DIGITAL FILTERS DESIGN WITH ARBITRARY MAGNITUDE AND PHASE SPECIFICATIONS

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Abstract: In many applications filters are required to satisfy certain amplitude specifications as well as to approximate linear-phase and/or constant group delay in the passband. This paper is dealing with the problem of digital filters design when certain amplitude and phase specifications are prescribed. Comparative review of existing design methods and criteria for synthesizing filters with simultaneously magnitude and phase constraints is given. The problem under consideration has been solved using different approximation methods for design of FIR and IIR filters. Several aspects of these approaches are considered and compared. Also, some new results concerning the design of recursive filters with constant group delay and Chebyshev attenuation are presented.

I. INTRODUCTION

Designing recursive digital filters satisfying both arbitrary loss and delay constraints is a classical problem. There is no known general analytical solution that can yield coefficient values. A number of papers have appeared in technical literature concerning the subject of linear-phase (or approximately linear-phase) digital filters. This problem has been solved using different methods and approximation criteria. Herewith presented paper gives a comprehensive review of such methods. In addition, some new results are shown after generalization and extension of Unbehauen's approach [4,6].

II. DESCRIPTION OF THE PROBLEM AND REVIEW

The design of **one-dimensional (1-D) IIR filters** with both maximally flat and Chebyshev group delay has been studied by Thiran and others [1, 2, 3]. In all these cases, the magnitude response is monotonic and therefore not highly selective for a given order. The work was extended to cover Chebyshev stopband attenuation by Unbehauen [4, 5, 6] and by Maria and Fahmy [7]. Also, an optimization procedure has been used to solve the problem by Deczky, Saramaki, Neuvo [8, 9]. This procedure leads to equiripple magnitude and group delay with minimized number of multiplications [9].

It is well known that IIR filters satisfying the desired amplitude and phase specifications can be designed in two parts [4, 10]: first, the denominator (IIR component) is determined to satisfy the phase requirements, than the mirror-image or antimirror-image numerator (linear-phase FIR component) is designed to achieve the magnitude requirements of the filter. The same approach is used in [10, 11]. Other investigations in this direction to design of different types of IIR filters (lowpass, highpass, bandpass, and bandstop) are reported also in [12].

Thajchayapong [13, 14] extended the capability of the indirect technique to provide the results equivalent to those of the direct techniques considered above. He proved that using bilinear transformation and known methods in the *s*-analog domain, it is also possible to derive numerator polynomial with order larger than that of denominator. This property was shown at first by Unbehauen [4].

New iterative least squares approach is given in [15, 16] based on the complex Chebyshev technique in the frequency domain. An open problem here is the convergence of the algorithm and stability of the resulting filter. The guarantee of stability in the case of a general phase characteristic is a problem to be addressed in the future. Other interesting methods are considered in [17, 18]. For example, Abo-Zahhad [18] designed selective bandpass IIR filter interpolating linear-phase and constant group delay. This is done by approximating the phase characteristic to be linear and its derivatives to be zero at a set of frequencies in the passband. As a result, the passband amplitude characteristic is equiripple, the stopband loss characteristic is maximally flat.

In recent years a variety of techniques for the design of **two-dimensional (2-D) IIR filters** that approximate not only the prescribed magnitude response but also a linear-phase characteristic have been developed [19-23]. Many applications in signal processing require 2-D digital filters with circular, elliptical or fan shaped passband magnitude response. It is more desirable to use IIR instead of FIR filters in view of computational efficiency and significant reduction in the number of multipliers. Several authors pointed out that the phase of a 2-D IIR filter used for restoration of images is an important factor. Among the commonly used approaches are these based on: (i) the application of transformations of 1-D filter function to generate the corresponding 2-D IIR filters, and (ii) use of H^{∞} optimization theory and computer-aided optimization. The first category includes the use of McClellan Transformation and DST (Digital Spectral Transformation). It is known [24, 25, 26] that some DST preserve the group delay response of the 1-D prototype filter when it is mapped to the 2-D plane. So, a 1-D IIR filter which has a maximally flat magnitude and simultaneously a constant group delay response in its passband yields to a 2-D IIR filter with the same kind of magnitude and group delay response in the passband region [25]. Other different 2-D methods are presented by Unbehauen *et al* [19, 20], Ahmadi *et al* [21] for design of 2-D IIR separable-denominator (SP) digital filters.

III. APPROXIMATION TECHNIQUE

3.1 Constant group delay.

In this section it will be shown [4] how the problem of realizing a constant group delay can be reduced to the task of rational approximation. We consider digital all-pole filter with a transfer function:

$$H(z) = \frac{z^m}{a_0 + a_1 z + \ldots + a_m z^m},$$

where the coefficients a_μ have to be adjusted in such a way that the phase $\theta(\omega)$, $0 \le \omega \le \pi/T$, approaches a prescribed function $\theta_0(\omega)$ as close as possible. An ideal phase function $\theta_0(\omega) = \tau.\omega$ with constant group delay $\tau = \text{const} > 0$. Here ω is the variable radian frequency and T the time period of the digital filter.

Replacing ω with $W=\cos(\omega T/2)$, it was proved [4] :

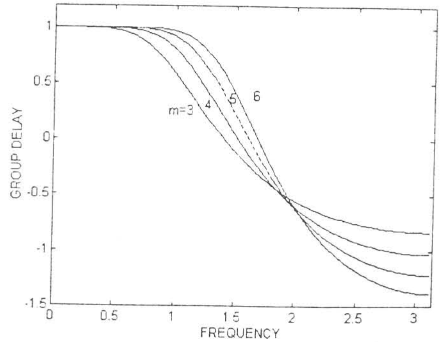
$$\frac{k.\prod_{\mu=1}^{m}(W - X_{2\mu-1})}{\prod_{\mu=1}^{m-1}(W - X_{2\mu})} = \sqrt{1-W^2}.\cot\left[\theta_0\left(\frac{2}{T}\cos^{-1}W\right) + m.\cos^{-1}W\right],$$

where $-1 \le W \le 1$. The left-hand function is approximating odd rational function $f_m(W)$ with parameters $X_{2\mu-1}$, $X_{2\mu}$, and k introduced in [4]. After some calculations and replacements we obtain the following function:

$$f(W) = \sqrt{1-W^2}.\cot\left[q.\tan^{-1}\frac{\sqrt{1-W^2}}{W}\right], \quad q = 2\tau/T + m,$$

which must be approximated by the function $f_m(W)$. According to Perron [27] $f(W)$ can be represented by a continued fraction and its m-th convergent is :

$$f_m(W) = \frac{W}{q}\left[1 + \frac{(q^2-1)\,\frac{W^2-1}{W^2}|}{|3} + \ldots + \frac{\{q^2-(m-1)^2\}\,\frac{W^2-1}{W^2}|}{|(2m-1)}\right].$$



It can be graphically determined that increasing the order m, the group delay becomes more flat in the passband of the filter. From the theory of continued fractions [27] function $\theta(\omega)$ of the resulting $H(z)$ approaches the ideal phase $\theta_0(\omega)$ at $\omega=0$ in the maximally flat sense.

Fig.1 Dependence of the group delay from degree m for a lowpass filter with $\tau/T=1$

3.2. Equiripple magnitude condition.

In this section our task is to summarize and extend the approach given in [4] with application of different squared-magnitude functions $Q(\zeta)$ [6]. We show that this idea works not only for lowpass but also for highpass, bandpass, and bandstop filters. For this purpose, the numerator of an all-pole transfer function will be substituted by a mirror-image polynomial which does not affect the group delay and guarantees equiripple magnitude in a prescribed stopband $\omega_c \le \omega \le \pi / T$.

$$Q(z) = H(z)H(z^{-1}) = \frac{1}{(a_0 + a_1 z + ... + a_m z^m)(a_0 + a_1 z^{-1} + ... + a_m z^{-m})}$$

and after transformation $\overline{w} = (z + z^{-1})/2$ we obtain:

$$Q(\overline{w}) = \frac{1}{B_0 + B_1 \overline{w} + ... + B_m \overline{w}^m}, \quad \text{where } Q(e^{j\omega T}) = \left|H(e^{j\omega T})\right|^2.$$

Then, two transformations of the frequency are applied consecutively : $\overline{w} = \alpha . w + \beta$ and $w = (\zeta + \zeta^{-1})/2$. Parameters α and β are selected so that the interval in which the squared-magnitude function must approximate a constant value in the equiripple sense, is stretched into the range $-1 \le w \le 1$ [28]. We apply two functions $Q(\zeta)$ [6]:

$$\text{Case 2a}: \; Q(\zeta) = \frac{(1+\varepsilon)P_s^2(\zeta) - P_a^2(\zeta)}{P_s^2(\zeta) - P_a^2(\zeta)}; \qquad \text{Case 2b}: \; Q(\zeta) = \frac{P_s^2(\zeta) - (1+\varepsilon)P_a^2(\zeta)}{P_s^2(\zeta) - P_a^2(\zeta)},$$

where $\varepsilon\,(<1)$ is a positive constant corresponding to the stopband ripple (Fig.2).

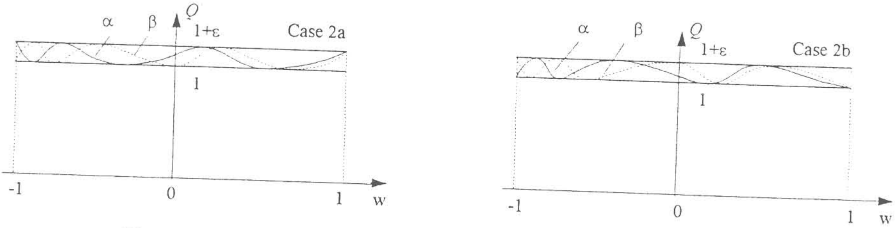


Fig.2 Squared-magnitude function Q with equiripple behavior in the range [-1,1]

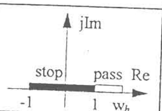
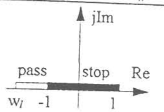
Type	$Q(\zeta)-1$	w - plane	$w_h, \zeta_h, w_l, \zeta_l$	α, β
Lowpass Case 2aβ	$\frac{\varepsilon . P_s^2(\zeta)}{P_s^2(\zeta)-P_a^2(\zeta)}$		$w_h = \frac{3-\overline{w}_c}{1+\overline{w}_c} > 1$	$\alpha = \frac{\overline{w}_c+1}{2} > 0$
Lowpass Case 2aα			$\zeta_h = w_h + \sqrt{w_h^2-1}$	$\beta = \frac{\overline{w}_c-1}{2} < 0$
Highpass Case 2bβ	$\frac{(-\varepsilon).P_a^2(\zeta)}{P_s^2(\zeta)-P_a^2(\zeta)}$		$w_l = \frac{-3-\overline{w}_c}{1-\overline{w}_c} < -1$	$\alpha = \frac{1-\overline{w}_c}{2} > 0$
Highpass Case 2aα	$\frac{\varepsilon . P_s^2(\zeta)}{P_s^2(\zeta)-P_a^2(\zeta)}$		$\zeta_l = w_l + \sqrt{w_l^2-1}$	$\beta = \frac{1+\overline{w}_c}{2} > 0$

Table1

Values of the parameters α and β for lowpass and highpass filters are given in Table1. All necessary frequency mappings $z \rightarrow \overline{w} \rightarrow w \rightarrow \zeta$ are graphically shown in [28]. Some graphical results from application of the method are shown in Fig.3 (highpass filters).

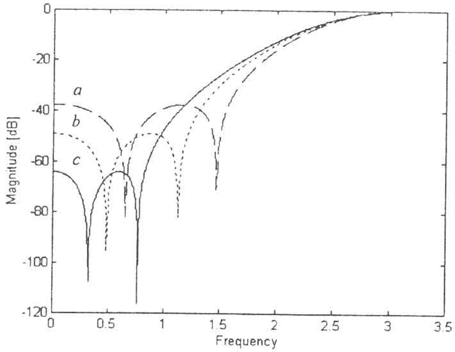


Fig. 3. **Case 2aα** amplitude response with m=3 and numerator degree 4:
a) ε=0.1 and $\omega_c=\pi/2$;
b) ε=0.07 and $\omega_c=\pi/2.6$;
c) ε=0.05 and $\omega_c=\pi/3.8$.

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