A SIMPLE UNIJUNCTION TRANSISTOR MODEL

Prof. Tomislav Dzhekov, Ph. D
Electrotechnical faculty - Skopje

I. Introduction

The exact analysis of any electronic device must be in compliance with the five basic semiconductor equations which, for steady-state conditions and one-dimensional geometry, read:

$$J_p = e\mu_p pF - eD_p \frac{dp}{dx} \tag{1}$$

$$J_n = e\mu_n nF + eD_n \frac{dn}{dx} \tag{2}$$

$$0 = g - R - \frac{1}{e}\frac{dJ_p}{dx} \tag{3}$$

$$0 = g - R + \frac{1}{e}\frac{dJ_n}{dx} \tag{4}$$

$$\frac{dF}{dx} = \frac{e}{\varepsilon\varepsilon_o}\left(p - p_o - n + n_o\right) \tag{5}$$

where g is the generation rate, R is the recombination rate and p_o and n_o are the equilibrium hole and electron concentrations. The basic characteristics of the unijunction transistor (UJT) can be obtained if these equations are applied to its EB_1 region, Fig.1.

To simplify matters, one usually assumes that in this region there is approximate space-charge neutrality, that is $dn/dx=dp/dx$ and $n\text{-}n_o= p\text{-}p_o$. This, when applied to (1) and (2), and J_p and J_n are replaced by I_p/A and $(I_{B1} - I_p)/A \equiv I_n/A$, respectively, yields [1]:

$$\frac{dp}{dx} = \frac{pI_{B1} - I_p\left[p(b+1) + b(n_o - p_o)\right]}{ebD_pA(2p + n_o - p_o)} \tag{6}$$

and

$$F = \frac{I_{B1} - eD_pA(b-1)dp/dx}{e\mu_pA\left[p(b+1) + b(n_o - p_o)\right]} \tag{7}$$

where $b=\mu_n/\mu_p$. These two relations, together with equation (3), which could be rewritten in

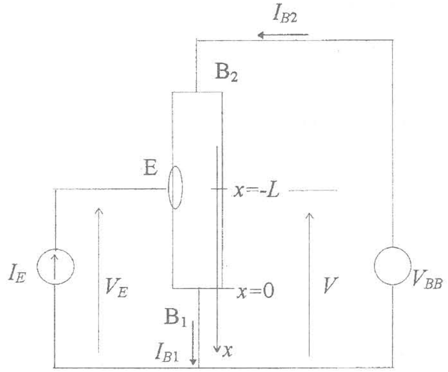

Fig. 1 Schematic diagram of a unijunction transistor

terms of I_p as:

$$\frac{dI_p}{dx} = e(g - R)A, \tag{8}$$

and with the well known expression for the voltage across the *pn* junction:

$$\phi_E = U_T \ln \frac{p(-L)}{p_o} \tag{9}$$

form a relatively precise, but very complicated, model for calculation of the steady-state characteristics of the UJT.

For instance, the procedure of calculating one point of the V_E-I_E characteristic, i.e., of finding V_E for given I_E, is the following. We note - since g - R is a known function of p - that relations (6) and (8) compose a system of two simultaneous differential equations of $p(x)$ and $I_p(x)$. They can be solved numerically for the corresponding initial conditions and an assumed value of I_{B1}. The hole current is negligible in the EB$_2$ region; thus, we have $I_p(-L) = \gamma I_E$, where γ is the emitter efficiency. Also, we can write $I_p(0) = esA[p(0) - p_o]$, where s is the recombination velocity of the B$_1$ contact. All we know about I_{B1} is that it must satisfy the equation:

$$I_{B1} = I_E + I_{B2} = I_E + \frac{V_{BB} - V}{R_{20}} \tag{10}$$

(R_{20} is the resistance of the EB$_2$ region!), but its exact value cannot be known until we find the potential V. Once the required solution $p(x)$ is obtained, we can use (7) to calculate an estimate of V, by integrating F from $-L$ to 0. This V will be correct if and only if it satisfies (10) for the assumed value of I_{B1}. Normally, we will have to repeat the calculation several times, each time assuming a new, more correct, value for I_{B1} until we obtain a satisfactorily precise value for V. Having the correct V and, therefore, the correct $p(x)$, we can finally calculate the emitter voltage:

$$V_E = V + \phi_E \tag{11}$$

Obviously, because of the complexity of the equations (6), (8) and (7), no exact analytical description of the steady-state characteristics of the UJT is possible. This is also true, of course, for its transient behavior. For many applications, however, even an approximate analytical description of the UJT could be useful. The development of such a model is the main concern of this paper.

II. STEADY- STATE MODEL

To simplify expression (6), we assume that no recombination is present within the EB_1 region. This implies:

$$I_p = \gamma I_E = \text{const.} \tag{12}$$

Next, we will replace pI_{B1} by $p\gamma I_E$, ignoring the influence of the term $p\left[(1-\gamma)I_E + I_{B2}\right]$ on the distribution $p(x)$, what is obviously true for larger values of I_E. This reduces expression (6) to:

$$\frac{dp}{dx} = \frac{-\gamma I_E}{eD_p A}\frac{p + n_o - p_o}{2p + n_o - p_o} \tag{13}$$

Since in most cases the recombination velocity at the contact B_1 is very high, we will further assume that $s \to \infty$. This implies $p(0) = p_o$.

The differential equation (13) is nonlinear and cannot be solved analytically in closed form. We note, however, that dp/dx is a monotone function of x, which, when the condition $p(0) = p_o$ is applied, becomes approximately equal to $-\gamma I_E / eD_p A$ at $x = 0$ and approaches $-\gamma I_E / 2eD_p A$ at $x \to -\infty$. Therefore, the distribution:

$$p = p_o - \frac{\gamma I_E}{eD_p A}x \tag{14}$$

will be a very good approximation of the exact solution of equation (13).

Combining (7) with (10) and (14), one obtains:

$$V = \int_{-L}^{0} F dx = \left[I_E(\gamma b + 1 - \gamma) + \frac{V_{BB} - V}{R_{20}}\right]R_1$$

or

$$V = \frac{V_{BB} R_1}{R_1 + R_{20}} + \frac{R_1 R_{20}}{R_1 + R_{20}} (\gamma b + 1 - \gamma) I_E \tag{15}$$

where

$$R_1 = \frac{U_T}{(b+1)\gamma I_E} \ln\left[1 + \frac{(b+1)\gamma I_E}{U_T / R_{10}}\right] \tag{15'}$$

In the last expression, $U_T = D_p / \mu_p \equiv kT / e$ and $R_{10} = L / eA(n_o \mu_n + p_o \mu_p)$ (R_{10} is the resistance of the EB_1 region !). Note that R_1 reduces to R_{10} for $I_E = 0$.

From (9) and (14) it follows:

$$\phi_E = U_T \ln\left[1 + \frac{\left(\frac{n_o}{p_o} b + 1\right)\gamma I_E}{\frac{U_T}{R_{10}}}\right] \tag{16}$$

Expressions (15) and (16), together with (10) and (11), form the required analytical steady-state model of the UJT.

A presentation of the V_E-I_E characteristics of our model is given in Fig. 2. We have

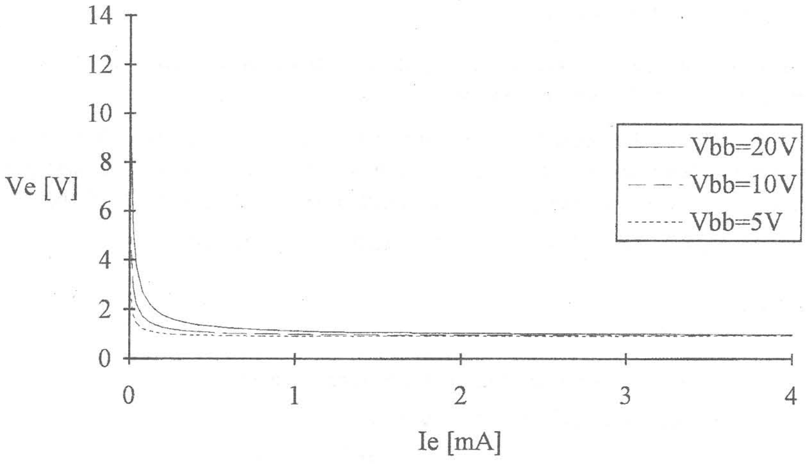

Fig. 2 V_E-I_E characteristics of the developed model. $R_{BB}=R_{10}+R_{20}=8$ kΩ, $\eta=R_{10}/R_{BB}=0.6$, $N_D= 10^{15}$ cm^{-3}, $\gamma=0.99$ and $b=2.8$. We assume silicon semiconductor and room temperature ($T=298$ K).

compared these characteristics with the measured characteristics of a real UJT of the same values of R_{BB} and η [2] and a very good conformity was found.

At the peak and the valley points, the slope of the V_E-I_E characteristics $dV_E/dI_E \equiv dV/dI_E + d\phi_E/dI_E = 0$. So, the currents I_{Ep} and I_{Ev}, corresponding to these points, are solutions of the equation:

$$\frac{R_{20}V_{BB}\dfrac{dR_1}{dI_E}}{(R_1+R_{20})^2}+(\gamma b+1-\gamma)R_{20}\frac{R_1(R_1+R_{20})+R_{20}\dfrac{dR_1}{dI_E}I_E}{(R_1+R_{20})^2}+\frac{U_T}{\dfrac{U_T/R_{10}}{\left(\dfrac{n_o}{p_o}b+1\right)}+I_E}=0 \tag{17}$$

where

$$\frac{dR_1}{dI_E}=-\frac{U_T}{(b+1)\gamma I_E^2}\ln\left(1+\frac{(b+1)\gamma I_E}{U_T/R_{10}}\right)+\frac{U_T/I_E}{U_T/R_{10}+(b+1)\gamma I_E} \tag{17'}$$

To find a simple expression for I_{Ep} we will assume:

$$\frac{U_T/R_{10}}{\left(\dfrac{n_o}{p_o}b+1\right)\gamma} << I_{Ep} << \frac{U_T/R_{10}}{(b+1)\gamma} \tag{18}$$

This allows to neglect $U_T\Big/R_{10}\left(\dfrac{n_o}{p_o}b+1\right)\gamma$ with respect to I_E and to use the approximations:

$$R_1 \cong R_1\big|_{I_E=0}=R_{10};\qquad dR_1/dI_{E1}\cong dR_1/dI_{E1}\big|_{I_E=0}=-\frac{R_{10}^2(b+1)\gamma}{U_T},$$

what reduces (17) to a simple linear equation. The expression obtained is:

$$I_{Ep}\cong\frac{U_T\left(\dfrac{1}{R_{10}}+\dfrac{1}{R_{20}}\right)}{\dfrac{V_{BB}}{U_T}\eta(b+1)\gamma-(b\gamma+1-\gamma)} \tag{19}$$

Obviously, this result is correct since it satisfies (18) for all cases of practical interest.

At the valley point, the emitter current is relatively large, so we can assume:

$$I_{Ev} >> \frac{U_T/R_{10}}{(b+1)\gamma} \tag{20}$$

For all practicable values of η, this implies $R_1 << R_{20}$. So, by neglecting R_1 with respect to R_{20} and having in mind (20), we can reduce (17) to the following simple transcendental equation:

$$I_E = \frac{V_{BB}}{R_{20}(2b\gamma+1)}\left[\ln\frac{(b+1)\gamma I_E}{U_T/R_{10}} - 1\right] \tag{21}$$

To obtain an explicit expression for I_{Ev}, we will replace I_E on the right side of (21) by a constant of the same order of magnitude as I_{Ev}. Such a constant could be the value of I_E at which both terms of V (equation (15)) become identical. Since this occurs at $I_E = V_{BB}/(b\gamma+1-\gamma)R_{20}$, we have:

$$I_{Ev} \cong \frac{V_{BB}}{R_{20}(2b\gamma+1)}\left[\ln\frac{V_{BB}}{U_T}\frac{\eta}{1-\eta}\frac{(b+1)\gamma}{b\gamma+1-\gamma} - 1\right] \tag{22}$$

III. TRANSIENT MODEL

Expression (14) indicates a linear relationship between the emitter current I_E and the excess hole charge in the EB_1 region, Q_p. Namely:

$$\gamma I_E = e\frac{p(-L)-p_o}{L}D_pA = \frac{2D_pQ_p}{L^2} \tag{23}$$

So relations (15) and (16) can be easily expressed in terms of Q_p as follows:

$$V = \frac{V_{BB}R_1}{R_1+R_{20}} + \frac{R_1R_{20}}{R_1+R_{20}}\frac{2D_pQ_p}{L^2}\left(b+\frac{1}{\gamma}-1\right) \tag{24}$$

$$R_1 = \frac{U_T}{(b+1)\frac{2D_pQ_p}{L^2}}\ln\left[1+\frac{(b+1)\frac{2D_pQ_p}{L^2}}{\frac{U_T}{R_{10}}}\right] \tag{24'}$$

$$\phi_E = U_T \ln\left[1 + \frac{\left(\frac{n_o}{p_o} b + 1\right)\frac{2D_p Q_p}{L^2}}{\frac{U_T}{R_{10}}}\right] \tag{25}$$

In steady-state conditions the hole current is independent of x, thus one can write: $i_p(-L) - i_p(0) = 0$. In dynamic conditions, however, $i_p(-L)$ differs from $i_p(0)$ by the rate the excess hole charge cumulates in the EB_1 region (remember that we ignore recombination!).

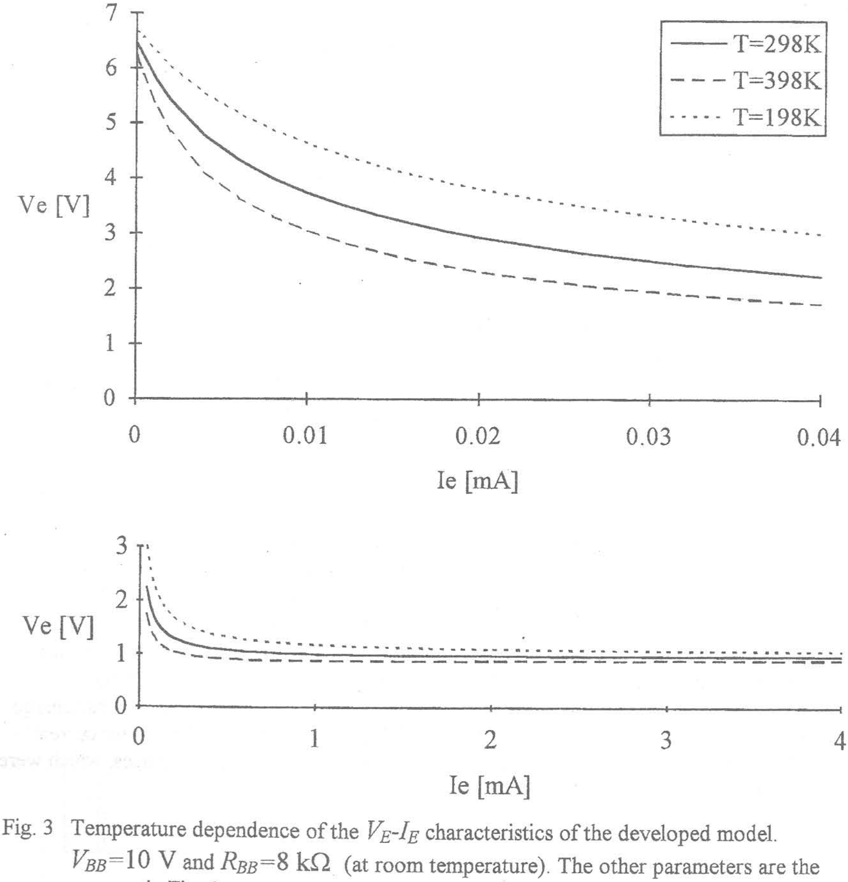

Fig. 3 Temperature dependence of the V_E-I_E characteristics of the developed model. V_{BB}=10 V and R_{BB}=8 kΩ (at room temperature). The other parameters are the same as in Fig. 2.

Thus:

$$i_p(-L) - i_p(0) = \frac{dQ_p}{dt} \tag{26}$$

Replacing $i_p(-L)$ by $\gamma\, i_E$ and $i_p(0)$ by $2D_pQ_p/L^2$ (in analogy with the charge control approach in the bipolar transistors), we obtain the following simple differential equation:

$$\frac{dQ_p}{dt} + \frac{Q_p}{L^2/2D_p} = \gamma\, i_E \tag{27}$$

Equation (27) and expressions (24) and (25), together with (11) and (10), complete the required simple transient model of the UJT. Here we have implicitly assumed that expressions (24) and (25) hold not only in steady-state but also in dynamic conditions, which is equivalent to assuming that the distribution p(x) depends only on Q_p, and not on the previous history. Obviously, the higher the rate of change of the emitter current the less true will be this assumption.

IV. Temperature dependence

The temperature dependence is easily incorporated by using the well known expressions: $n_i^2 = CT^3 e^{-E_G/kT}$ and $\mu = \mu_o(T_o/T)^{5/2}$ (or: $R = R_o/(T/T_o)^{5/2}$). As can be seen from Fig. 3, the voltage temperature coefficient $\alpha_v = \partial V_E/\partial T$ of our model is negative and becomes smaller at larger currents. This is in good agreement with the experiments referred in [2]. There is a certain disagreement, though, in the saturation region, where the experimental α_v assumes small positive values for larger currents.

V. Conclusion

The developed model of the UJT is general and complete. Although simple, it describes satisfactorily well the steady-state, transient and temperature characteristics of this device. This makes it suitable for educational purposes and gives it a practicable value. It is far more productive than the simple model given in [2], which simulates, very purely, only the negative resistance region of the V_E-I_E characteristic. It should be clear, however, that the applicability of the model will be limited to only approximate calculations. This refers especially to the transient regime - not only because of the assumption of nearly linear charge distribution (which cannot be quite true for very high rates of change of the emitter current) - but also because of the effects of the barrier and the existing parasite capacitances, which were not taken into account.

References

[1] van der Ziel, *Solid State Physical Electronics*, Prentice-Hall, 1968

[2] S. M. Sze, *Physics of Electronic Devices*, Wiley -Interscience, 1969