INFLUENCE OF DELAY-FREE-LOOP ELIMINATION ON THE TRANSFER FUNCTION SENSITIVITY OF DIGITAL FILTERS

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Abstract. Some results concerning the elimination of delay-free loops in the structures of digital filters are obtained. On the base of them formulae for simplified determination of first-order transfer function sensitivities of the of equivalently transformed digital filter structure are derived.

1. Introduction

From the digital signal processing theory it is well known that the existence of delay-free loops in the structure of a digital filter makes its practical realisation impossible [1]. That is why one of specific problems that arise in the synthesis of digital filters is the elimination of eventual obtained initially delay-free loops. Because the structure of a digital filter normally is a signal-flow graph, the elimination procedure can be implemented according to the rules for equivalent transformations of signal-flow graphs [2]. These transformations however do not influence the graph transfer function but change the filter first-order (and higher-order too) transfer-function sensitivities. Because the sensitivity is very important from practical point of view parameter and its calculation is necessary in every transformation step.

In the paper presented some results concerning the simplified determination of first-order sensitivity of digital filters are given. For this purpose the attention is concentrated on the elimination of loops in the signal-flow graph representation of the filter transfer function. We suppose that the signal-flow graph transfer-function sensitivities with respect to the transmission coefficients of the incident to such a loop edges are determinate on the previous transformation step. Then the sensitivities with respect to the transmission coefficients of the obtained after the equivalent transformation new edges can be directly calculated in a way similar to the one described in [4]:

2. Equivalent Transformation of a Signal-Flow-Graph by Loop Elimination

Let us assume that the signal-flow graph G is given and that it contains the subgraph SG . In the case when G describes a digital filter structure some of its edge transmission coefficients depend on the complex variable z^{-1}. Further, for simplicity, we will denote all coefficients briefly as t_{ji} .

The subgraph SG includes one or more incident loops and it is connected with the remaining part of G by incoming and outgoing edges.

According to the conditions given above, we can form the sets:

(1) $\alpha = \{1,2,\dots,M\};\ \beta = \{M+1, M+2,\dots,N\};\ \gamma = \{N+1, N+2,\dots,P\}$

from the numbers of the input vertices of G, from the numbers of the mixed and output vertices of G and from the numbers of these vertices of SG that are incident to its loop(s), respectively.

Obviously we have

(2) $(\alpha \cup \beta) \cap \gamma = \varnothing$.

If we number the edges of G starting from $P+1$ until V one obtains the edge sets:

(3) $\delta = \{P+1, P+2,\dots,Q\};\varepsilon = \{Q+1, Q+2,\dots,R\};\theta = \{R+1, R+2,\dots,S\};$
$\psi = \{S+1, S+2,\dots,T\};\varphi = \{T+1, T+2,\dots,U\};\mu = \{U+1, U+2,\dots,V\}$

from the numbers of edges going out from the vertices of α and directed towards the vertices of γ, from the numbers of edges going out from the vertices of β and directed towards the vertices of γ, from the numbers of edges going out from the vertices of γ and directed towards the vertices of β, from the numbers of edges going out from the vertices of α and directed towards the vertices of β, from the numbers of edges between the vertices of β, from the numbers of edges between the vertices of γ, respectively.

One forms the vectors of vertice signals:

(4) $\mathbf{x}_\alpha = [x_1 \dots x_M]_t;\mathbf{x}_\beta = [x_{M+1} \dots x_N]_t;\mathbf{x}_\gamma = [x_{N+1} \dots x_P]_t$.

For $V - U = v$ it holds

(5) $\begin{cases} U - T = u; T - S = t; S - R = s; R - Q = r; \\ Q - P = q; P - N = p; N - M = n; M = m . \end{cases}$

The elements of the sets δ , ε , θ, ψ, φ and μ correspond to edge transmission coefficients which can be arranged in the transfer matrices $\mathbf{T}_{\gamma\alpha}$ $(p \times m)$; $\mathbf{T}_{\gamma\beta}$ $(p \times n)$; $\mathbf{T}_{\beta\gamma}$ $(n \times p)$; $\mathbf{T}_{\beta\alpha}$ $(n \times m)$; $\mathbf{T}_{\beta\beta}$ $(n \times n)$; $\mathbf{T}_{\gamma\gamma}$ $(p \times p)$, respectively.

Then the matrix equations are valid

(6) $\mathbf{x}_\gamma = \mathbf{T}_{\gamma\alpha}\mathbf{x}_\alpha + \mathbf{T}_{\gamma\beta}\mathbf{x}_\beta + \mathbf{T}_{\gamma\gamma}\mathbf{x}_\gamma\ ;\ \mathbf{x}_\beta = \mathbf{T}_{\beta\alpha}\mathbf{x}_\alpha + \mathbf{T}_{\beta\beta}\mathbf{x}_\beta + \mathbf{T}_{\beta\gamma}\mathbf{x}_\gamma$.

From here we obtain

(7) $\mathbf{x}_\gamma = \left(\mathbf{1} - \mathbf{T}_{\gamma\gamma}\right)^{-1}\left(\mathbf{T}_{\gamma\alpha}\mathbf{x}_\alpha + \mathbf{T}_{\gamma\beta}\mathbf{x}_\beta\right)$

and after substituting this result in (6) it follows

(8) $\mathbf{x}_\beta = \mathbf{T}_{\beta\alpha}\mathbf{x}_\alpha + \mathbf{T}_{\beta\beta}\mathbf{x}_\beta + \mathbf{x}'_\beta$,

where

(9) $\mathbf{x}'_\beta = \mathbf{T}_{\beta\gamma}\left(\mathbf{1} - \mathbf{T}_{\gamma\gamma}\right)^{-1}\mathbf{T}_{\gamma\alpha}\mathbf{x}_\alpha + \mathbf{T}_{\beta\gamma}\left(\mathbf{1} - \mathbf{T}_{\gamma\gamma}\right)^{-1}\mathbf{T}_{\gamma\beta}\mathbf{x}_\beta$.

Our main task hereafter is to find the direct relationships between the vertex signals of α and β which means an elimination the loop(s) in the subgraph SG. For this purpose it is necessarily to analyse the transfer matrix products in (9) as follows:

The elements of the matrix $\mathbf{T}_{\gamma\gamma}$ are only the transmission coefficients of the loop edges in SG . Every row (column) of $\mathbf{T}_{\gamma\gamma}$ contains only on nonzero element when the subgraph SG contains only one loop. In the case when SG contains several but touched loops, the number of nonzero elements in every row (column) may be greater then one.

The matrix multiplier $(\mathbf{1} - \mathbf{T}_{\gamma\gamma})^{-1}$ which takes part in both terms of (9) is in fact a transfer matrix and it describes the relationships between the signals in $\mathbf{x}_\gamma$. The common denominator of all elements of this matrix is the determinant of SG .

(10) $\Delta = det\left(\mathbf{1} - \mathbf{T}_{\gamma\gamma}\right).$

From the other hand, the numerators of these elements are the path transmission coefficients between the corresponding vertices of the subgraph SG .

The expression of the determinant (10) can be written according to the well known Mason"s formula [2] and all paths mentioned above can be detected by inspection.

Having in mind these results and the relationships (9) we reach to the following simple

Rule

The subgraph SG in G can be replaced by a set of edges between the vertices belonging to the set α and the set β. This procedure removes simultaneously all vertices belonging to the set γ.

The transmission coefficient of every new edge between the vertices of set α and set β is the expression of the kind

(11) $t_{ji} = \dfrac{P_{ji}\Delta_{ji}}{\Delta}\ ;\ i \in \alpha \cup \beta\ ;\ j \in \beta\ ,$

where P_{ji} is the transmission coefficient of the paths between the vertices i and j , Δ_{ji} is the adjunct of SG for P_{ji} and Δ is the determinant of the subgraph SG .

When we concern a selfloop of SG the so formulated rule coincides with the rule for the removing a mixed incident to the selfloop vertex [2].

One illustrates the application of this rule by the example given below.

Example

A signal-flow graph G with a subgraph SG is shown in Fig. 1. We wont to remove the loop of the subgraph SG .

Here it holds

$\alpha = \{1,2\}; \beta = \{3,4,5,6\}; \gamma = \{7,8,9\}; m = 2; n = 4; p = 3;$
$\delta = \{t_{92}\}; \varepsilon = \{t_{73}, t_{96}\}; \theta = \{t_{58}\}; \psi = \{t_{31}, t_{32}, t_{41}, t_{62}\};$
$\varphi = \{t_{34}, t_{45}, t_{56}\}; \mu = \{t_{79}, t_{87}, t_{98}, t_{89}\};$
$q = 1; r = 2; s = 1; t = 4; u = 3; v = 4.$

The transmission coefficients of the all paths between of the sets $\alpha \cup \beta$ and γ are

$P_{52} = P_{52}' + P_{52}'' = t_{92}t_{79}t_{87}t_{58} + t_{92}t_{89}t_{58}; P_{53} = t_{73}t_{87}t_{58};$

$P_{56} = P_{56}' + P_{56}'' = t_{96}t_{89}t_{58} + t_{96}t_{79}t_{87}t_{58}$

and the determinant Δ of the subgraph SG is

$\Delta = 1 - t_{79}t_{87}t_{98} - t_{89}t_{98}.$

All adjuncts of the founded paths equal 1 because there are no untouching loops

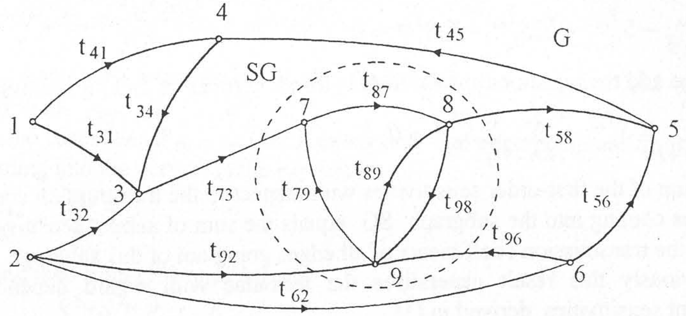

Fig.1

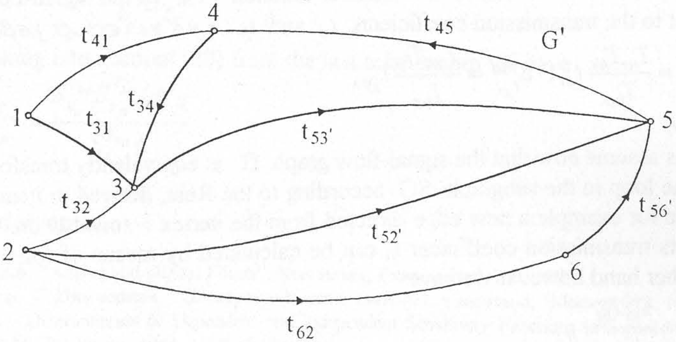

Fig.2

of G to the paths P_{52}, P_{53} and P_{56}.

Consequently we have

$t_{53} = P_{53} / \Delta; \; t_{52} = P_{52} / \Delta; \; t_{56}' = t_{56} + P_{56} / \Delta.$

The equivalently transformed signal-flow graph G' is shown in Fig. 2.

3. Expressing of First-Order Transfer Function Sensitivities of Equivalently Transformed by Loop Elimination Signal-Flow Graphs

Let us consider again the signal-flow graph G . Our task here will be to find some relationships between the signal-flow-graph transfer function sensitivities with respect to the edge transmission coefficients of the sets δ , ε , θ and of the new set

(12) $\nu = \{V+1, V+2, \dots, W\}$,

which corresponds to the edges obtained after removing the subgraph SG .

First we suppose that the subgraph SG contains only one loop and the vertices that are incident to this loop. According to [3] for every vertex l , $l \in \gamma$, the algebraic sum of the signal-flow-graph transfer function first-order sensitivities with respect to the transmission coefficients incident to this vertex equals zero. Then for $a \in \alpha;\ b \in \beta$ we can write

(13) $$S_{t_{l+1,l}}^{T_{ba}} - S_{t_{l-1,l}}^{T_{ba}} + \sum_{l} S_{t_{li}}^{T_{ba}} - \sum_{l} S_{t_{jl}}^{T_{ba}} = 0 .$$

If we add the equations similar to (13) for all vertices of SG , it follows

(14) $$\sum_{l=N+1}^{P} \sum_{i} S_{t_{li}}^{T_{ba}} - \sum_{l=N+1}^{P} \sum_{j} S_{t_{jl}}^{T_{ba}} = 0$$

- i.e. the sum of the first-order sensitivities with respect to the transmission coefficients of all edges coming into the subgraph SG equals the sum of similar sensitivities with respect to the transmission coefficients of all edges going out of this subgraph.

Obviously this result generalises the formulae with regard dependent and independent sensitivities, derived in [3].

The first-order sensitivities of a transfer function T_{ba} of the signal-flow graph with respect to the transmission coefficients t_{xi} and t_{jy} ; $x,y \in \gamma$; $i \in \alpha \cup \beta$; $j \in \beta$ are

(15) $$S_{t_{xi}}^{T_{ba}} = \frac{T_{ia}T_{bx}}{T_{ba}} t_{xi}; \quad S_{t_{jy}}^{T_{ba}} = \frac{T_{ya}T_{bj}}{T_{ba}} t_{jy},$$

respectively.

Let us assume now that the signal-flow graph G is equivalently transformed by removing the loop in the subgraph SG according to the Rule, derived in item 1. As a consequence for example a new edge directed from the vertex i towards the vertex j arises and its transmission coefficient t_{ji} can be calculated by means of formula (11). From the other hand however we have

(16) $$S_{t_{ji}}^{T_{ba}} = \frac{T_{ia}T_{bj}}{T_{ba}} t_{ji} .$$

By expressing the partial transfer functions T_{ia} and T_{bj} from (15) and after substituting them in (16) one obtains

(17) $$S_{t_{ji}}^{T_{ba}} = \frac{S_{t_{xi}}^{T_{ba}} S_{t_{jy}}^{T_{ba}} t_{ji} T_{ba}}{T_{bx} T_{ya} t_{xi} t_{jy}} .$$

Taking into account (11) we find

$$(18)\quad t_{ji} = \frac{t_{xi}P_{ji}t_{jy}\Delta_{ji}}{\Delta},$$

where P_{ji} is the transmission coefficient of the path between the vertices i and j (in the common case this path includes more than one consequent edges of the loop in SG and then the expression (17) takes the form

$$(19)\quad S_{t_{ji}}^{T_{ba}} = \frac{S_{t_{xi}}^{T_{ba}} S_{t_{jy}}^{T_{ba}} P_{yx} T_{ba} \Delta_{ji}}{T_{bx} T_{ya} \Delta}.$$

where P_{yx} is the transmission coefficient of this part of the path P_{ji} which is between the vertices y and x of SG. In addition for the loop transmission coefficient we can write

$$(20)\quad L = P_{yx}P_{xy} = \frac{P_{ji}P_{xy}}{t_{ia}t_{bj}},$$

where P_{xy} is the path transmission coefficient between the vertices x and y along the examined loop.

In the case when $P_{xy} = t_{xy}$ (i.e. it exists only one edge going out from the vertex y and coming into the vertex x) the expression

$$(21)\quad S_{t_{xy}}^{T_{ba}} = \frac{T_{bx}T_{ya}}{T_{ba}} t_{xy}$$

is valid and then from (19) one obtains

$$(22)\quad S_{t_{ji}}^{T_{ba}} = \frac{S_{t_{xi}}^{T_{ba}} S_{t_{jy}}^{T_{ba}} P_{yx} t_{xy} \Delta_{ji}}{S_{t_{xy}}^{T_{ba}} \Delta}.$$

Taking into account (20) from the last relationship we have

$$(23)\quad S_{t_{ji}}^{T_{ba}} = \frac{S_{t_{xi}}^{T_{ba}} S_{t_{jy}}^{T_{ba}} L \Delta_{ji}}{S_{t_{xy}}^{T_{ba}} \Delta}.$$

References

1. H.I-F.Lam, "Analog and Digital Filters", New Jersey, Prentice-Hall, Inc., 1979
2. С. Мэзон, Г. Циммерман, "Электронныe цепи, сигналы, и системы, "Москва, ИЛ, 1963
3. C. Acar, "Determination of Dependent and Independent Sensitivity Functions in Signal-Flow Graphs", ECCTD 81, The Hague, 1981, 1039-1042
4. G.A.Nenov, "On Some Equivalent Transformations of Signal-Flow Graph and Related First-Order Transfer Function Sensitivities", Third IEEE International Conference on Electronics, Circuits and Systems, Rhodes, 1996, Greece

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