Methods for improving the selectivity of spectral analysis and their implementation with LabView

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ABSTRACT: The paper proposes to analyze the influence of window functions on the evaluated spectrum and proposes some improvement methods by using the PSpice and LabView programs. The theoretical analysis is checked by simulation and tested in a real application.

Introduction

In numerous practical applications, appears the necessity of determining as exactly as possible the frequency spectrum of the analyzed signal. Generally, for the calculation of the spectrum, the FFT algorithm is used.
The calculation is made on the basis of a finite number of samples (N), taken from the analyzed signal during a finite time of its observation (T_F).
This way of calculation is accompanied by the appearance of some errors.
In this article we will focus on the influence of window functions.

The Fourier Transformation of the time limited signals

For a given function f(t), the Discrete Fourier Transformation (DFT) is given by the following expression [1]:

$$F(m\Delta\omega) = \Delta t \cdot \sum_{n=0}^{N-1} f(n\Delta t) \cdot \exp(-jm\Delta\omega \cdot n\Delta t)$$
$$f(n\Delta t) = \frac{1}{T_F} \cdot \sum_{n=0}^{N-1} F(m\Delta\omega) \cdot \exp(jm\Delta\omega \cdot n\Delta t) \qquad (1)$$

It is very well known that in Fourier analyses, the discretisation in one domain involves periodisation in another domain. In the case of sampled signals, observed in a finite period of time T_F ,we can determine its spectral components only in a finite number of points f_K. This discrete spectrum suggests the existence of a periodisation in the time domain.

We can say, that, because of the DFT algorithms, the analyzed signal will be considered periodic with a T_F period, even though it is not really so.

This imposed periodisation of the signal generally leads to the appearance of discontinuity at the ends of the observed interval.

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In order to stress the influence of the finite value of the observation time T_F, we shall replace the samples of the original function s(t) with the samples of a modified function $s_m(t) = s(t) * w(t)$ where we noted with w(t) the window function.

This product in the time domain means a convolution in the frequency domain and therefore the Fourier Transformation of this relation will be a convolution in the frequency domain:

$$S_m(\omega) = S(\omega) \cdot W(\omega) = \int_{-\infty}^{\infty} S(\omega) \cdot W(\omega - \omega')d\omega' \qquad (2)$$

In order to better understand the significance of the above statements, we shall consider the case of a sinusoidal signal and a rectangular window illustrated in fig. 1.

In figure 1, two cases have been presented, namely: $s_{m1}(t)$ signal, characterized by the fact that the observation period is a multiple of the period of sinusoidal signal ($T_F = 4*T_1$), and the signal $s_{m2}(t)$, characterized by the fact that the observation period does not comprise a whole number of periods from the sinusoidal signal ($T_F = 4.5*T_2$).

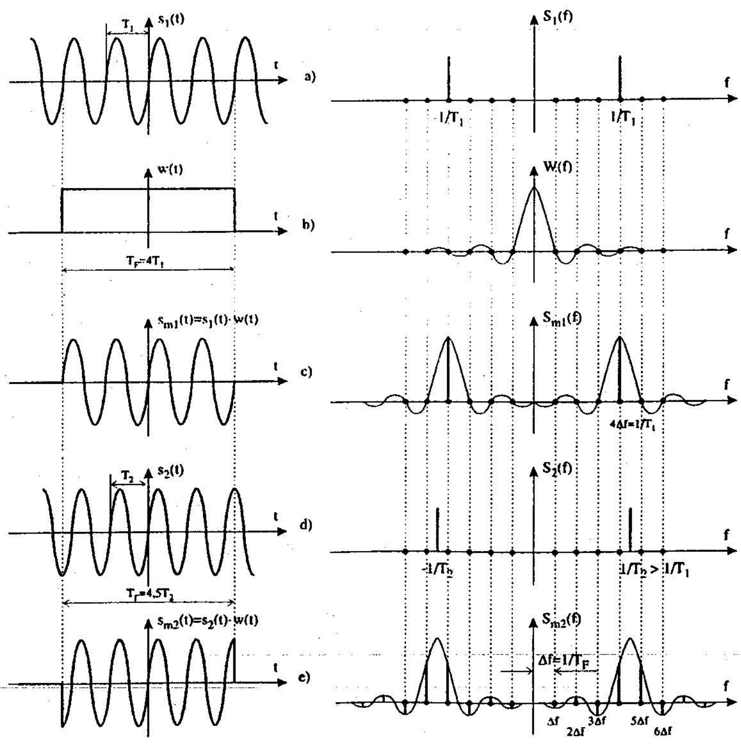

***Fig. 1** The influence of a rectangular window function on the spectrum*

The Fourier analysis shows that in the first case the $S_{m1}(\omega)$ spectrum coincides with $S(\omega)$, while in the second case the $S_{m2}(\omega)$ spectrum does not coincide, but, on the contrary, we notice the appearance of a whole series of spectral components.

In order to eliminate the above mentioned effect, special window functions are used.

From the mathematical point of view the window functions are built so that their presence may eliminate the finite jumps of the time limited signals at the end of the observation time.

In consequence, they will ponderate the signal which will be analyzed on the observation interval T_F with coefficients attending towards zero at the end of this interval; for example, the Hanning window in time domain is given by the following equation:

$$w_{FH}(t) = \begin{cases} 0{,}5 + 0{,}5\cos\left(\dfrac{2\pi t}{T_F}\right) & pentru\ |t| < \dfrac{T_F}{2} \\ 0 & \text{\`n rest} \end{cases}$$

$$W_{FH}(\omega) = \left\{0{,}5W_0(\omega) + 0{,}25\left[W_0\left(\omega + \frac{2\pi}{T_F}\right) + W_0\left(\omega - \frac{2\pi}{T_F}\right)\right]\right\} \tag{3}$$

where we noted $W_0(\omega) = T_F \cdot \left|\dfrac{\sin(\pi f T_F)}{\pi f T_F}\right|$.

Using PSpice, three types of window functions were simulated in the time domain and then their Fourier Transformations were calculated. The results are

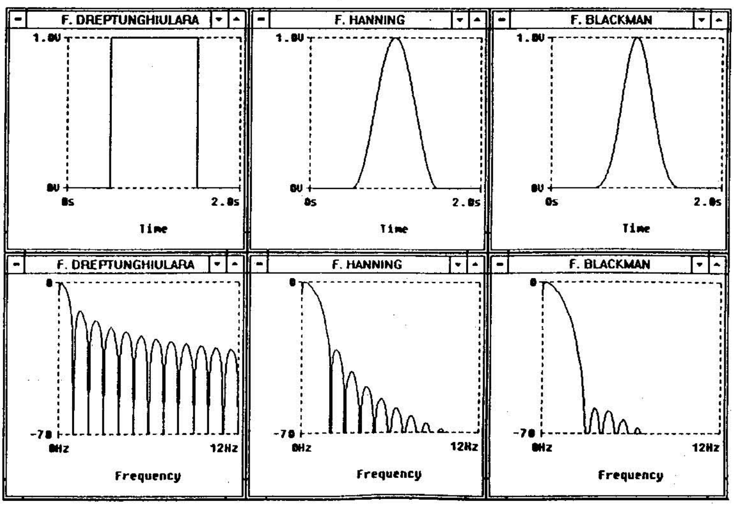

Fig.2 Different type of window functions and their spectrum

presented in figure 2.

If we compare the three types of windows, we find the following:

- the rectangular window has a good selectivity and a very low attenuation of the lateral spectral components;
- the Blackman window has a very strong attenuation for lateral spectral components but also the highest frequency band at 3 dB
- the Hanning window has intermediate performances.

The selectivity properties of the window functions

In order to illustrate the influence of the window function and to stress clear the spectral components of a given signal, a simulation with PSpice was done.

The analyzed signal s(t) is obtained by adding the fundamental component to the second and third order harmonics. In order to increase the difficulty of identifying the second order harmonic, it's amplitude was chosen as a 100 times smaller than the fundamental amplitude.

The results of the analysis are presented in figure 3.

In a practical application there are no general rules for choosing the right window function for a given signal. The decision depends essentially on the signal properties and the expected results; for instance, if we are interested in detecting the spectral components in a stationary signal, it is recommended to use a window function with a very strong attenuation of the lateral spectral components (see figure 2 and 3).

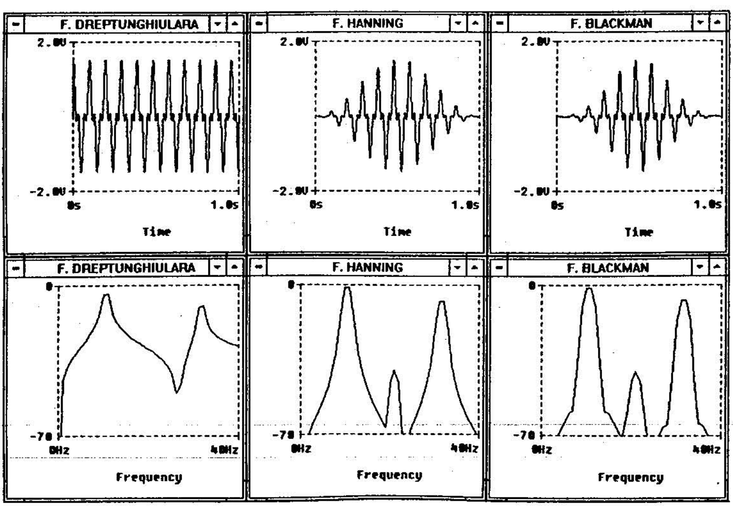

Fig. 3 Detecting capabilities of the different type of window functions

Improving the selectivity by an averaging technique in the frequency domain

This technique is commonly used in the processing of stationary or quasistationary signal in the presence of wide noise [1] and the obtained performances are proportional with the number of averaging.

The effect of the proposed averaging became obvious analyzing the parameters presented in Table 1.

Table 1

Window function	Bandwith at 3dB	Maximum error in dB	The highest lateral lobe attenuation	Attenuation rate of lateral lobes in dB/oct
Rectangular	$0.89 \cdot \Delta f$	3.92	-13	6
Hanning	$1.44 \cdot \Delta f$	1.42	-32	18
Averaged	$1.07 \cdot \Delta f$	2.67	-25	3.7

As it can be observed, the averaging leads to an intermediate performance, which means that we could combine the resolution properties of the rectangular window with the good selectivity properties of the Hanning window function.

In case of the presence of a gaussian noise in the analyzed signal, with this averaging technique in the spectral domain we can achieve a reduction of noise effect. It can be proved that the standard deviation in case of establishing of the spectral components is $\sigma = 1/\sqrt{M}$ where M is the number of the averages made.

For improving the selectivity, in order to be able to detect spectral components in a stationary periodical signal, a modified averaging techniques was used, as it is presented in the block diagram from figure 4.

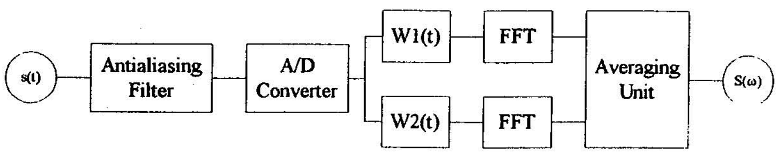

***Fig. 4** The block diagram of the data acquisition and processing system*

The proposed technique consists in using a different type of window function in time domain, the evaluation of the respective Fourier transformation and the calculation of its average value.

The propose method was used for detecting the spectral components of the current delivered by a static converter supplying a asynchronous motor. We chose the following window functions: $w_1(t)$ a rectangular window function, having a good selectivity but very bad attenuation properties, and $w_2(t)$ a Hanning window with a grate attenuation properties. Because of this, is very useful for detecting spectral components.

The acquired data was processed by a LabView program [2] and the obtained results are illustrated in figure 5.

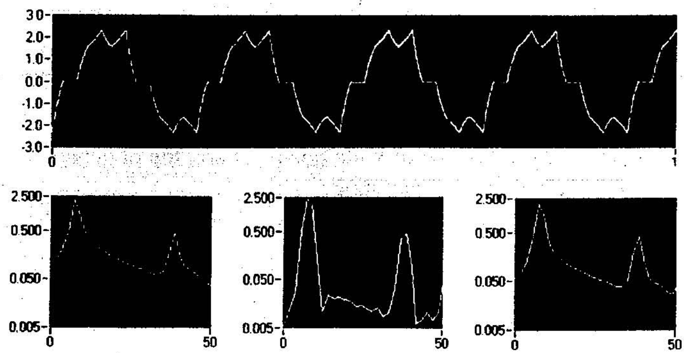

Fig. 5 The results given by the averaging technique

Conclusion

In many practical application, it is very important to evaluate as exactly as possible the frequency spectrum of the analyzed signal. An adequate window function can lead to the improvement of the selectivity for a certain class of function. The proposed average technique in the frequency domain can lead to an additional improvement of the performances.

References

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