+1}\right)\right\}$ varying the number of whole periods $r^*_{k(v+1)}$ of the carrier frequency ($T_0 = 1/f_0$) is performed [2].

Therefore, the time delay of the signal from the k-th ES becomes [2]

(5)
$$T^*_{k(v+1)} = \hat{T}_{dk(v+1)} + r^*_{k(v+1)} T_0.$$

In this way, the lack of uniqueness in measuring the time delay of the HFF is avoided and the higher accuracy of these measurements is utilized.

A final step of the mentioned recursive algorithm uses the estimates of the signals time delays $T^*_{k(\nu+1)}$ $\left(k=\overline{1,K}\right)$ to determinate the estimates $x^*_{\nu+1}, y^*_{\nu+1}, z^*_{\nu+1}, \Delta^*_{\nu+1}$. By statistical averaging of these estimates, their final corrected values are obtained.

3. Algorithm for accuracy estimating

The equation describing the covariance matrix of the filtered errors $\boldsymbol{R}_\nu$ in the extended Kalman filter is given by [1, 4]

(6) $$\boldsymbol{R}^{-1}_{\nu+1}=\widetilde{\boldsymbol{R}}^{-1}_{\nu+1}+\frac{2}{N}\int_{t_\nu}^{t_{\nu+1}}\left[\frac{\partial s\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial(\lambda_d)^T}\right]^T\left[\frac{\partial s\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial(\lambda_d)^T}\right]dt$$

where $\widetilde{\boldsymbol{R}}_{\nu+1}=\Phi_d\boldsymbol{R}_\nu\Phi_d^T+\Psi_d$ is the prediction-error covariance matrix;

$$\boldsymbol{R}_\nu=\begin{vmatrix}\boldsymbol{R}_{\lambda_\nu} & \boldsymbol{R}_{\lambda T_{d\nu}}\\ \boldsymbol{R}_{T_d\lambda_\nu} & \boldsymbol{R}_{T_{d\nu}}\end{vmatrix};\qquad \boldsymbol{R}_{\lambda T_{d\nu}}=\boldsymbol{R}^T_{T_d\lambda_\nu};$$

$\dfrac{\partial s\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\lambda_d}$ is the matrix of the derivatives of the signals from each ES on the components of the state vector λ_d:

$$\frac{\partial s\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\lambda_d}=\begin{vmatrix}\dfrac{\partial s_1\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\lambda} & \dfrac{\partial s_2\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\lambda} & \dots & \dfrac{\partial s_K\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\lambda}\\ \dfrac{\partial s_1\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\boldsymbol{T}_d} & \dfrac{\partial s_2\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\boldsymbol{T}_d} & \dots & \dfrac{\partial s_K\left(t,\widetilde{\lambda}_{d(\nu+1)}\right)}{\partial\boldsymbol{T}_d}\end{vmatrix};$$

$$\frac{\partial s_k\left(t,\widetilde{\lambda}_{\nu+1},\widetilde{T}_{dk(\nu+1)}\right)}{\partial\lambda}=\frac{\partial s_k\left(t,\widetilde{\lambda}_{\nu+1},\widetilde{T}_{dk(\nu+1)}\right)}{\partial T_k(\lambda)}\cdot\frac{dT_k\left(\widetilde{\lambda}_{\nu+1}\right)}{d\lambda}=$$

$$=-\frac{\partial f_k\left(t-T_k\left(\widetilde{\lambda}_{\nu+1}\right)\right)}{\partial t}cos\left[\omega_0\left(t-\widetilde{T}_{dk(\nu+1)}\right)\right]\cdot\frac{dT_k\left(\widetilde{\lambda}_{\nu+1}\right)}{d\lambda};$$

$$\frac{\partial s_k\left(t,\widetilde{\lambda}_{\nu+1},\widetilde{T}_{dk(\nu+1)}\right)}{\partial\boldsymbol{T}_d}=\frac{\partial s_k\left(t,\widetilde{\lambda}_{\nu+1},\widetilde{T}_{dk(\nu+1)}\right)}{\partial T_{dk}}\cdot\frac{d\widetilde{T}_{dk(\nu+1)}}{d\boldsymbol{T}_d}=$$

$$=\omega_0 f_k\left[t-T_k\left(\widetilde{\lambda}_{\nu+1}\right)\right]sin\left[\omega_0\left(t-\widetilde{T}_{dk(\nu+1)}\right)\right]\cdot\frac{d\widetilde{T}_{dk(\nu+1)}}{d\boldsymbol{T}_d};\qquad k=\overline{1,K}.$$

The equation of covariance matrix of the errors takes the form

(7) $$R_{\nu+1}^{-1} = \left(\Phi_d R_\nu \Phi_d^T + \Psi_d\right)^{-1} + H^T V^{-1} H$$

where

$$H = \begin{vmatrix} H_\lambda & O_{(K\times K)} \\ O_{(K\times 8)} & H_{T_d} \end{vmatrix}; \qquad V = \begin{vmatrix} D_\tau E_{(K\times K)} & O_{(K\times K)} \\ O_{(K\times K)} & D_{\tau_d} E_{(K\times K)} \end{vmatrix};$$

$E_{(K\times K)}$ is a unitary matrix containing K-rows and K- columns;

$O_{(K\times K)}$ is a zero matrix containing K-rows and K- columns;

$$H_\lambda = \begin{vmatrix} c^{-1}\cos\alpha_1 & 0 & c^{-1}\cos\beta_1 & 0 & c^{-1}\cos\gamma_1 & 0 & 1 & 0 \\ c^{-1}\cos\alpha_2 & 0 & c^{-1}\cos\beta_2 & 0 & c^{-1}\cos\gamma_2 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c^{-1}\cos\alpha_K & 0 & c^{-1}\cos\beta_K & 0 & c^{-1}\cos\gamma_K & 0 & 1 & 0 \end{vmatrix};$$

$$c^{-1}\cos\alpha_k = \frac{\partial T_k(\lambda)}{\partial x}; \qquad c^{-1}\cos\beta_k = \frac{\partial T_k(\lambda)}{\partial y}; \qquad c^{-1}\cos\gamma_k = \frac{\partial T_k(\lambda)}{\partial z};$$

$$H_{T_d} = \begin{vmatrix} 1 & \dfrac{\partial \tilde{T}_{d1(\nu+1)}}{\partial T_{d2}} & \cdots & \dfrac{\partial \tilde{T}_{d1(\nu+1)}}{\partial T_{d(K-1)}} & \dfrac{\partial \tilde{T}_{d1(\nu+1)}}{\partial T_{dK}} \\ \dfrac{\partial \tilde{T}_{d2(\nu+1)}}{\partial T_{d1}} & 1 & \cdots & \dfrac{\partial \tilde{T}_{d2(\nu+1)}}{\partial T_{d(K-1)}} & \dfrac{\partial \tilde{T}_{d2(\nu+1)}}{\partial T_{dK}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \dfrac{\partial \tilde{T}_{d(K-1)(\nu+1)}}{\partial T_{d1}} & \dfrac{\partial \tilde{T}_{d(K-1)(\nu+1)}}{\partial T_{d2}} & \cdots & 1 & \dfrac{\partial \tilde{T}_{d(K-1)(\nu+1)}}{\partial T_{dK}} \\ \dfrac{\partial \tilde{T}_{dK(\nu+1)}}{\partial T_{d1}} & \dfrac{\partial \tilde{T}_{dK(\nu+1)}}{\partial T_{d2}} & \cdots & \dfrac{\partial \tilde{T}_{dK(\nu+1)}}{\partial T_{d(K-1)}} & 1 \end{vmatrix};$$

$\cos\alpha_k, \cos\beta_k, \cos\gamma_k$ are direction cosines of the k-th ES $\left(k = \overline{1,K}\right)$; $\boldsymbol{H}$ is a matrix of the derivatives of the measured parameters $T_k(\lambda)$ and T_{dk} on the estimated parameters λ_d ($\boldsymbol{H}$ determines the system geometry); V is a covariance matrix of the errors of the measured parameters $T_k(\lambda)$ and T_{dk}; D_τ and D_{τ_d} are variances of the errors of the estimate of the time position respectively of the range-finder code and of the HFF of the signal emitted from each ES

$$D_\tau = \frac{N}{2}\left\{\int_{t_N}^{t_{N+1}}\left[\frac{\partial f(t)}{\partial t}\right]^2 \left[cos(\omega_0 t)\right]^2 dt\right\}^{-1}; \quad D_{\tau_d} = \frac{N}{2\omega_0^2}\left\{\int_{t_N}^{t_{N+1}}\left[f(t)\right]^2 \left[sin(\omega_0 t)\right]^2 dt\right\}^{-1}.$$

At the integration above the member, containing double frequency $2\omega_0$ is ignored. The expressions for D_τ and D_{τ_d} take the form [3]

$$D_\tau = \frac{\tau_e t_\phi}{2q}; \qquad D_{\tau_d} \approx \frac{1}{q\omega_0^2}$$

where q is signal-to-noise ratio: τ_e is a time-duration of one element of the range-finder code of the received signal; t_ϕ is a time-duration of the transition process when the state of the range-finder code changes from +1 to -1 or conversely.

Conclusion

The algorithm of optimal processing with time delays separation is represented in the paper. The algorithm for accuracy estimating of the filtration with time delays separation is developed.

The receiver based on the synthesized above algorithm provides higher accuracy in comparison with the conventional one that provide measurements of the time delays only by using the range-finder code. The increase of the positioning accuracy is achieved through more complete utilization of the information, which is contained in the received radio-signal.

References

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