

# ACCURACY CHARACTERISTICS OF SATELLITE NAVIGATION SYSTEM WITH MULTIPLEX SYNCHRONIZATION

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## Summary

A quantitative analysis of the positioning accuracy of the algorithm for processing of satellite navigation signals with multiplex synchronization is done. The filtration with multiplex synchronization of the signals time delays of the envelope and the signals carrier is compared to the conventional algorithm without taking into account of the time delay of the carrier frequency. The obtained results represent the superiority in the positioning accuracy of multiplex synchronization method.

## Introduction

An increasing of positioning accuracy of a mobile object (MO) with the help of satellite radionavigation systems can be achieved owing to more completely extracting of the information from high frequency structure of the received signal. With applying differential mode of working of NAVSTAR satellite navigation system and using C/A-code combined with closed system for tracking of the phase of the signal carrier frequency considerably higher accuracy is achieved that is typical of P-code.

## Accuracy characteristics

The frame of reference  $O_0XYZ$  is rectangular and has an origin  $O_0$  that is fixed with respect to the Earth and lies on the ground surface. The axis  $O_0Y$  is oriented upward in the local vertical line, the axes  $O_0X$  and  $O_0Z$  lie on the horizontal plane.

A quantitative estimating of the accuracy characteristics of optimal processing of signals received from four satellites of NAVSTAR system is carried out. The NAVSTAR system is assumed to work in differential mode. The estimate of the accuracy is done for multiplex synchronization of the signals time delays (MSTD) in accordance to the algorithm represented in [1]. The accuracy characteristics of two processing algorithms are compared: 1) with using multiplex synchronization; 2) filtration without multiplex synchronization (FWMS) of the signals time delays. The signal-to-noise ratios ( $q$ ) for the signals received from all emission sources (ES) are assumed to be equal.

The computations are carried out for  $q = 1 \div 100$ ;  $T = 0,1$  s; and typical values of the parameters of motion model of MO [1]:  $\sigma_V = \sigma_{V_{x,y,z}} = 0,1 \div 10$  m/s;  $\alpha = \alpha_{x,y,z} = 0,1 \div 0,001$  Hz; model of the time disagreement of the supporting generator (SG) on board the MO with respect to the system time of NAVSTAR [1]:  $\sigma_{V_{\Delta}} = D_f/f = 10^{-6} \div 10^{-10}$ ;  $\alpha_{\Delta} = 0,1 \div 0,001$  Hz; carrier frequency of the received signal  $f_0 = \omega_0/2\pi = 1575,42$  MHz.

The rest initial data which are not shown for fig. 1 ÷ 18 has following values:  $q=100$ ,  $\sigma_V = \sigma_{V_{x,y,z}} = 1$  m/s;  $\alpha = \alpha_{x,y,z} = 0,1$  Hz;  $\sigma_{V_{\Delta}} = D_f/f = 10^{-8}$ ;  $\alpha_{\Delta} = 0,01$  Hz, initial values of standard deviation of the estimating parameters:  $\sigma_{x,y,z=0} = 300$  m;  $\sigma_{V_{x,y,z=0}} = 300$  m/s;  $\sigma_{\Delta 0} = 10^{-6}$  s;  $\sigma_{V_{\Delta 0}} = 10^{-6}$ ; time-duration of the transition process when the state of the range-finder code changes from +1 to -1 or conversely  $t_{\Phi} = \frac{\tau_e}{4}$  where  $\tau_e$  is a time-duration of one element of the range-finder code. The computations are carried out for Geometric Dilution of Precision:  $GDOP = 3,95$ .

Fig. 1 ÷ 18 represent the results of the computations. Fig. 1 ÷ 10 shows the time diagrams of the maximal error ( $2\sigma$ ) of the main estimating parameters  $x$ ,  $y$  and  $\Delta$  for C/A- and P-code for different signal-to-noise ratios  $q$  and different models parameters:  $\alpha, \alpha_{\Delta}, \sigma_V$  и  $\sigma_f/f$  [1]. Fig. 11 ÷ 18 represents the accuracy characteristics of NAVSTAR system in the case of applying multiplex synchronization one hour after starting the algorithm.

Fig. 1 and fig. 5 represent the accuracy characteristics of the coordinates  $x$ ,  $y$  at MSTD and at algorithm without time delays separation for different signal noise ratios for C/A-code. Fig. 4 and fig. 8 shows the accuracy characteristics for P-code. With the increasing of the signal-to-noise ratio from 1 to 10 and 100 the positioning accuracy of the MO rises respectively 2 ÷ 2,3 and 4,6 ÷ 5,5 times for filtration without time delays separation and for MSTD 3,1 ÷ 3,2 and 9,8 ÷ 10 times.

Fig. 2 and fig. 6 represent the accuracy characteristics of the coordinates for different values of the parameter  $\alpha$  for the C/A-code. With decreasing of  $\alpha$  from 0,1 Hz to 0,01 Hz and 0,001 Hz the accuracy of positioning of MO raises respectively 1,2 ÷ 1,3 and 1,6 ÷ 1,7 times for FWMS but for MSTD the variation of  $\alpha$  in the frame of the interval 0,1 ÷ 0,001 Hz has no influence on the accuracy characteristics of the system.

Fig. 3 and fig. 7 represent the positioning accuracy characteristics for different values of the parameter  $\sigma_V$  for the C/A-code. With decreasing of  $\sigma_V$  from 10 m/s to 1 m/s and 0,1 m/s the accuracy of positioning of the MO rises respectively 1,6 ÷ 1,8 and 2,9 ÷ 3,9 times for filtration without the time delays separation and

for MSTD the variation of  $\sigma_f$  in the frame of the interval  $\sigma_f = 0,1 \div 10$  m/s has no influence on the accuracy characteristics of the system.

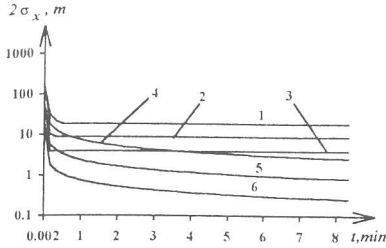


Fig. 1. Horizontal error ( $2\sigma_x$ ) for C/A-code:  
 1, 2, 3 -FWMS; 4, 5, 6 -MSTD;  
 1, 4 -  $q = 1$ ; 2, 5 -  $q = 10$ ; 3, 6 -  $q = 100$

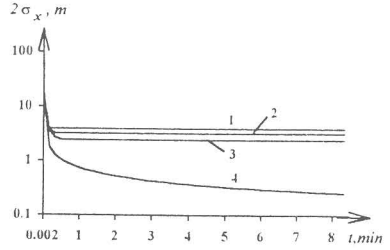


Fig. 2. Horizontal error ( $2\sigma_x$ ) for C/A-code:  
 1, 2, 3 -FWMS; 4 - MSTD  $\alpha = 0,1 \div 0,001$  Hz;  
 1 -  $\alpha = 0,1$  Hz; 2 -  $\alpha = 0,01$  Hz; 3 -  $\alpha = 0,001$  Hz

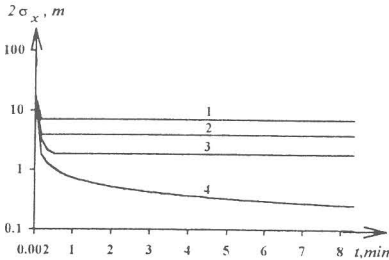


Fig. 3. Horizontal error ( $2\sigma_x$ ) for C/A-code:  
 1, 2, 3 -FWMS; 4 - MSTD  $\sigma_f = 0,1 \div 10$  m/s;  
 1 -  $\sigma_f = 10$  m/s; 2 -  $\sigma_f = 1$  m/s; 3 -  $\sigma_f = 0,1$  m/s

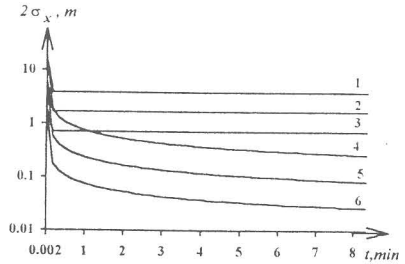


Fig. 4. Horizontal error ( $2\sigma_x$ ) for P-code:  
 1, 2, 3 -FWMS; 4, 5, 6 -MSTD;  
 1, 4 -  $q = 1$ ; 2, 5 -  $q = 10$ ; 3, 6 -  $q = 100$

With the variation of  $\alpha_\Delta$  in the frame of the interval  $0,001 \div 0,1$  Hz and  $\sigma_f/f$  in the frame of the interval  $10^{-6} \div 10^{-10}$  the positioning accuracy of the MO doesn't vary practically for both methods. Therefore, it is advisable to use SG with non-high frequency stability within the limits of  $10^{-6} \div 10^{-8}$ .

Fig. 9 represents the accuracy characteristics of the parameter  $\Delta$  for different values of the parameter  $\alpha_\Delta$  for the C/A-code. With decreasing of  $\alpha_\Delta$  from  $0,1$  Hz to  $0,01$  Hz and  $0,001$  Hz: the accuracy of determining of  $\Delta$  increases respectively 1,2 and 1,5 times for FWMS but for MSTD the variation of  $\alpha_\Delta$  in the frame of the interval  $0,1 \div 0,001$  Hz doesn't influence the accuracy characteristics of the system. Fig. 10 shows the accuracy characteristics of the parameter  $\Delta$  for different values of the frequency instability of the SG ( $\sigma_f/f$ ) for C/A-code. With decreasing of  $\sigma_f/f$  from  $10^{-6}$  to  $10^{-8}$  and  $10^{-10}$  the accuracy of determining

of  $\Delta$  increases respectively 1,9 ÷ 2,4 and 5,4 ÷ 7,5 times for FWMS but for MSTD the variation of  $\sigma_f/f$  in the frame of the interval  $10^{-10} \div 10^{-6}$  does not influence on the accuracy characteristics of the system.

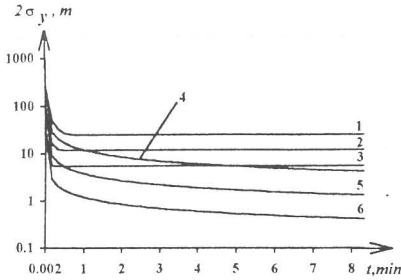


Fig. 5. Vertical error ( $2\sigma_y$ ) for C/A-code:

1, 2, 3 - FWMS; 4, 5, 6 - MSTD;  
1, 4 -  $q=1$ ; 2, 5 -  $q=10$ ; 3, 6 -  $q=100$

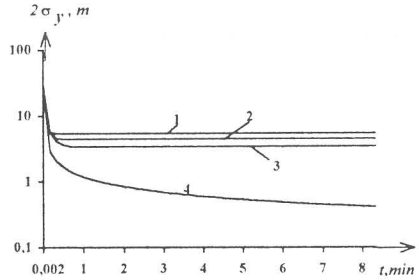


Fig. 6. Vertical error ( $2\sigma_y$ ) for C/A-code:

1, 2, 3 - FWMS; 4 - MSTD  $\alpha = 0,1 \div 0,001$  Hz;  
1 -  $\alpha = 0,1$  Hz; 2 -  $\alpha = 0,01$  Hz; 3 -  $\alpha = 0,001$  Hz

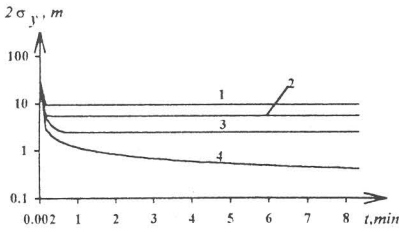


Fig. 7. Vertical error ( $2\sigma_y$ ) for C/A-code:

1, 2, 3 - FWMS; 4 - MSTD  $\sigma_f = 0,1 \div 10$  m/s;  
1 -  $\sigma_f = 10$  m/s; 2 -  $\sigma_f = 1$  m/s; 3 -  $\sigma_f = 0,1$  m/s

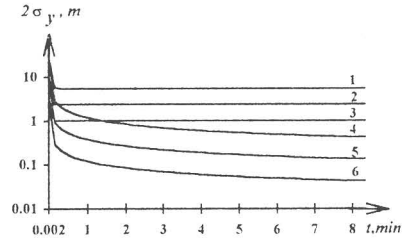


Fig. 8. Vertical error ( $2\sigma_y$ ) for P-code:

1, 2, 3 - FWMS; 4, 5, 6 - MSTD;  
1, 4 -  $q=1$ ; 2, 5 -  $q=10$ ; 3, 6 -  $q=100$

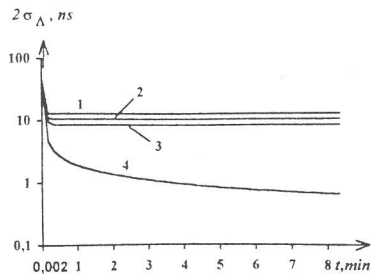


Fig. 9. Estimating error of  $\Delta$  ( $2\sigma_\Delta$ ) for C/A-code:

1, 2, 3 - FWMS; 4 - MSTD  $\alpha_s = (10^3 \div 10^1)$  Hz;  
1 -  $\alpha_s = 10^1$  Hz; 2 -  $\alpha_s = 10^2$  Hz; 3 -  $\alpha_s = 10^3$  Hz

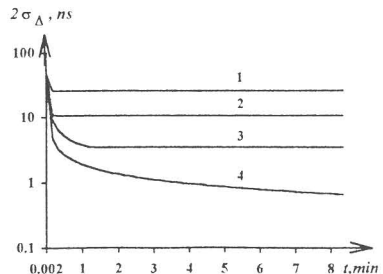


Fig. 10. Estimating error of  $\Delta$  ( $2\sigma_\Delta$ ) for C/A-code:

1, 2, 3 - FWMS; 4 - MSTD  $\sigma_f/f = 10^{10} \div 10^6$ ;  
1 -  $\sigma_f/f = 10^6$ ; 2 -  $\sigma_f/f = 10^8$ ; 3 -  $\sigma_f/f = 10^{10}$

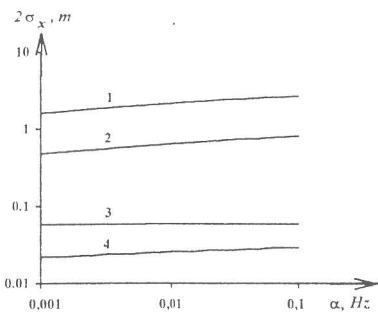


Fig. 11. Horizontal error ( $2\sigma_x$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

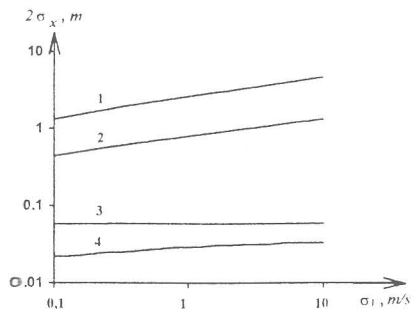


Fig. 12. Horizontal error ( $2\sigma_x$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

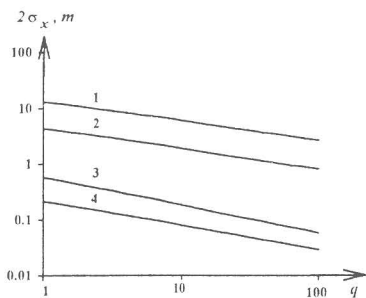


Fig. 13. Horizontal error ( $2\sigma_x$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

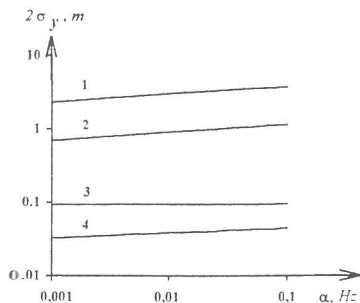


Fig. 14. Vertical error ( $2\sigma_y$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

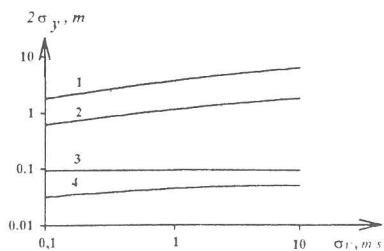


Fig. 15. Vertical error ( $2\sigma_y$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

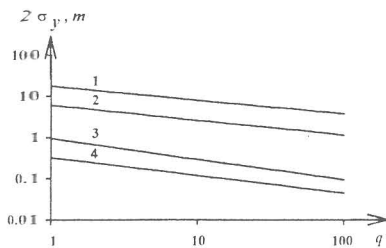


Fig. 16. Vertical error ( $2\sigma_y$ ):  
 1, 2 - FWMS; 3, 4 - MSTD;  
 1, 3 - for C/A-code; 2, 4 - for P-code

With the increasing of the signal-to-noise ratio from 1 to 10 and 100 the accuracy of determining of  $\Delta$  rises respectively  $2,3 \div 2,4$  and  $5,4 \div 5,7$  times for fil-

tration without time delays separation and for MSTD 3,1 ÷ 3,2 and 9,9 ÷ 10 times.

With the variation of  $\alpha$  in the frame of the interval  $0,001 \div 0,1$  Hz and  $\sigma_f$  in the frame of the interval  $0,1 \div 10$  m/s the accuracy of determining of  $\Delta$  does not practically vary. The positioning accuracy is not influenced by the parameter  $\alpha_\Delta$  and weakly depends on the instability  $\sigma_f/f$  for FWMS. The accuracy of determining of the parameter  $\Delta$  almost is not influenced by the value of the parameter  $\alpha$  at FWMS. At MSTD the positioning accuracy too weakly depends on the values of the parameters  $\alpha$ ,  $\alpha_\Delta$ ,  $\sigma_f$  and  $\sigma_f/f$ .

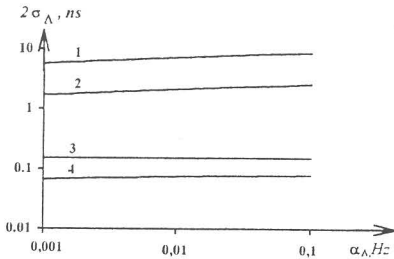


Fig. 17. Estimating error of  $\Delta$  ( $2\sigma_\Delta$ ):

1, 2 - FWMS; 3, 4 - MSTD;  
1, 3 - for C/A-code; 2, 4 - for P-code

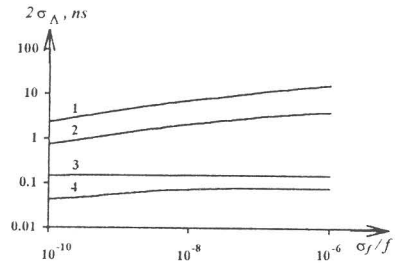


Fig. 18. Estimating error of  $\Delta$  ( $2\sigma_\Delta$ ):

1, 2 - FWMS; 3, 4 - MSTD;  
1, 3 - for C/A-code; 2, 4 - for P-code

## Conclusions

Quantitative results of the positioning accuracy when using multiplex synchronization of the signal time delays of envelope and carrier frequency are achieved.

Comparison with the positioning accuracy of the algorithm without taking into account of the time delay of the carrier frequency is done. Even for most unfavorable circumstances of working, the algorithm with multiplex synchronization provides high positioning accuracy.

The results analysis shows that even for using C/A-code in differential mode of working of NAVSTAR system and at availability of SG with non-high frequency stability ( $10^{-6} \div 10^{-8}$ ) the processing algorithm with multiplex synchronization provides high accuracy characteristics.

The positioning accuracy is increased with one order only within the first 5 ÷ 10 minutes filter working with multiplex synchronization in comparison to the conventional algorithm without multiplex synchronization of the time delays.

## References

1. Rashkov V. L. Optimum processing of satellite navigation signals with time delays separation. - The Seventh National Scientific and Applied Science Conference "Electronics - ET'98" - September 23-25 1998, Sozopol, Bulgaria, 1998.